Investigation of the Ring Spur Assignment Problem

Paula Carroll, Seán McGarraghy

Management Information Systems, Quinn Business School, UCD, Dublin, Ireland.

Abstract: This paper describes the Ring Spur Assignment Problem (RSAP), an interesting new problem arising in the design of Next Generation Networks. We describe the problem, position it in relation to problems previously addressed in the literature and give an IP model suitable for solving small problem instances. We outline a cutting plane algorithm for larger problem instances and present some computational results.

Keywords: Telecommunications Network Topology Design, Integer Programming formulation, Cutting Plane Algorithm

1 Introduction and Telecommunications Background

We address an interesting new problem not, to our knowledge, previously discussed in the literature. An operator seeks to identify a logical Next Generation Network (NGN) topology that can be overlaid on existing physical infrastructure.

The higher the transmission speed, the greater the loss if an individual cable or piece of equipment develops a fault. So it has become mandatory for backbone networks to be designed with survivability in mind. Survivability issues are reviewed in [7] and [8].

Wavelength Division Multiplexing (WDM) is used on Fibre Optic networks to achieve even higher bandwidths. Synchronous Digital Hierarchy (SDH) is a transmission standard which allows for ease of access to individual channels. It promotes the use of Self Healing Rings (SHR) to increase reliability. [14] gives an introduction to optical networking issues and indicates that Internet Protocol (IP) over WDM may be the preferred option for NGNs. This protocol can be implemented over the physical SDH layer.

2 Telecommunications Topological design Problems

We summarise some related literature on topology design problems focusing on ring-based topologies. The Two Connected Bounded Ring problem (2CNBR) and its variants that arise in the design of SDH/SONET and WDM networks are described in [2], [3] and [4]. The SDH Ring Loading problem (SRLP), which can be tackled after the high level NP-Hard SDH Ring Assignment Problem (SRAP) has been solved, is described in [5]. Another formulation of SRAP as a set-partitioning model with additional knapsack constraints is given in [11]. [1] focuses on the impact of node costs on network design. They describe the Discrete Node Cost Network Design Problem (DNNDP). These are just some of the wide variety of approaches and topologies used in survivable network design.

We now describe our design problem. Communities of interest are identified and their traffic demands are estimated. If such communities can be clustered on node disjoint rings, no wavelength conversion is required eliminating the cost of wavelength conversion and /or opto-electronic conversion equipment for intra-ring demand. We call these rings local rings. Local rings can then be connected by a special ring, which we call the tertiary ring. The tertiary ring facilitates inter-ring demand; wavelength converters are required where local rings connect to the tertiary ring. Practitioners require that local rings be disjoint as shared nodes or edges increase the reliance on particular pieces of infrastructure. We seek a logical topology using spare capacity in the physical SDH network. The objective is to achieve a highly resilient topology at minimum cost.

So far the problem described is similar to the SRAP problem. However, in some real world instances, no SRAP solution is possible. A node with (exactly) *n* incident edges is said to be (exactly) *n*-connected. All ring nodes must be exactly two-connected to two other ring nodes. Let $e_{i,j}$ represent the edge joining node *i* to node *j*.

Figure 1 on the left shows bhv1c.nd, a benchmark problem from [1]. We can see that nodes 5,6 and 14 are two-connected and share node 1 as an adjacent node. A SRAP solution could start by connecting edges $e_{1,5}$ and $e_{1,6}$ to attempt to form a ring. Since node 14 is exactly two-connected and adjacent to node 1, edge $e_{1,14}$ must also be included in the ring but that violates the requirement that ring nodes be exactly two-connected since node 1 is now three-connected, hence no SRAP solution exists.

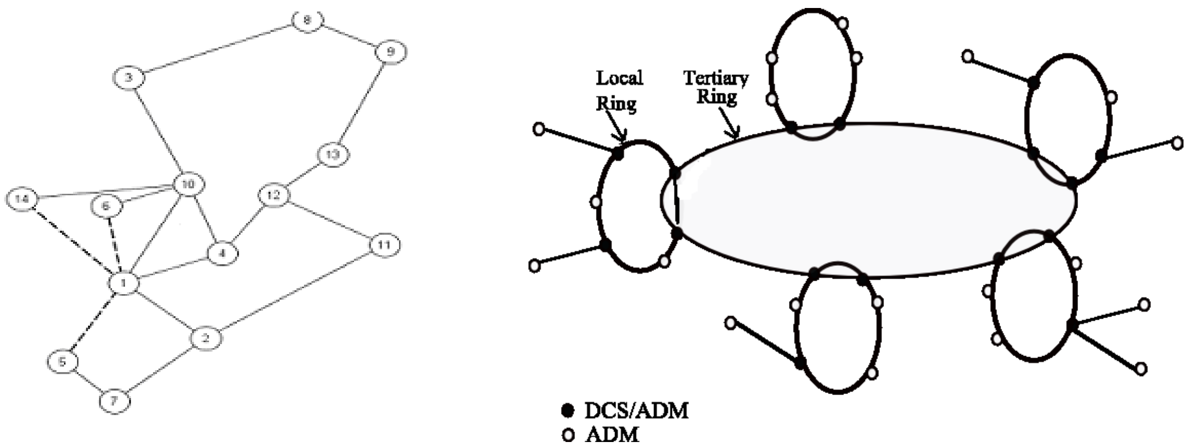

Figure 1 Left; Benchmark problem with no SRAP solution. Right: RSAP topology

As an alternative, where no SRAP solution exists, we allow locations that have insufficient spare capacity or no possible physical route due to limitations of geography, to be connected to SHRs by spurs off the rings. Ring capacities must be sufficient to carry this additional demand. We call this problem the Ring Spur Assignment Problem, RSAP. Since SRAP is a special case of RSAP, it follows that RSAP is also NP-Hard.

We also mention the Ring Star problem [9]. A Ring Star is used to connect terminals to concentrators where not all nodes are required to be 2-connected. A solution to our problem is a set of ring stars interconnected by a Tertiary ring as shown on the right of Fig 1.

3 IP Formulation

We describe an exact IP formulation that can be used on small problem instances to identify the optimal local ring (spur) partitions. An instance of a problem is specified by:

- an undirected graph $G = (V, E)$ defined on a set V of nodes, labeled from 1 to n where $n = |V|$, and a set of undirected edges E; the underlying set A of oriented arcs contains, for each edge $\{i, j\} \in E$, two arcs (i, j) and (j, i), one in each directions;
- a set Q of traffic demands determining, for each $q \in Q$, a source-sink node pair (s_q, t_q) and a demand volume $d_q \in \Re$, the real numbers;
- a *non-negative routing cost* giving the cost of a link dependant on its length and capacity.

Let $c_{ij} \geq 0$ be the cost coefficient of edge $\{i, j\} \in E$ for ring edges and $b_{ij} \geq 0$ that for spur arcs.

Let x_{ijr} be a binary variable equal to 1 if and only if edge $\{i, j\}$ appears on ring r, and equal to 0 otherwise; i.e. both i and j are assigned to the same ring $r \in R$, the set of all possible rings. For each arc $(i, j) \in A$, let y_{ij} be a binary variable equal to 1 if vertex i is assigned to vertex j as a spur; we set $y_{ii} = 1$ for any vertex i that is on a ring. The set of nodes adjacent to node i is denoted by $adj(i)$ and the cut of $S \subset V$ is denoted by $\delta(S) := \{\{i, j\} \in E : i \in S, j \notin S\}$, i.e. the set of edges having only one endpoint in S.

We wish to foster high resilience by having locations assigned to rings where possible. By assigning a sufficiently high weight, b, to links that are spurs, we achieve this objective. We have quantified the b value in terms of other network parameters. For simplicity we set, b_{ij}, the linear coefficient of arc $(i, j) \in A$, to be bc_{ij}. We wish to find a minimal cost solution. We first address the topology considerations and specify the formulation as follows;

$$\min \sum_{\{i,j\}\in E} \sum_{r\in R} c_{ij} x_{ijr} + \sum_{(i,j)\in A} b_{ij} y_{ij} \qquad (1)$$

subject to Topological Constraints:

$\sum_{j=1}^{n} y_{ij} = 1$	$\forall\, i \in V$	Assignment Constraint	(2)
$\sum_{j\in adj(i)} \sum_{r\in R} x_{ijr} \ge 2y_{ii}$	$\forall\, i \in V$	Connectivity Constraint	(3)
$\sum_{\substack{k\in adj(i)\\ k\ne j}} x_{ikr} + \sum_{\substack{l\in adj(j)\\ l\ne i}} x_{ljr} \ge 2x_{ijr}$	$\forall\, \{i,j\} \in E, r \in R$	Connectivity on ring constraint	(4)
$\sum_{j\in adj(i)} \sum_{r\in R} x_{ijr} \le 2$	$\forall\, i \in V$	Node Use Constraint	(5)
$\sum_{\{i,j\}\in E} x_{ijr} \le 8$	$\forall\, r \in R$	Ring Bound Constraint	(6)
$y_{ii} \ge y_{ji}$	$\forall\, i \in V$	Spur Assignment Constraint	(7)
$x_{ijr}, y_{ij} \in \{0,1\}$		Binary Integer Constraint	(8)

Note: if no nodes are assigned as spurs, the result is an SRAP solution. The assignment constraints (2) ensure that each site is assigned, either as a ring node (assigned to itself) or as a spur (assigned to another site). If only constraints (2) are used, the result is an assignment with all y_{ii} values set to 1 and all other values set to 0. So, Connectivity constraints (3) are used to ensure that any node assigned as a ring node with $y_{ii} = 1$ is at least two-connected, i.e. at least two incident ring edges x_{ijr} are active.

Using Constraints (2) and (3) only, allows an assignment $x_{ijr_1} = 1 = x_{ijr_2}$, so Connectivity on ring constraints (4) are added to force the cut of any active ring edge to be at least two. Unfortunately constraints (4) can be satisfied by two edges incident at one end so the Node Use constraints (5) are added to ensure that any node i is used on a ring at most twice.

SONET standards set the maximum number of ADMs per ring at sixteen but, in this practical application, rings are restricted to having no more than eight nodes by the Ring Bound Constraints (6). We do not predetermine the number of rings to be used, but [5] propose an upper bound of n on the number of rings. Since any valid ring must contain at least 3 nodes, we propose an upper bound of $\lfloor n/3 \rfloor$.

The Spur Assignment Constraints (7) ensure that any node i is on a ring by setting $y_{ii} = 1$ if a node j is assigned to i as a spur; otherwise $y_{ij} = 1 = y_{ji}$ satisfies the Assignment constraint (2). Finally, constraints (8) ensure that the decision variables are binary integers.

In this paper, we address only the topological formulation for the local rings. In other work, we check whether the resulting topology can support the demand requirements. If this check fails we can backtrack through the sub-optimal integer solutions of the Branch and Bound search tree to find one that does allow for the creation of the tertiary ring.

4 Polyhedral Investigation and Cutting Plane Approach

As we would expect with any NP Hard problem, once the problem size increases, the exact IP approach becomes impractical. Therefore we attempt to solver larger problems using a cutting plane approach. We relax the strict integer requirement for the decision variables of the IP model and solve the resulting LP. We then seek out any violated constraints in the fractional LP solution that can be added to the model and solve the amended LP problem.

We first identify additional inequalities that are valid for the LP formulation of the RSAP but were not required in the original IP description.

$$\sum_{r \in R} x_{ijr} + y_{ij} + y_{ji} \leq 1 \qquad \forall \{i,j\} \in E \qquad \text{Edge Use Constraint} \qquad (9)$$

We can also strengthen the Spur Assignment Constraints (7) as follows:

$$\sum_{r \in R} x_{ijr} + y_{ji} \leq y_{ii} \qquad \forall\, i \in V, j \in V \qquad \text{Strengthened Spur Assignment Constraint} \quad (7a)$$

We search for further inequalities by examining known Facet Defining Inequalities (FDIs) for related problems, [10] and [12], and verify if these inequalities are valid for the RSAP. The degree constraints of the TSP force each node to be exactly 2-connected while the Subtour Elimination Constraints (SECs) require that the cut of any subset of nodes be at least two. Subtour elimination (and degree) constraints are valid for ring subsets, i.e., each ring must form a single simple cycle so we need modified versions of known inequalities to accommodate the multiple rings of the RSAP. We use the following ring-modified SEC:

$$\sum_{j \in adj(i)} x_{ijr} \geq 2 y_{ii} \qquad \forall\, r \in R, \forall\, i \text{ on } r \qquad \text{Ring-modified SECs} \qquad (10)$$

Ring node i is 2-connected and active on ring r: this partially determines which rings are active. This is a refinement of (3). Unfortunately, node i may be assigned to ring r' and node j to r'', which may be sub-optimal, so our cutting plane algorithm only adds one ring-modified SEC. We also add another type of modified SEC for any ring $r \in R$ that has a min cut of zero. This forces at least two edges $\{i,j\}$ to join the ring partitions of r; r_a and r_b, with i on r_a and j on r_b. Where no such edges (or at most 1) exist, we restrict the use of the set of edges that form ring r to two less than the current set on r. These are SEC-zero constraints.

The Gomory-Hu tree of min cuts can be generated and used to identify violated subtour constraints. Since we only need to identify violated subtours on any ring rather than generate all min-cuts in the solution, we use a modified version of [16].

The 2-matching constraints enforce that the number of edges across a cut is even [12]. We write a form of the 2-Matching constraints to ensure that an odd ring edge ij on ring r is balanced with another ring edge kl on ring r where $i \in H \cap \hat{E}$, $j \in \hat{E} \setminus H$. $H \subset V$ is called the handle and $\hat{E} \subset E$ is an odd set of disjoint edges (known as teeth), each with exactly one end in H. We use the following constraint (11) as a refinement of the Connectivity on ring constraints (4). In (11), $r \in R$ is any ring and ij is any unmatched edge on r.

$$\sum_{\substack{k \in G \setminus H \\ k \neq j}} \sum_{l \in H} x_{klr} \geq x_{ijr} \quad \forall\, r \in R \qquad \text{RSAP 2-Matching Constraints} \qquad (11)$$

[13] gives a polynomial time exact algorithm for the separation of 2-matching inequalities. We follow a procedure similar to [13] and use a modified version of [16] to identify odd cuts. For each ring with fractional edges we find the min cut of the ring. If the min cut has an odd number of edges, an Odd Min Cut, this set of edges is used as a possible violated 2-Matching. If the min cut is even, an Even Min Cut, but an odd cut has been found during the search, the odd cut is returned. This is not necessarily the min odd cut, but any odd cut found on the fractional ring edges is a violation of some sort. Finally, if no odd cuts are identified, the even min cut is returned and the ring-modified SEC added.

2-Matchings with only one tooth are dominated by subtour elimination constraints in the traditional TSP formulation. However in our formulation, we may have a result with a one connected ring node. We check each fractional ring edge to identify a possibly violated 1-Conn elimination constraint (12). Note that this is a dis-aggregation of the Connectivity on ring constraints (4). In (12), $r \in R$ is a ring and ik is an edge on r with i 1-connected:

$$\sum_{ij \in \delta(i) \setminus \{ik\}} x_{ijr} \geq x_{ikr} \quad \forall\, r \in R \qquad \text{1-Conn Elimination Constraints} \qquad (12)$$

Since the ring edges of the LP solution may be fractional, local rings of the LP solution might not be disjoint and we observe a 'ghosting' effect as in Figure 2 below, where local rings, r' and r'', coincide. Each ring satisfies the ring bound constraint (6) but there are more than the allowed number of sites on the ring. Such ghosts can be eliminated by observing that at most $|r'| - 2$ of the active edges on the ghost ring r' can be used in the optimal solution.

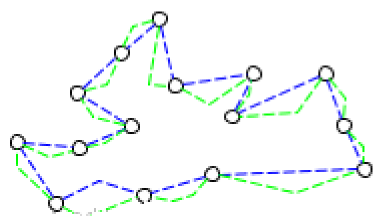

Figure 2. Ghost rings: $y_{ii} = 1$ for all nodes on ghost rings, $x_{ijr'} = 0.533$ for all edges on ring r', $x_{ijr''} = 0.467$ for all edges on ring r''. In this example, $|r'| = |r''| = 15$, greater than the ring bound.

$$\sum_{\{i,j\}\ \text{on}\ r'} x_{ijr} \leq |r'| - 2 \quad \forall\, r \in R \qquad \text{Ghost Elimination Constraints (GECs)} \qquad (13)$$

In summary, our cutting plane algorithm is as follows. Having solved the relaxed LP, if the result is fractional, we identify violated 1-Conn constraints, 2-Matchings, a ring-modified SEC, SEC-zero and ghost elimination constraints and add these constraints to the cut pool. The amended LP is then solved and the process repeated for a maximum number of iterations or until no further cuts are found or the solution is integer.

5 Results

We present promising computational results. Small problems can be solved to optimality in a reasonable amount of time using the exact IP and we achieve the desired effect of favouring ring topologies where possible. All code was written in ANSI C and run on a Dell Latitude D600 with an Intel Pentium 1.5GHz clock and 256M of RAM under Windows XP. Xpress-MP 2008A was used as the LP solver with Xpress-Optimizer version 19.00.00 and Xpress-BCL version 4.0.0 Builder Component library routines.

The test data used was SNDlib (2008) [15] since it provides many real world problem instances with both a network model and a set of demand requirements.

Problem	n,m,d	IP (s)	b	LP & cut (s)	Cuts	Rounds	Integer Y/N	IP opt	CP Obj	Comment
dfn-bwin	10,45,90	0.04	3	0.03	0	1	y	81,333	81,333	CP Obj = IP Opt
Atlanta	15,22,210	0.751	17	0.07	0,0,1,3,0	3	y	33,582,500	33,582,500	CP Obj = IP Opt
France	25,45,300	0.36	5	1.88	16,20,1, 2,8	11	y	11,800	11,800	CP Obj = IP Opt
Cost266 **	37,57,1332	685.15	8	39.64	199,40,1,7,24	47	n	7,929,000	7,595,780	CP Obj < IP Opt
Germany50 *	50,88,662	7200*	100	69.96	92,231,1,8,9	12	y	*	3,548,170	no IP solution

Table 1 Cutting Plane Reults. *IP stopped after 2 hrs with no integer solution, ** fractional result

A condensed set of our cutting plane algorithm results for this short paper is shown in Table 1. Columns from left to right show the problem name and size, time to run the exact IP algorithm, *b* value, time to run the cutting plane algorithm, number of cuts found (1-Conn, 2-Matching, ring-modified SEC, SEC-zero and GEC). The next columns show how many rounds of the cutting plane were run, an indication of whether the solution is integer, the IP then Cutting Plane (CP) objective function values and a comment field.

The algorithm runs relatively quickly on the larger problems (compared to the IP) but not all problems are solved to integrality. Results for these cases, while still fractional, yield a cutting plane objective (CP Obj) which is a lower bound for the IP optimal objective value known for small problems. We are currently investigating further cuts and hope that with these additional cuts the optimal integer solution can be found.

References

[1] Belotti P, Malucelli F, Brunetta L (2007) Multicommodity network design with discrete node costs, Networks, Vol. 49, Iss. 1:90-99.

[2] Fortz B, Soriano P, Wyants C (2003) A tabu search algorithm for self-healing ring network design, European Journal of Operational Research 151:280-295.

[3] Fortz B, Labbé M (2004) Two-Connected Networks with Rings of Bounded Cardinality, Computational Optimization and Applications, 27:123-148.

[4] Fortz B, Mahjoub AR, McCormick ST, Pesneau P (2006) Two-edge connected subgraphs with bounded rings: Polyhedral results and Branch-and-Cut, Math Program. Ser. A 105:85-111.

[5] Goldschmidt O, Laugier A, Olinick E (2003) SONET/SDH ring assignment with capacity constraints, Discrete Applied Mathematics, Vol. 129, Iss. 1:99 – 128.

[6] Grötschel M, Monma C, Stoer M (1995) Polyhedral and Computational Investigations for Designing Communication Networks with High Survivability Requirements, Operations Research, Vol. 43, No. 6: 1012 – 1024.

[7] Grover W, Doucette J, Clouqueur M, Leung D (2002) New Options and Insights for Survivable Transport Networks, IEEE Communications Magazine, 02:34-41

[8] Kerivin H, Mahjoub AR (2005) Design of Survivable Networks, Networks, Vol.46, Iss. 1:1-21.

[9] Labbé M, Laporte G, Martin IR, Salazar-Gonzalez JJ (2004) The Ring Star Problem: Polyhedral Analysis and Exact Algorithm, Networks, Vol. 43, Iss. 3:177 – 189.

[10] Lawler EL, Lenstra JK, Rinnooy Kan AHG, Shmoys DB. (eds.) (1985) The Travelling Salesman Problem: A Guided Tour of Combinatorial Optimization, Wiley, Chichester, ISBN 0-471-90413-9.

[11] Macambira EM, Maculan N, de Souza CC (2006) A column generation approach for SONET ring assignment, Networks, Vol. 47, Iss. 3:157-171.

[12] Nemhauser G, Wolsey L (1988) Integer and Combinatorial Optimization, John Wiley and sons.

[13] Padberg MW, Rao MR (1982) Odd Minimum cut-sets and b-matchings, Mathematics of Operations Research, Vol. 7, No. 1:67-80.

[14] Papadimitriou GI, Obaidat MS, Pomportsis AS (2001) Advances in Optical Networking, Int. J. Commun. Sys., Vol. 15:101-113.

[15] SNDlib www.zib.de/

[16] Stoer M, Wagner F (1997) A Simpe Min-Cut Algorithm, Journal of the ACM, Vol. 44, No. 4:585-591