A Hierarchical Optimization Approach to Optical Network Design where Traffic Grooming and Routing is Solved by Column Generation

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Abstract

The paper deals with a two layer optical network design problem: The virtual layer where nodes are logically connected by optical hops and the physical layer where optical hops have to be routed. The optical hops are used to route a very large number of commodities that direct approaches have difficulties to handle efficiently. We propose a hierarchical optimization procedure where we first take care of the grooming and routing decisions and then we perform wavelength assignment. The fist stage is solved by column generation and a rounding heuristic provides integer solutions. The second stage was always feasible in our computational tests. As shown in the computational tests, the two stage optimization scheme is very efficient for solving large traffic instances in optical core/metropolitan networks both from a computing time and an accuracy points of view.

Keywords *: Network design problem, Decomposition techniques, Integer rounding heuristic.*

Introduction

Today's optical networks have huge capacity ranging up to 10 Gb/s thanks to *wavelength division multiplexing* (WDM). This technique allows one to pack many streams on the same link. However each link can carry at most one stream for each wavelength (wavelength clash constraints). A link can carry tens of wavelengths of capacity OC-192 (1 OC $\simeq$ 52Mb/s). Traffic requests are expressed as bandwidth reservation of standard granularities: OC-1, 3, 12, or 48. Each request must be single-path routed from origin to destination. Packing multiple requests together on the same optical stream in a given wavelength is so-called traffic grooming [8]. Each aggregation of traffic or disaggregation at an origin, destination or intermediate node requires to convert the signal in the electrical domain. Hence, a request route decomposes into *optical hops* defined by a physical path along which the signal optically *bypasses* the intermediate nodes and thus creates a virtual connection between the end-nodes. The sequence of optical hops followed by a request defines a *lightpath*. To limit the end-to-end delay caused by the O/E/O conversions, we bound the number of optical hops in a lightpath to 2 as in [9] (so-called hop constraints). Optical-electrical conversions requires the installation of expensive ports at each end-node of the optical hops. Hence, one must optimize traffic *grooming, routing and wavelength assignment* (GRWA) to reduce opto-electronic system installation costs or equivalently, the overall number of optical hops that have to be set-up to grant the set of traffic requests.

Different variants of the GRWA problem variants have been discussed [3]: Maximizing the throughput was previously studied by, e.g., [11, 10], and solving the GRWA problem under simplifying assumptions was investigated, e.g., with uniform request bandwidth [6], or with a fixed set of optical hops [5] or with infinite wavelength capacity [6] in order to reduce the GRWA solution complexity or in order to guarantee a scalable GRWA solution scheme. To the best of our knowledge, existing studies (except [9] for IP over WDM networks and throughput maximization) do not enforce a maximum number of hops for each request.

The GRWA problem is NP-hard. Several heuristic approaches have been proposed in the literature, such as greedy heuristics [11, 10, 10] or genetic algorithms [7]. The GRWA problem can be formulated as a single-path multi-commodity multi-layer capacitated network design problem on a complete multi-graph where nodes are network nodes and arcs are optical hops [11]. Solving directly such a formulation can only be performed for networks with a small number of nodes and wavelengths [11, 6, 5]. A first hierarchical optimization procedure was proposed in [5] where the solution of the GRWA problem is decomposed into two stages, request grooming and routing (GR) in the first stage and wavelength assignment (WA) in the second stage. The GR problem corresponds to a single-path multicommodity capacitated network design problem for which branch-and-price [2], branch-and-cut [1] and heuristic [4, 5] algorithms have been developed. However, the hierarchical approach was only applied to instances with less than 500 commodities, while typical optical networks need to route thousands of requests, where routes can be either single hop or multi-hop ones.

We adopt the hierarchical optimization approach of [5] whereby request grooming and routing is decided first and wavelength assignment is done in a second stage. The second stage problem happens to be always feasible in our numerical tests. For the first stage, we develop a rounding heuristic based on a restricted formulation: Ee take advantage of the 2-hop restrictions to model the GR problem in terms of grooming patterns (GP) that correspond to the traffic loading of some basic optical hop patterns. As the number of GPs is exponential, the linear programming (LP) relaxation of the GR formulation is solved by column generation. We also allow for implicit exchanges among traffic granularities in order to enhance stability of the column generation procedure. Last, valid inequalities are used to improve the dual bounds.

The paper is organized as follows. First, in Section 1, we discuss the mathematical formulations of the GRWA, GR and WA problems. Secondly, we deal with their efficient solution in Section 2. Last, computational results are presented in Section 3.

1 Descriptions of the GRWA, GR, and WA Problems

Let graph $G = (V, A)$ represents the physical network where V is the node set and A are the directional fiber links. Each optical fiber link can carry up to W wavelengths (let $\Lambda = \{1, \dots, W\}$), each of which has capacity $U = 192$. Demand D_{sdt} is the number of requests of granularity $t \in T = \{1, 3, 12, 48\}$ on the origin destination pair $(s, d) \in V^2$. Let $D_{sd} = \sum_{t \in T} t\, D_{sdt}$ and $\mathcal{SD} = \{(s, d) \in V^2 : D_{sd} > 0\}$ denote the set of origin and destination pairs of the set of traffic requests. Let $\mathcal{P}_{sd}$ be the set of elementary paths in G from s to d, for $(s, d) \in V^2$ (in practice we restrict this set to the first three shortest paths), and $\mathcal{P}$ be the overall collection of paths: $\mathcal{P} = \bigcup_{(s,d) \in V^2} \mathcal{P}_{sd}$. Each physical path $p \in \mathcal{P}$ is defined by its source node s_p, by its destination node d_p, and by an arc indicator: $\delta^p_a = 1$ if $a \in A$ belongs to the path p. Let $\mathcal{H}$ be the set of optical hops, set which is in one to one correspondence with the set of physical paths: Each optical hop $h \in \mathcal{H}$ is defined by a source s_h and a destination d_h and is associated to a path $p \in \mathcal{P}$ in the physical network represented by an arc indicator vector: $\delta^h_a = 1$ if $a \in A$ is in the path p. Optical hops define a *virtual network*. Let $\mathcal{L}$ be the set of lightpaths with no more than 2 hops. Each $\ell \in \mathcal{L}$ is defined by an optical hop indicator ξ^ℓ_h, for $h \in \mathcal{H}$ (which implicitly defines the physical path), a source node s_ℓ and a destination node d_ℓ.

We present a formulation of the GRWA problem adapted from that of [11], where we use path flow variables to model single-path routing constraint for each (s, d, t) demand. The variables are $y^\lambda_h = 1$ iff optical path $h \in \mathcal{H}$ carries a stream on wavelength $\lambda \in \Lambda$; and x^ℓ_{sdt} = the number of requests (s, d, t)

routed on the lightpath $\ell \in \mathcal{L}_{sd}$.

$$[GRWA] \qquad \min \sum_{h \in \mathcal{H}} \sum_{\lambda \in \Lambda} y_h^\lambda \tag{1}$$

$$\sum_{\ell \in \mathcal{L}_{sd}} x_{sdt}^\ell = D_{sdt} \qquad (s,d) \in \mathcal{SD}, t \in T \tag{2}$$

$$\sum_{h \in \mathcal{H}} \delta_a^h y_h^\lambda \leq 1 \qquad a \in A, \lambda \in \Lambda \tag{3}$$

$$\sum_{(s,d) \in \mathcal{SD}} \sum_{\ell \in \mathcal{L}_{sd}} \sum_{t \in T} \xi_h^\ell \, t \, x_{sdt}^\ell \leq U \sum_{\lambda \in \Lambda} y_h^\lambda \qquad h \in \mathcal{H} \tag{4}$$

$$x_{sdt}^\ell \in I\!N \qquad (s,d) \in \mathcal{SD},\ t \in T,\ \ell \in \mathcal{L}_{sd} \tag{5}$$

$$y_h^\lambda \in \{0,1\} \qquad \lambda \in \Lambda,\ h \in \mathcal{H} \tag{6}$$

The objective function counts the number of optical hops. Constraints (2) insure that each request is supported. Constraints(3) are wavelength clash constraints. Constraints (4) are wavelength transport capacity constraints on each optical hop. One would expect that the model should have one capacity constraint for each $(h, \lambda) \in \mathcal{H} \times \Lambda$. However, because divisibility property of the set $T \cup \{U\}$ (dividing an element of this set by a lower element always give an integer), one can replace the individual knapsack capacity constraints by a surrogate aggregate capacity constraint: Any solution to the aggregate constraint can be trivially disaggregated per wavelength by solving the associated bin packing problem using a best fit decreasing approach which is exact in this case.

We aggregate wavelength assignment variables y_h^λ into

$$y_h = \sum_{\lambda \in \Lambda} y_h^\lambda$$

to reduce the size of the problem and avoid a symmetry drawback of formulation (1-6) where permuting the λ index value defines a symmetric solution. Then, we also apply a surrogate relaxation of constraints (3). The resulting relaxation defines our first stage problem:

$$\min\{\sum_{h \in \mathcal{H}} y_h : \sum_{\ell \in \mathcal{L}} x_{sdt}^\ell = D_{sdt}, \quad (s,d) \in \mathcal{SD},\ t \in T; \sum_{\ell \in \mathcal{L}} \sum_{(s,d) \in \mathcal{SD}} \sum_{t \in T} \delta_h^\ell \, t \, x_{sdt}^\ell \leq U y_h \quad h \in \mathcal{H};$$
$$\sum_{h \in \mathcal{H}} \delta_a^h y_h \leq W \quad a \in A;\ x_{sdt}^\ell \in I\!N, \quad (s,d) \in \mathcal{SD},\ t \in T,\ \ell \in \mathcal{L};\ y_h \in I\!N \quad h \in \mathcal{H}\} \tag{7}$$

called the *grooming and routing* (GR) problem. Given a first stage solution, $(\bar{x}, \bar{y})$, it remains to check whether there exists an associated feasible *wavelength assignment* (WA). This second stage problem takes the form:

$$\{y \in \{0,1\}^{\mathcal{H} \times \Lambda} : \sum_{\lambda \in \Lambda} y_h^\lambda = \bar{y}_h \quad h \in \mathcal{H}_r,\ \sum_{h \in \mathcal{H}} \delta_a^h \, y_h^\lambda \leq 1 \quad a \in A, \lambda \in \Lambda\} \tag{8}$$

If a feasible (optimal) solution of the GR leads to a feasible WA problem, then the 2-stage procedure gives a feasible (optimal) solution of the GRWA (this is always the case in our numerical tests). Otherwise, one could attempt to increase the number of wavelengths and return to the RWA problem, as proposed by [5], or one could implement a Benders decomposition approach (returning an infeasibility cut in RWA and iterating).

2 Solving the Grooming and Routing by Column Generation

Problem (7) involves a large number of requests, optical hops, and lightpaths. Hence, the methods previously proposed in the literature for such single-path multicommodity capacitated network design

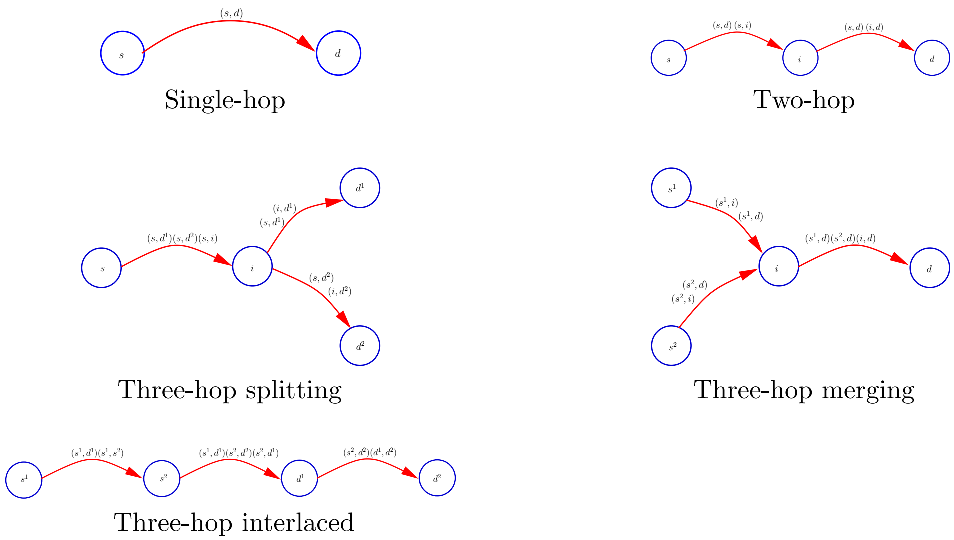

Figure 1: Optical hop configurations that are considered for the support of GP with the potential traffic on each optical hop.

problem cannot be applied to our GR problem. We simplify the problem by restricting the solution set: we consider only a limited set of grooming scenarios, as illustrated in Figure 1, that satisfy the 2-hop constraint. We consider: (i) *single-hop configurations* where the stream only carries a single (s, d)-traffic; (ii) *two-hop configurations* with an intermediate node i where (s, d)-traffic can be groomed with (s, i) and (i, d)-traffic; (iii) *three-hop merging* where (s_1, d)-traffic and (s_2, d)-traffic are groomed with traffic arriving or starting from intermediate node i and (iv) the symmetric *three-hop splitting*; (v) *interlacing configurations* where (s_1, d_1)-traffic, (s_2, d_2)-traffic and intermediate nodes traffic can be groomed together.

A column generation approach is used to compute dual bounds for the restricted model; then we apply a rounding heuristic to obtain primal solutions. To this end, the GR problem is reformulated as a selection of *grooming patterns*: A grooming pattern $\nabla \in \mathcal{GP}$ is defined for one of the above grooming scenarios by specifying the traffic vector x^∇: x^∇_{sdt} gives the number of requests (s, d, t) that are routed over ∇, and an optical hop indicator δ^∇_h: $\delta^\nabla_h = 1$ iff ∇ uses optical hop $h \in \mathcal{H}$. Then GR problem can be reformulated in terms of integer variables μ_∇ that count the number of times the *grooming pattern* ∇ is used:

$$[GR_{GP}] \qquad \min \sum_{\nabla \in \mathcal{GP}} \sum_{h \in \mathcal{H}} \delta^\nabla_h \mu_\nabla \tag{9}$$

$$\sum_{\nabla \in \mathcal{GP}} x^\nabla_{sdt}\, \mu_\nabla = D_{sdt} \qquad (s, d) \in \mathcal{SD},\ t \in T \tag{10}$$

$$\sum_{\nabla \in \mathcal{GP}} \sum_{h \in \mathcal{H}} \delta^h_a\, \delta^\nabla_h\, \mu_\nabla \leq W \qquad a \in A \tag{11}$$

$$\mu_\nabla \in I\!N \qquad \nabla \in \mathcal{GP} \tag{12}$$

In solving the LP relaxation of $[GR_{GP}]$, we generate *grooming patterns* dynamically. The minimum reduced cost is obtained by solving the following pricing problem:

$$\min_{\nabla \in \mathcal{GP}} \{ \sum_{h \in \mathcal{H}} (1 - \sum_{a \in A} \sigma_a\, \delta^h_a)\, \delta^\nabla_h - \sum_{(s,d) \in \mathcal{SD}} \sum_{t \in T} \pi_{sdt} x^\nabla_{sdt} \},$$

where π_{sdt} and σ_a are dual variables associated with constraints (10) and (11), respectively. Pricing is done by enumerating the optical hop configurations that are depicted in Figure 1, for all possible sets of extreme and intermediate nodes.

For each configuration, we solve a traffic loading problem. Consider the loading problem on a single-hop for a given (s, d) pair. For any integer $u \in \{1, \ldots, U\}$, let $X^{sd}_{=u} = \{x_{sdt} \in I\!N^{|T|} : \sum_{t \in T} t\, x_{sdt} = u\}$ and $X^{sd}_{\leq u} = \{x_{sdt} \in I\!N^{|T|} : \sum_{t \in T} t\, x_{sdt} \leq u\}$. The core sub-problem for loading is:

$$z^{sd}_{\text{LOAD}}(u) = \max \left\{ \sum_{t \in T} \pi_{sdt}\, x_{sdt} \,:\, x \in X^{sd}_{\leq u} \right\} = \max_{u' : 0 \leq u' \leq u} \left\{ \sum_{t \in T} \pi_{sdt}\, x_{sdt} \,:\, x \in X^{sd}_{=u'} \right\} \qquad (13)$$

To compute $z^{sd}_{\text{LOAD}}(u)$ for $u \in [1, U]$, we first determine the cost of every solution $x \in X^{sd}_{=u} \neq \emptyset$, which can be done in $O(U|T|)$, and then z^{sd}_{LOAD} can be get in $O(U)$. With this approach we can compute the best reduced cost for each GP, we can compute the best reduced grooming pattern in $O(1)$ for single hops, in $O(U)$ for two hops, and in $O(U^2)$ for three hops by iterating on all traffic partitions along the optical hops.

The dual bound given by the LP solution of the restricted model $[G_{GP}]$ is further improved by adding cuts dynamically. We use *cut-set* based inequalities that set a lower bound on the number of optical hops incoming or outgoing any node, called port cuts. We also use inequalities based on the minimal number of lightpaths that carry (s, d) demand, called (s, d)-cuts. The same idea is applied to lightpaths carrying (s, d, t) traffic, called (s, d, t)-cuts. The last inequalities enforce upper bounds on the number of lightpaths with high traffic load, and are called load cuts. These inequalities do not modify the pricing procedure.

The primal bounds are obtained by rounding the LP solution. At each iteration of our rounding procedure, the fractional variables with a value larger than 1 are set to their closest lower integer value. Then, the column with the largest fractional μ_∇ is rounded up (a variant is to round up the column with the largest demand covered $\sum t\, x^\nabla_{sdt}$ or largest ratio of demand covered over cost $(\sum t\, x^\nabla_{sdt})/(\sum \delta^\nabla_h)$ among those with largest μ_∇ value).

3 Computational Results and Conclusion

We tested our hierarchical optimization procedure on the NSF (14 nodes and 21 links) and the EON (20 nodes and 39 links) networks. For each network, we define four traffic instances, NSFi and EONi, $i = 1, 2, 3, 4$, that has traffic between each pair of nodes. The first one is randomly generated with a total traffic for each (s, d) in $[48, 2 \times 192]$. The second one (resp. the third one) is with an overall traffic for each (s, d) lower (resp. larger) than OC$-$192, the fourth one follows usual backbone granularity distribution, with a large number of requests with low granularity. The number of wavelengths is set to the minimum number of wavelengths needed to insure that a GRWA solution using only single-hop lightpaths is feasible: It ranges between 13 to 67 in our tests.

Table 1 reports: *(i)* the number of requests in each instance ; the gaps between the primal and dual bounds (assuming the primal bound corresponds to the integer solution obtained with the rounding heuristic) when *(ii)* using no cuts, *(iii)* adding the port cuts, *(iv)* adding the (s, d) cuts, *(v)* adding the (s, d, t) cuts, *(vi)* adding load cuts, *(vii)* adding all cuts ; and the same gaps assuming we use the rounding procedure in order to get an integer solution when *(viii)* using no cuts, *(ix)* using all cuts. Average values are given in the last row.

The results show that using a single class of cut is not sufficient to get a good improvement of the dual bounds. The larger improvement when using a single class of cut is when we use (s, d) cuts, however when using all the cuts simultaneously, the improvement is twice better. The primal bounds are also better when using all cut classes, as a fractional μ_∇ is a more accurate indicator of which column should be selected in the integer solution.

We have presented a hierarchical optimization procedure for a network design problem where the number of commodities is very large. To narrow the solution space, we have restrict the way traffic

		Optimality gaps with respect to cut classes						Rounding	
Instances	# Requests	No cuts	Port	(s, d)	(s, d, t)	Load	All	No cuts	All cuts
NSF1	1,667	12.89	12.89	12.31	10.61	11.17	9.50	10.00	9.50
NSF2	1,332	6.21	6.21	6.21	5.56	6.21	5.56	9.88	5.56
NSF3	1,949	20.00	18.89	13.16	14.67	16.22	8.86	9.28	8.86
NSF4	600,303	7.05	5.80	7.05	7.05	6.16	5.63	5.96	5.63
EON1	6,639	14.67	14.67	13.9	13.14	12.39	10.92	12.88	10.92
EON2	2,292	11.11	11.11	11.11	9.85	10.27	9.43	9.81	9.43
EON3	3,667	23.76	23.19	11.21	20.09	20.64	10.27	12.16	10.27
EON4	120,676	7.59	6.65	7.50	7.50	6.28	6.18	7.58	6.18
average	78,453.75	12.62	12.13	10.04	10.77	10.88	8.03	9.69	8.03

Table 1: Comparison of the impact of the different cut classes on the optimality gap.

can be groomed on the same optical hop. The first stage is solved by a rounding heuristic based on a decomposition formulation solved by column generation. This method produces feasible solution with optimality gap that is lower that those reported in recent research [4]. In addition, the hierarchical approach allows solving significantly larger instances than the largest solved ones in the literature, with, in addition, an indication (i.e., optimality gap) of the solution accuracy.

References

[1] A. Atamtürk and D. Rajan, *On splittable and unsplittable capacitated network design arc-set polyhedra*, Mathematical Programming **92** (2002) pp. 315–333.

[2] Cy. Barnhart, C.A. Hane, and P.H. Vance, *Using branch-and-price-and-cut to solve origin-destination integer multicommodity flow problems*, Operations Research **48** (2000) pp. 318–326.

[3] Richard S. Barr, M. Scott Kingsley, and Raymond A. Patterson, *Grooming telecommunications networks: Optimization models and methods*, technical report 05-EMIS-03, Southern Methodist University, Department of Engineering Management, Information, and Systems, Dallas, TX 75275 (2005).

[4] E. Bartolini and A. Mingozzi, *Algorithms for the non-bifurcated network design problem*, Journal of Heuristics (2008).

[5] J.Q. Hu and B. Leida, *Traffic grooming, routing and wavelength assignment in optical WDM mesh networks*, In IEEE Infocom (2004).

[6] V.R. Konda and T.Y. Chow. *Algorithm for traffic grooming in optical networks to minimize the number of transceivers*, IEEE Workshop Hight Performance Switching and Rooting (2001) pp. 218–221.

[7] C. Lee and E.K. Park. *A multistart randomized greedy algorithm for traffic grooming on mesh logical topologies*, In IEEE International Conference on Systems, Man and Cybernetics **7** (2002) pp. 7–12.

[8] B. Mukherjee. *Optical WDM Network*, Springer (2006).

[9] H. Yen and S. Lee. *Qos aware traffic grooming and integrated routing on ip over wdm networks*, Photonic Network Communications **14** (2007) pp. 1–10.

[10] H. Zhu, H. Zang, K. Zhu, and B. Mukherjee. *A novel generic graph model for traffic grooming in heterogeneous WDM mesh networks*, IEEE/ACM Trans. on Networking **11** (2003) pp. 285–299.

[11] K. Zhu and B. Mukherjee. *Traffic grooming in an optical WDM mesh network*, IEEE Journal on Selected Areas in Communications **20** (2002) pp. 122–133.