Equipment replacement planning in a telecommunication network with a decreasing number of clients

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Abstract

Frequent upgrades of equipments in the telecommunications industry occur due to the emergence of new services or technologic breakthroughs. In this work, we consider a network where each client is linked to a site and handled by a card located on that site. A technological migration has to be operated and it consists in replacing all the existing cards by cards of new generation within a fixed number of years. For practical considerations, all the cards of a site must be replaced on the same year. Furthermore, because of new offers, the number of clients per site is decreasing during the period. This enables us to reuse the new cards that are not used anymore once some clients have left. The optimization problem consists in deciding for each year, which sites are migrated and how many cards are bought or reused, in order to minimize the total cost. We present a formulation by integer linear programming and an exact solution for this problem.

Keywords: *Planning optimization, Telecommunications, Integer programming, Heuristic*

1 Introduction: An Industrial case study

A telecommunication operator such as France Telecom is often confronted with the emergence of new technologies that implies adaptation of the existing networks. This problem occurred recently with the growth of Voice Over Internet Protocol (or VoIP), which is now one of the most important ways of conveying information. In the initial state of the network for fixed telecommunications, France Telecom is using PSTN technology (Public Switched Telephone Network) and in order to reduce operating costs, the operator wishes to replace this technology by VoIP. This means that in each site of the network, the PSTN equipment connected to the client has to be replaced by a VoIP compatible card. The overall operation has to be realized in five years and at minimal cost. So, we have to determine the optimal replacement year for each site in order to reduce the total cost. We will call migration the equipment replacement process in a site and we will consider that we need one VoIP card per client.

Furthermore, a particular feature has to be considered. Indeed, with competition between providers and emergence of new offers inside France Telecom as well, clients are more and more subject to cancel their standard contract for offers that combine TV, Internet and phone services. These new offers are directly using VoIP technology. As a consequence, the number of clients concerned by the technological change is decreasing. This raises an issue: when a new expensive VoIP card has been set up for a client who leaves afterwards, the investment is lost. To secure a return on this investment, cards that become unused before the end of the time period can be reused. Such cards are disconnected and installed again for other clients on a site not yet migrated to VoIP. Taking into account this possibility, we have to determine for each year of the period, how many VoIP cards are bought and how many are reused. As reusing a card is less expensive than buying a new one, we will try to maximize the amount of reused cards. This specificity is responsible for the problem's difficulty. In fact, without the possibility of reusing cards, we could determine the optimal year of migration for each site independently and would be closer from classical equipment replacement problems (see [1], [2] and [3]).

Another specificity is that a tax has to be paid for each kind of equipment set up in the network: on old PSTN equipments as well as on new VoIP cards. This tax represents a percentage of the equipment's purchasing price. It means that when a site is migrated to VoIP, tax on the PSTN equipments is no more paid, which represents a gain.

To conclude, we have two types of decisions to make: the year at which each site is migrated, and the number of cards bought or reused every year. Using the Partition problem, we have proved that this optimization problem was NP-hard.

2 Integer linear programming formulation

We have chosen to formulate this equipment replacement problem by an integer linear program.

We are given the following positive integers:
m : number of years of the time period dedicated to the migration
p : number of sites to be migrated
$n_{i,t}$: number of clients on site i at year t
PT_t : price of a VoIP card bought at year t and the corresponding amounts of tax to pay during the rest of the period
I_t : cost of installing a VoIP card at year t
IA_t : cost of reusing a VoIP card at year t
$GT_{i,t}$: gain of tax obtained when site i is migrated at year t

We denote by M the set $\{1, ..., m\}$ and by P the set $\{1, ..., p\}$. The aim is to plan the migration of p sites in m years. At year t, if one site i has to be migrated, it is currently connected to $n_{i,t}$ clients. And this number of new VoIP cards has to be bought at price PT_t or reused for a cost IA_t and then connected on the site for a cost of I_t. The reuse cost corresponds to the previous disconnection and the carrying of the card.

As prices might change from one year to another, it is possible that sometimes buying cards in advance would be interesting. Such cards could be stocked (at no cost) and used for the first time when needed (however tax is paid as soon as they have been bought). We allow this to happen and it represents an easy calculation to be done before specifying the model. We do not explain it here and we consider only the number of new VoIP cards (first installed) needed at year t, no matter the year of their purchase. This transformation implies a different cost for the purchase of a new card first installed at t, including its actual purchase date: we will call this coefficient PR_t.

We now introduce three sets of variables:
$x_{i,t} \in \{0, 1\}$: Binary variable equal to 1 if and only if site i is migrated at year t
$a_t \in \mathbb{N}$: Number of new VoIP cards needed at year t
$r_t \in \mathbb{N}$: Number of reused VoIP cards at year t

Thus the objective function is now composed of:

- a cost for the purchase and tax of cards: $\sum_{t \in M} PR_t \times a_t$
- a cost for reusing some cards: $\sum_{t \in M} IA_t \times r_t$

- a cost for installation of cards: $\sum_{t \in M} I_t \times \sum_{i \in P} n_{i,t} x_{i,t}$

- a gain related to the no more paid tax on PSTN equipments: $\sum_{t \in M} \sum_{k \geq t} GT_{i,t} \times x_{i,t}$

The main constraint is affecting the year of migration of the sites: each site has to be migrated during the period: $\sum_{t \in M} x_{i,t} = 1 \quad \forall i \in P \quad (C1)$.

At the beginning of the time period, the stock of available cards is empty, so we cannot reuse cards on the first year: $r_1 = 0 \quad (C2)$.

At each year, the number of VoIP cards needed is equal to the sum of new and reused cards installed: $a_t + r_t = \sum_{i \in P} n_{i,t} x_{i,t} \quad \forall t \in M \quad (C3)$.

Eventually, we cannot reuse more cards than those available in the stock. This means that the amount of cards reused each year must be lower than the amount of cards disconnected since the beginning of the period: $\sum_{k \leq t} r_k \leq \sum_{k<t} \sum_{i \in P} (n_{i,k} - n_{i,t}) x_{i,k} \quad \forall t \geq 2 \quad (C4)$.

In order to lighten our formulation, we suppress in constraint $(C3)$ variables a_t that can be viewed as slack variables and we substitute $(\sum_{i \in P} n_{i,t} x_{i,t} - r_t)$ to a_t in the objective function. Thus, constraint $(C3)$ become: $r_t \leq \sum_{i \in P} n_{i,t} x_{i,t} \quad \forall t \in M \quad (C5)$. We obtain the following linear program:

$$
(PL\ 1) \left\{
\begin{array}{llll}
Min & \sum_{t \in M} (IA_t - PR_t) \times r_t + \sum_{t \in M} \sum_{i \in P} [\ (PR_t + I_t)\ n_{i,t} & & \\
 & \quad - \sum_{k \geq t} Tax_k\ (CS_i + CI_i \times n_{i,k})\ ]\ \times\ x_{i,t} & & \\
s.c. & \sum_{t \in M} x_{i,t} = 1 & \forall i \in P & (C1) \\
 & r_1 = 0 & & (C2) \\
 & r_t \leq \sum_{i \in P} n_{i,t} x_{i,t} & \forall t \in M & (C5) \\
 & \sum_{k \leq t} r_k \leq \sum_{k<t} \sum_{i \in P} (n_{i,k} - n_{i,t}) x_{i,k} & \forall t \in M - \{1\} & (C4) \\
 & x_{i,t} \in \{0,1\} & \forall i \in P, \forall t \in M & (C6) \\
 & r_t \in \mathbb{N} & \forall t \in M & (C7)
\end{array}
\right.
$$

We have now a compact linear integer programming formulation for our problem.

3 Exact and approximate solution

We focus now more precisely on the problem solution.

All the experiments presented in this paper have been carried out on a Linux server Intel Xeon (2.8 GHz, 4Go RAM). For exact and relaxed solutions, we used the MIP solver of CPLEX 11.

Instances have been generated randomly, but their structure was inspired from real-life instances. We define the number of years for the migration period: m, the number of sites p to be migrated, and intervals for each data.

For each site i, we draw values for $n_{i,1}$, CS_i and CI_i inside their respective intervals. To obtain a decreasing number of clients on this site i, we then apply a random percentage of decrease each year t to the value $n_{i,t-1}$. Initial costs (PT_1, I_1, IA_1 and $GT_{i,1}$) are equally drawn inside their bounds, costs for the following years are obtained by applying small random perturbations on the previous year's value.

We have carried out experiments for different sizes of instances (m, p). For each size, 10 random instances have been generated and solved. We give the average values of the results in Table 1. Even if the actual number of years of the period is 5, we tried to consider a 10 years period to observe the influence of this parameter. The largest instance includes about 10 000 sites for a 5 years period but we also tried other smaller numbers of sites.

Formulated as (PL 1), the problem can be directly solved by CPLEX. The results are presented in Table 1. We denote by *TimeOpt* the time in seconds to obtain the optimal solution, by *NodeOpt* the number of nodes developed in the branch and cut algorithm, and by *RootGap* the relative gap between the optimal value and the continuous relaxation value.

m	**p**	**Time Opt**	**Node Opt**	**Root Gap**
5	10	0.040s	19	0.695
5	100	0.507s	1080	0.028
5	1000	4.22s	208	0.006
10	10	0.126s	304	45.448
10	100	716.4s	1 367 697	2.804
5	10 000	30.1s	230	0.004

Table 1: Exact results

We notice a relatively quick solution even for the largest instances, that correspond to real-life instances. However, for these instances results are quite dissimilar: while some of them are immediately solved, others can take several hours. We observe also the relationship between the increase of parameter m and the solution time. In order to speed up the solution we also propose an efficient heuristic, based on the continuous relaxation of (PL 1) (see [4]).

4 Conclusion and future prospects

To sum up, we have studied a very specific industrial problem arising in telecommunication networks that is NP-hard and we proposed an integer linear programming formulation. This formulation is very efficient, since it provides an exact solution for real-life instances within an average of 30 seconds.

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