An Interior-Point Algorithm for Routing in Data Telecommunications Networks

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Abstract

The routing of data in packet-switched networks plays an important role in the optimization of network performance. This problem is formulated as a (likely very large) convex multicommodity flow problem, each commodity related to the demand from some origin to some destination node. Like most previous approaches, we consider the Kleinrock average delay objective function, augmented with a quadratic reliability term by Stern. This results in a separable nonlinear convex multicommodity problem, which is solved through a specialized interior-point method. This method combines direct and preconditioned iterative solvers for the solution of normal equations. The reliability term is shown to be instrumental for the quality of the preconditioner. If such a term is removed, it is even possible to efficiently solve the resulting problem through a regularized version of the algorithm. The computational results with a set of real and artificial instances show the efficiency of the approach.

Keywords: *interior-point methods, preconditioned conjugate gradient, routing in telecommunications, average delay*

1 Extended abstract

Let $G = (V, E)$ be an undirected graph with n edges and $m+1$ nodes representing a telecommunication network designed to satisfy a number κ of requirements (represented by commodities). Each link $j \in E$ of the network is designed with a given capacity b_j^0. We consider an arbitrary orientation of the edges and two vectors x^{k+}, $x^{k-} \geq 0$ in $\mathbb{R}^n$ are used to represent a commodity k (x^{k+} for the orientation and x^{k-} for the reverse). The demand of commodity k is represented by a vector $b^k \in \mathbb{R}^{m+1}$, such that $\sum_{i=1}^{m+1} b_i^k = 0$. We are interested in the following multicommodity flow problem

$$\begin{array}{ll} \min & f(x) \triangleq \sum_{j \in E} \frac{x_j^0}{b_j^0 - x_j^0} \\ \text{s.t.} & Nx^{k+} - Nx^{k-} = b^k, \quad k = 1, \ldots, \kappa, \\ & \sum_{k=1}^{\kappa} (x^{k+} + x^{k-}) = x^0, \\ & x^0 \leq b^0, \\ & x^{k+},\ x^{k-} \geq 0, \quad k = 1, \ldots, \kappa. \end{array} \tag{1}$$

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$N \in \mathbb{R}^{m\times n}$ is the node-arc incidence matrix; one node has been removed to guarantee full row-rank. This problem has been studied and plays an important role in the performance of packet-switched networks. Let us introduce the slack vector s to rewrite the mutual constraint as

$$\sum_{k=1}^{\kappa} x^k + s = b^0$$

As the flow vectors are positive, we may consider that s is bounded by b^0:

$$0 \le s \le b^0.$$

Note that f is strictly convex with respect to only x^0. To make it convex for any component, and thus the system more reliable, we follows Stern [7] and add the quadratic terms

$$\frac{\alpha}{2}\sum_{k=1}^{\kappa}(\|x^{k+}\|^2 + \|x^{k-}\|^2)$$

to the objective function, where α is a small positive parameter. It can be shown that x_j^{k+} and x_j^{k-} cannot be simultaneously positive. Hence

$$\frac{\alpha}{2}\sum_{k=1}^{\kappa}\|x^{k+} + x^{k-}\|^2 = \frac{\alpha}{2}\sum_{k=1}^{\kappa}(\|x^{k+}\|^2 + \|x^{k-}\|^2).$$

As in [6], we express the delay function in terms of the slack variable s and consider the problem

$$\begin{array}{ll} \min & f(x) \triangleq \sum_{j\in E} \frac{b_j^0 - s_j}{s_j} + \frac{\alpha}{2}\sum_{k=1}^{\kappa} \|x^{k+}\|^2 + \|x^{k-}\|^2 \\ \text{s.t.} & Nx^{k+} - Nx^{k-} = b^k, \quad k = 1,\dots,\kappa, \\ & \sum_{k=1}^{\kappa}(x^{k+} + x^{k-}) + s = b^0, \\ & 0 < s \le b^0, \\ & x^{k+},\ x^{k-} \ge 0, \quad k = 1,\dots,\kappa. \end{array} \tag{2}$$

Vector x includes all the variables x^{k+}, x^{k-} and s. Flows x^{k+} and x^{k-} for each commodity can be bounded by the arc capacity b^0 if required.

$f(x)$ is a convex separable function in the feasible set. Denoting as M the diagonal matrix obtained from any vector m, and by e a vector of 1's of appropriate dimension, the components of the gradient of $f(x)$ are

$$\begin{array}{rcl} \nabla_{x^{k+}} f(x) & = & \alpha X^{k+} e \\ \nabla_{x^{k-}} f(x) & = & \alpha X^{k-} e \\ \nabla_s f(x) & = & -B^0 S^{-2} e, \end{array} \tag{3}$$

while the diagonal submatrices of the diagonal Hessian are

$$\begin{array}{rcl} \nabla^2_{x^{k+}} f(x) & = & \alpha I \\ \nabla^2_{x^{k-}} f(x) & = & \alpha I \\ \nabla^2_s f(x) & = & 2B^0 S^{-3}. \end{array} \tag{4}$$

Problem (2) can be written in standard form as

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & Ax = b \\ & 0 \le x \le u. \end{array} \tag{5}$$

Table 1: Comparison of an interior point method with two state-of-the-art active sets codes

Instance	rows	columns	CPU IPM	CPU MINOS	CPU SNOPT
3	6040	28140	5.43	255	246.86
4	7468	36260	11.44	4123.71	2180.5
5	2591	17088	2.74	37.91	87.44

Results on Linux PC with AMD Athlon 64 bits Dual Core 4400+, and 2 Gb RAM

$A \in \mathbb{R}^{m' \times n'}$ has full row-rank, and $f(x) : \mathbb{R}^{n'} \to \mathbb{R}$ is convex. According to (2), we have $n' = (2\kappa + 1)n$, and $m' = \kappa m + n$. Problem (5) is convex, and it can be efficiently solved by interior-point methods [4, 8]. In particular the objective function of (2) provides a barrier with small parameter [4], which guarantees a low (polynomial) computational complexity of the algorithm. This is empirically shown in Table 1, that reports the CPU time in the solution of three small instances from [6] with a home-made interior-point implementation, and two state-of-the art active set codes: MINOS and SNOPT. Instance 3 is a random network, instance 4 is a nonoriented versions of problem "ndo148", and instance 5 is a real network. However, for large problems a general interior-point solver may be even inefficient, if the particular multicommodity (i.e., primal block-angular) structure of (2) is not exploited. This can be done by applying the specialized procedure of [2, 3]. In this work this procedure is extended to convex objectives for the efficient solution of (2). Preliminary computational results show the efficiency of this approach, being competitive against current state-of-the-art methods [1, 5] for (2).

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