A shortest path algorithm in multimodal networks: a case study with time varying costs

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Abstract

In this paper we deal with the problem of finding optimal routes in multimodal networks. Since modal change nodes play a relevant role in the choice of origin – destination paths, and consequently in the computation of multimodal shortest paths, we evaluate the performance of such nodes with the aim of increasing their attractivity. We propose an algorithm that focuses on the modal change nodes and forces as much as possible routings through those nodes that could be profitably selected as commuting points.

Preliminary results of a computational experimentation aimed at validating the proposed algorithm with randomly generated multimodal networks are reported together with a case study related to the city of Genoa, Italy.

Keywords: *multimodal networks, shortest path problems, modal change nodes, generalized cost function.*

1 Introduction and problem description

Intermodality, that consists of a change of carrier during the travel, one or more times, using different connections, is nowadays almost necessary in many kinds of transports, both concerning freight and passenger mobility. There is hence the need of establishing the proper sequence of means and commuting points that could allow advantages for the travellers.

The costs of multimodal routes are given by the cost of the travels related to each modality plus the cost implied by the change of modality, that is the so called transition costs. When using multimodal routes some travelling costs can be reduced since economies of scales can be obtained when using different transportation modalities, especially those related to the mass transit modality. Moreover, the usage of a mass transit modality allows a great saving in terms of cost and pollution; unfortunately, this is not always available and can rarely reach many points of a multimodal network. Thus we have higher monetary and time costs due to the change of modality, associated with a greater discomfort paid by the users. When a user reaches an intermodal node, the corresponding transition time is usually given by the time spent for finding a free and legal parking, the walking time required for reaching the train / bus stop, and the waiting time at the bus stop or train station. Generally these terms are functions consisting of a deterministic part plus a stochastic deviation (i.e. drivers don't know exactly the time required for looking for an available car park and reaching it once they arrived at the modal change node). The monetary cost component is given by the parking fee.

Most of the papers presented in the literature about intermodality focus on urban mobility, and in particular on the shortest path problem in urban multimodal transportation networks; see among others [8], [7], [6] and [3]. In this paper we focus on urban passenger mobility networks; however, the algorithm here described can be applied also to any kind of multimodal networks. The main goal of the proposed approach is to find origin – destination (*o-d*) multimodal least cost paths forcing as much as possible routings through those nodes that could be profitably selected as commuting points. Our objective function considers both time and monetary cost components; the required optimal paths are thus referred to herein as Pareto-optimal paths. Note that since such solutions are not generally equivalent to the decision makers, they are evaluated also on the basis of a sort of rating of the modal change nodes. Of course, different priorities or orders in the selection of modal change nodes can be used, according to several criteria, such as the buffer queue or some structural characteristics of the nodes like the connectivity, as defined in [2]; another selection criterion can be to consider only primary an secondary modal change nodes, as they have been defined in [1], that is modal change nodes that guarantee very easy connections to the mass transit transportation network.

In the search for optimal multimodal routes we first compute the required shortest paths by looking for the most well performing modal change nodes from both the origin and the destination nodes; successively, we evaluate the corresponding possible multimodal paths with the aim of finding more convenient the intermodal solutions than the mono modal ones. We based our computations on the Dijkstra's algorithm [4], but any of the classical shortest path algorithms proposed in literature can be used as well (see e.g. [5] for a survey of such algorithms). Our goal is to compute multimodal *o-d* paths in a reasonable amount of CPU time, without having the need of verifying exhaustively every possible *o-d* multimodal connections. We are also interested in analysing how the proposed approach is sensible to any kind of upgrading of the data concerning the network; in particular, we consider some perturbation in the data about travelling times. Moreover, this analysis allows to face possible travel time changes over a day due to a variation of the traffic congestion in transportation networks.

2 The network model

In order to be able to represent all the features of multimodal transportation problems we have to deal with models that enable us to include all the parameters that can impact on the corresponding decisional process, especially those related to the modal change points. For this purpose, focusing on urban transportation networks, we consider digraph $G = (V, A, C, I, D, M, R)$ with the following specifications. V is the set of nodes that defines, as usual, relevant locations within the network; great attention will be devoted to $N \subseteq V$, that represents the subset of possible modal change nodes. Let $|V| = m$ and $|N| = n$.

Set A defines the oriented arcs, or connections, between pair of nodes; C is the set of weights (travelling times) associated with each arc $(i,j) \in A$, while I is the set of weights (transition costs) associated with each node belonging to N.

D is the set of attributes associated with the modal change nodes that will be evaluated in the selection phase of the algorithm for determining the shortest multimodal paths. A better explanation of costs and attributes of the network model will be given in Subsections 2.1 and 2.2.

M is the set of the transportation modalities that are allowable in the network under consideration; we may split them into two main classes, namely:

a) restricted modalities, that is private ones, such as car or motorcar, which can be used only by the decision maker;

b) mass transit modalities, that is shared ones, such as train or bus, which can allow a multiple transport with a single carrier.

We have to consider that people generally don't like more than one changes of mean in the same trip; in fact, modal changes are perceived "uncomfortable" by users and usually are not free. For this reason, once the change of modality from the private to a mass transit one is selected, the presented algorithm tries to do up to two modal changes, that is we assume that no more than two means for each *o-d* path is allowable; more precisely, we say that node j is connected to node i in a mass transit transportation network if and only if there exists a path from i to j in G in which at most one change of a mass transit mean may occur.

Finally, R is the set of the possible decisional criteria used to evaluate the generalized cost of the whole journey in the multimodal network. The specification of any element $r \in R$ that we consider in this work is follows: $r = 1$ is related to the traveling time, $r = 2$ concerns the monetary cost, $r = 3$ can refer to some discomfort factors, such as weather conditions in the case of urban mobility, reliability in case of data transfer and so on.

2.1 Definition of the (arc and node) costs in *G*

We consider different kinds of weights, associated, respectively, with the arcs and nodes of the network.

Set C is the set of the costs associated with the arcs of the network; every element $c(i,j)_{t,r} \in C$ is the cost for travelling arc $(i,j) \in A$ with modality t according to a given decisional criterion r.

In order to better represent the user's preferences, we use a weighted value for each criterion, that is we compare the cost of different routes by computing for every considered *o-d* path travelled with modality t the generalized cost function $C(o,d)_t$ given in (1) as

$$C(o,d)_t = \sum_{r=1}^{|R|} c(i,j)_{t,r} \varpi_r \quad (1)$$

where ω_r is the weight that users assign to the *r-th* evaluation criterion, such that $\sum_{r=1}^{|R|} \omega_r = 1$.

In literature, a lot of works deal with travelling time functions; assuming that every problem should have its own cost function, we legitimately assume that the travelling cost is a linear function; for example, we may suppose that, in a road transportation network, the time function depends on the length of the street, the width, the traffic condition, the grade line and the level of bow-street (see e.g. [9]).

I a relevant set in our model and for our algorithm too, since it defines the transitions costs that users have to pay at the modal change nodes. Set I of the cost associated with the nodes of the network is a $|M| \times |R|$ matrix.

Let $\tau(i)_{ab,r}$ be a generic element of set I representing the cost that the users have to pay for moving from modality a, that is the restricted one, to modality b, that is one of the possible mass transit modality, at the modal change node $i \in N$ according to a given decisional criterion r. Note that we imply that $\tau(i)_{ab,r} = +\infty$ $\forall$ $i \notin N$, that is we assume that a change of modality is not allowed anywhere.

2.2 Attributes

D is a particular set that is different from the others since it contains the measure of the qualitative characteristics of the modal change nodes. We think that for the decisional process concerning the transportation modalities it is relevant the evaluation of the modal change nodes from a structural point of view, that is to verify their connection capability to the mass transit transportation network from the restricted one. Therefore, we consider the following attributes.

Connectivity. The connectivity is the first attribute that we verify in the evaluation of every possible modal change node. This index (δ_i^1) gives a measure of how a given modal change node $i \in N$ is connected to the other nodes of the network; it is hence given by the ratio between the number $x_{i,t}$ of nodes that can be reached from i without any modal or vehicle change over the total number of nodes of V. Note that if more than one modality is available, the connectivity index is relative to the modality with higher connectivity. Usually the connectivity index is computed for evaluating, as in our case, the goodness of the connection between the restricted modality, that is chosen at the origin node, and the mass transit one, that can be selected for reaching the destination node.

Accessibility. This index (δ_i^2) concerns the distance from all nodes in V to node i, $\forall$ $i \in N$. That is, the accessibility index gives a measure of how easily a modal change node $i \in N$ can be reached by the other nodes of the network.

Expected time. The expected time (e.g. queue or transfer) spent at a modal change node may be calculated by an ad-hoc function specifically for each case. Note that a too long waiting time may often discourage the user and make him/her chooses another node, thus implying a change in the strategy. For this reason, we may think to do not consider as possible commuting points those modal change nodes that have an excessive waiting time to enter to. Generally, the waiting time associated with each node may be computed as the product between the number of elements actually in the queue and the mean processing time; in this case the expected queuing time at a modal change node $i \in N$ (δ_i^3) can be given by the product between the average waiting time and the number of users in the queue at node i.

Remember that set D is a component of the network and it is defined a priori at the beginning of the process, therefore users don't know precisely the queue time associated with the modal change nodes. Thus set D can be now generally defined as $D = \{\delta_i^1, \delta_i^2, ..., \delta_i^d\}$, $\forall$ $i \in N$; in our case we have $d = 3$.

As previously described, the selection of the modal change nodes is our main issue in finding the best intermodal path. We suppose that users don't know the effective generalized cost function associated with the commuting nodes and thus make their choice in low information conditions. In fact, when users reach a modal change node, they know the network status of a node and the available connections from such node to the others; however, they may only suppose the average waiting time at it basing the choice on a previous knowledge instead of calculating the real cost. For this reason, attributes are a set of information that users know and use to reduce the number of modal change nodes to be considered in the computation of the shortest paths, as we will see in the next paragraph.

3 The proposed algorithm

The presented algorithm is a heuristic approach to the problem of finding optimal multimodal *o-d* routes in urban transportation networks. The algorithm looks for the best modal change nodes and computes the minimum cost path using such nodes for the given pair of *o-d* nodes. In particular, by using a Dijkstra like algorithm, we first compute the required shortest path by looking for the most well performing modal change nodes from both the origin and the destination nodes; successively, we evaluate the generalized cost of possible alternative multimodal paths with the aim of forcing as much as possible routings through those nodes that could be selected as commuting points while reducing the previously determined mono-modal solution.

At the beginning of the decision process, the user defines the upper and lower bounds for attribute δ_i^d , $d = 1, \ldots, 3$; these bounds are the threshold value for selecting a modal change node in the search for the shortest path. In particular, let $\alpha_1, \alpha_{2, \ldots,} \alpha_d$ and $\beta_1, \beta_{2, \ldots,} \beta_d$ be the upper and lower bounds, respectively, of values δ_i^d , $d = 1, \ldots, 3$; if $i \in N$ and $\beta_d \le \delta_i^d \le \alpha_d \; \forall \; d \in D$ then node *i* will be added to the set of possible commuting points. For each kind of networks we may have different evaluation parameters, as to better describe the user's decision process. We assume that attributes δ_i^d , $d = 1, \ldots, 3$, are already known at the beginning of the algorithm.

In more details, the proposed algorithm proceeds by first executing the Dijkstra algorithm starting from the given origin node. The selection process of the modal change nodes occurs within the labelling phase of the algorithm. At each iteration every selected node is analysed; if the node is a modal change one and the values of its considered attributes fit into the allowable ranges then the node is considered as possible commuting nodes. The Dijkstra algorithm stops as soon as a given number $k \le n$ of reachable modal change nodes are positively evaluated or we are not able to find any other candidate node. Finally, we execute the Dijkstra algorithm having the destination node as our starting node, as before, until *k* modal change nodes are selected.

Let Ω and Δ denote the sets of the identified modal change nodes starting from the origin and destination nodes, respectively. The Dijkstra algorithm is executed in both direction for every restricted modality.

Note that, due to the multi objective nature of the problem, every cost comparison has to be made relatively to the generalized cost function; that is, we look for the path having the minimum value of equation (1) considering as interchanging nodes only those belonging to Ω and Δ with respect to the chosen criterion r, $r = 1, \ldots, 3$. Note that in the selection of modal change nodes different priorities or orders can be considered.

The algorithm takes into account only modal change nodes that guarantee very easy connections from the private network to the mass transit one, and vice versa. For setting up our attributes before the execution of the algorithm, we fix the minimum acceptable value of δ_i^1 and δ_i^2 , to 40%; this means that those nodes with both connectivity and attractivity values lower than 40% are not considered as commuting points and hence as possible origin nodes. Note that the connectivity value is evaluated for the selection of the origin interchange nodes belonging to Ω, while the attractivity is relevant for the selection of the destination interchange nodes belonging to Δ.

4 Computational experimentation

An extensive computational effort devoted at validating the proposed algorithm is in progress. A massive number of trials with randomly generated instances have been used as a test bed for verifying both the goodness of the obtained solutions and the corresponding CPU time. We are also applying our algorithm to a urban passenger transportation network related to the central area of the city of Genova, Italy. The algorithm has been implemented in C++ and we are testing it on a 3200 Mhz dual core with 512 Mb RAM platform.

In Table 1 we report some preliminary computational results related to random instances of different sizes. Each entry of Table 1 is the average value of the objective function (1) computed on the basis of 100 different randomly generated graphs of the same size.

Values in column "Optimality gap" has been obtained as the difference of the generalized cost (1) between the optimal solution found with the exhaustive algorithm and the solution found by our algorithm. Note that on the average the optimality gap is only about 6,75% for the non optimal solutions; more precisely, in at most 34% of the instances the non optimal paths are averagely between 5% and 8,5% more expensive than the optimal

solutions. In fact, the proposed algorithm was able to find the Pareto optimal solution in a very high percentage of cases; in particular, from the observed results, we can see that the algorithm finds the optimal solutions about two times every three trials. Combined with the number of successes, this means that the algorithm is able to find on the average a solution that is less than 5% worst than the Pareto-optimal solution.
As far as the CPU time is concerned, we can note that, while the CPU time of the exact algorithm grows very quickly, our heuristic algorithm reaches the solution in a very small CPU time.

Table 1. Computational results with random instances

Instance type	# nodes	# arcs	CPU time our algorithm	CPU time exaustive algorithm	# optimal solutions (%)	Optimality gap (%)
1	100	2500	<0,01”	1,49”	66%	6.52%
2	200	5000	<0,01”	7,17”	64%	7.91%
3	300	11000	0,016"	35,89"	64%	8.40%
4	400	17000	0,04"	1'07"	68%	5.51%
5	500	23000	0,055”	2'16”	62%	5.32%

Referring to the city of Genova and assuming that the mass transportation demand level is given, we look for the best intermodal solution for the most travelled *o-d* paths. Starting from the definition of *G* and the available *o-d* matrices referring to a rush hour time period, i.e. from 6.30 to 9.30 a.m., we have derived the corresponding network model that represents the central area of the city and is related to a linear extension of about 5 kilometres. The size of the network is the following: $m = 56$, $n = 46$, $|A| = 1141$, whose 997 refer to the mass transit modalities, that consist in turn of a multimodal network since it is split into bus, subway, train, elevator and cableway connections.

We focus on 3 nodes that are the most relevant origin nodes in terms of traffic flow, since they represent the access to the centre for drivers coming from the west, east and north side of the city, respectively, and on 2 nodes in the centre of the town where it is concentrated the commercial heart of the city, that represent the most frequent destinations.

In Tables 2-3 we present the values of the most relevant *o-d* shortest paths; columns “Ω” and “Δ” represent the possible modal change nodes from the origin and the destination nodes, respectively, in the considered path; the remaining columns report the total cost and that related to the corresponding modality, including the walking time. In the case of the path from 56 to 18, synthesised in Table 2, the solution found by our algorithm is Pareto-optimal. Note that even if node 56 is a modal change node, it has been discarded by the algorithm due to its bad attributes; in this case, the modal change node is 27.

Table 2. Cost of the 56-18 multimodal path

Ω	Δ	***Total***	***Private***	***Mass transit***	***Interchange***	***Walking***
28	18	21,31	1,53	9,55	10,23	0
28	24	24,84	1,53	11,06	10,23	2
28	19	22,97	1,53	8,54	10,23	2,67
27	18	14,17	1,63	7,85	4,69	0
27	24	17,68	1,63	9,36	4,69	2
27	19	15,83	1,63	6,84	4,69	2,67
22	18	15,03	2,19	5,94	6,9	0
22	24	18,54	2,19	7,45	6,9	2
22	19	16,09	2,19	4,93	6,9	2,67

A different situation arises in the case of the path from 42 to 18, reported in Table 4; this case is interesting to be analyzed, since node 42 is a modal change node and it is not discarded by the algorithm. Having a look at the solution, that is the Pareto-optimal one, we can see that node 42 is not the interchanging node selected by the solution; indeed the change of modality occurs at node 27.

Table 3. Cost of the 42-18 multimodal path

Ω	Δ	***Total***	***Private***	***Mass transit***	***Interchange***	***Walking***
42	18	17,96	0	9,96	0	0
42	24	21,53	0	11,53	0	2
42	19	19,37	0	8,7	0	2,67
28	18	20,66	0,88	9,55	10,23	0
28	24	24,17	0,88	11,06	10,23	2
28	19	22,32	0,88	8,54	10,23	2,67
27	18	14,08	1,54	7,85	4,69	0
27	24	17,59	1,54	9,36	4,69	2

27	19	15,74	1,54	6,84	4,69	2,67

5 Sensitivity analysis

As a final validation of the proposed algorithm we test the stability of the founded optimal solutions in the presence of noise in the data, and more precisely of the weight of the arc set *A*. In fact, considering the technical difficulties usually encountered in getting precise data, in particular information about data flows, it is important that the algorithm is stable enough to allow to find good solutions also when a high data accuracy level cannot be guaranteed.

For this purpose, we implemented a random noise generator that perturbed the arc costs. More precisely, we implemented the function “addNoise X” that adds to the arc weights a perturbation up to X% of the corresponding given input value; that is every arc cost has been modified up to the defined percentage of noise (more or less) before running again the shortes path algorithm.

Table 4 reports the synthesis of our computational results relative to different levels of noise on different kinds of randomly generated graphs.

Table 4. Percentage of optimal solution in presence of perturbation on the arc cost

Noise	# optimal solutions (%)	Optimality gap (%)
5%	100%	0%
10%	90%	3.52%
15%	90%	3.52%
20%	80%	5.23%
25%	75%	6.21%

Each row of Table 4 reports the average value of 10 different networks, each one of the type given in Table 1 for every noise level, and the corresponding comparison with the solution obtained with the exact algorithm without any noise. We can see that results are very good up to 15% of noise, while starting from a 20% noise level, the solution’s reliability degenerates quickly.

Preliminary results show that the algorithm has a good tolerance to the noise. The main reason is that the node selection is done also on the basis of the qualitative information about the structural characteristics of the same nodes.

6 Conclusions and outlines for future works

In this paper we have presented an algorithm for finding optimal paths in multimodal networks having decisional weights on both arcs and nodes. Preliminary tests shown that the algorithm has good performances both in terms of CPU time and optimality gap, and it is able to find modal change nodes when such nodes are well connected to the other modalities. The authors intend to extend the computational experiments and the sensitivity analysis.

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