Network Design, Survivable Routing and Channel Assignment in WDM and TDM Optical Networks

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Abstract

In this paper WDM and TDM networks design problems are considered. The problem statement includes elements of network design, routing traffic demands with different protection types, disjointness types of working and protection paths of demands, existence of diversity groups of demands, and channel assignment. For this general problem, we propose an exact algorithm based on combination of branch-and-bound approach and integer linear programming formulation.

Keywords: *Network design; Survivability; Routing and channel assignment; Wavelength division multiplexing (WDM); Time division multiplexing (TDM); Optical network; Integer linear programming (ILP).*

1 Introduction

This work is prepared in the framework of the project on elaborating an integrated network planning system OptiX MDS Planner of the company `VPIsystems` (`www.vpisystems.com`) which is a provider of network planning software and services for the global telecommunications industry.

OptiX MDS 6600 Planner (MDS 6600 for short) provides algorithms in intuitive software to support swift and effective design of metro network scenarios. It supports optical transport network design based on WDM, TDM or SONET and covers various network topologies.

We present the results of our work on the exact algorithms for WDM and TDM networks design. Optical networks using wavelength division multiplexing (WDM) technique is a powerful tool to provide the increasing user requirements for modern communication [4, 5, 8]. In such networks large capacity of optical fiber can be used by means of transmitting multiple signals on different wavelengths (channels). Time division multiplexing (TDM) is a method of transforming multiple data streams into a single signal by separating the signal into many segments, each having a very short duration. Each individual data stream is assembled again at the receiving end by means of the timing. The main difference between TDM and WDM techniques is that the former simply manipulates data streams, while the latter manipulates channels.

We consider an optimization problem that can be formulated as follows. Given: 1) physical topology (or *fiber topology* or *fiber layer*); and 2) a list of *traffic demands* of different *protection types*. Find an optimal: 1) network (by installing multiple *facilities* on fibers); 2) routing for the traffic demands over the network; and 3) channel assignment for the demands. A common optimization objective of the problem is to minimize the total cost of the network (the sum of costs of all facilities).

The similar problems were considered, for instance, in [1],[2],[3],[6],[7].

The authors' task consisted in the development and implementation of a heuristic algorithm for the problem in general case and an exact algorithm for relatively small-size problems. In this paper we consider the second task. Actually the whole problem could be decomposed into two stages:

Rt) routing of demands on fiber layer;

IA) facilities installation and channel assignment.

Every stage taken separately is known to be NP-hard. The most accepted way to solve the problem of that kind exactly is using mixed integer programming (MIP). However, in our case the problem contains too many requirements and constraints. Thus the MIP formulation for the problem contains too many variables and constraints even for relatively small networks. The test showed that for our problem this approach is rather ineffective.

Our approach consists in using branch-and-bound method for **Rt**) and solving ILP for **IA**) in leaves of the branch-and-bound tree.

This approach proved itself to be quite effective. Tests showed that our algorithm is able to exactly solve the problems with up to 15 traffic sites, 25 fibers and 25 demands in reasonable time. In the overwhelming majority of test cases with large numbers of feasible solutions the algorithm solves the ILP problem for at most 0.15 percent of feasible solutions.

2 Problems statements

Consider the WDM networks first. The input of the problem consists of the following parts:

1) $G = (V, E, w)$ is a weighted graph representing the *fiber topology* (or *fiber layer*) of the optical network. It consists of the node set V (the traffic sites), the set of fibers E, and the non-negative weight function $w : E \to \mathbb{R}_+$ that assigns the weight $w(e) \geq 0$ to each fiber e of G.

2) The family $D = \{(u, v) : u, v \in V,\ u \neq v\}$ of point-to-point unit wavelength *demands*. Each demand has one of four protection types: unprotected, 1+1-protected client, 1+1-protected network, share protected. The sets of unprotected, 1+1-protected client, 1+1-protected network, share protected demands are denoted by $D_{un}, D_{cl}, D_{net}, D_{sh}$, respectively.

3) The family $D_{ex} \subseteq D$ of *existing demands* of two types. For every demand of the first type its *existing routing* on fiber layer is given and the numbers of channels which it should use are specified. For existing demands of the second type only their routing on fiber layer is given.

4) The set $DG = \{DG_1, ..., DG_r : DG_i \subseteq D\}$ of *diversity groups* of demands.

5) The capacity N of facilities.

The problem consists in finding a minimal-weight multigraph $G_{lo} = (V, E_{lo})$ representing a facility topology, the routing in G_{lo} and channel assignment for every demand from D according to its protection type. The elements of E_{lo} are called *facilities*. Each facility $e = uv \in E_{lo}$ joins two vertices only if they are joined by some fiber $f = uv \in E$. In this case we say that e is *installed* on f and write $f = \psi(e)$. The weight of the facility f is equal to the weight of the corresponding fiber plus *termination cost* c_t. For every fiber $e \in E$ the number of facilities installed on e should not exceed the given value M_e.

For every unprotected demand d the *working path* T_w^d in G_{lo} should be specified and for every protected demand d the pair (T_w^d, T_p^d) of link- or node-disjoint *on the fiber layer* paths in G_{lo} should be specified. T_w^d and T_p^d are *working and protection paths*, respectively; we say that T_w^d and T_p^d are link- or node-disjoint on the fiber layer, if the paths $P_w^d = \psi(T_w^d)$ and $P_p^d = \psi(T_p^d)$ are link- or node-disjoint in G. For every diversity group $DG_i \in DG$ and every $d_1, d_2 \in DG_i$ the paths $P_w^{d_1}$ and $P_w^{d_2}$ should be link- or node-disjoint. The type of disjointness in both cases is set as an input. For every existing demand $d \in D_{ex}$ the paths P_w^d and P_p^d should coincide with its existing routing.

For every demand d the same channels $\lambda_w^d, \lambda_p^d \in \{1, ..., N\}$ on all facilities of its working and protection path, respectively, should be assigned (*channel continuity requirement*). For 1+1-protected network demands $\lambda_w^d = \lambda_p^d$. Different demands which are routed along the same facility should use different channels on the corresponding paths, with the exception of share protected demands. In the last case the equality $\lambda_p^{d_1} = \lambda_p^{d_2}$ for two share protected demands d_1, d_2 with facility-intersecting protection paths is possible, if these demands have disjoint on the fiber layer working paths.

The problem statement for TDM network design problem differs from the statement for WDM networks in the following aspects:

1) The sizes of demands (number of channels that should be occupied by the demand) could be arbitrary. Denote by $sz(d)$ the size of the demand $d \in D$.

2) There are t different types of facilities with capacities $cap(1), ..., cap(t)$ (instead of one type with capacity N for WDM). Every fiber $e \in E$ has t weights $w_1(e), ..., w_t(e)$ and the cost of installing a facility of type i on fiber e equals to $u_i(e) = w_i(e) + c_t$.

3) each demand can occupy channels with different numbers in different facilities, but in each facility it should use the channels with consecutive numbers (*channels adjacency* requirement)

4) there is only one type of 1+1 protected demands - client demands. There are two types of share protected demands: type A and type B. Share protected demand could share channels only with demands of the same type. Moreover, demands with type A could share channels only with demands of the same size. For every share protected demand d, the sets of channels, simultaneously used by d and another share protected demands, should be ordered by inclusion.

3 Algorithm for TDM networks

Let us start with the main idea of the branch-and-bound algorithm for the stage **Rt**). It is based on the search of all routing assignments on fiber layer for the demands, the pruning is realized either on the base of the generated lower bound or when solutions a priori do not satisfy the disjointness requirements.

The lower bound is calculated in the following way. Suppose in some iteration of the algorithm we are located in the node of the branch-and-bound tree which is defined by assigning some fibers to the working or protection paths of the demands. Let E^y be the set of fibers which are assigning to the working or protection paths of the demands. For $e \in E^y$ let $L_e = \{d \in D : e \in P_w^d \cup P_p^d\}$, $S_e = \{d \in L_e \cap D_{sh} : e \in P_p^d\}$, S_e^A and S_e^B be the sets of demands from S_e having types A and B, respectively.

For $f \in E^y \setminus \{e\}$ and $d \in S_e$ let $v_{e,f}^d = 1$, if $f \in P_w^d$ and $v_{e,f}^d = 0$, otherwise. Define the following parameters.

$\xi_e^A := \max\limits_{f \in E^y \setminus \{e\}} \left\{ \sum\limits_{d \in S_e^A} sz(d) v_{e,f}^d \right\}$;

$\xi_e^B := \max\limits_{f \in E^y \setminus \{e\}} \left\{ \sum\limits_{d \in S_e^B} sz(d) v_{e,f}^d \right\}$;

$y_e := \sum\limits_{d \in L_e \setminus S_e} sz(d) + \xi_e^A + \xi_e^B$.

Let $p, q \in \mathbb{Z}_+^t$, $y \in \mathbb{Z}_+$. Denote by $AC(p, q, y)$ a solution of the following auxiliary problem:

$$AC(p, q, y) = \sum_{i=1}^{t} p_i x_i \to \min$$

$$\sum_{i=1}^{t} q_i x_i \geq y, \quad x \in \mathbb{Z}_+^t \tag{1}$$

Let $z_e = AC(u, cap, y_e)$ and $t_e = AC(\mathbf{1}, cap, y_e)$, where $\mathbf{1} = \{1\}^t$. It is easy to see that z_e and t_e are lower bounds for the minimal cost and minimal number of facilities needed to install on e in order to assign channels to all demands from L_e. Therefore the pruning is realized if $t_e > M_e$ or $\sum\limits_{e \in E^y} z_e$ is greater or equal to the current record. The values of z_e and t_e could be easily computed using dynamic programming.

It is clear that the subproblem **IA**) in the case of TDM network could be solved separately for every fiber $e \in E$. Subject to the existence of share protected demands in the input our problem could be reduced to different problems. Those problems are rather different by the sizes of their ILP formulations and therefore in order to speed up the performance of our algorithm we used three different models for channel assignment and adding multiple facilities.

If there is no share protected demands routed along e, then the corresponding problem is the modification of the classical bin packing problem with different types of bins available:

$$\sum_{f \in B_e} u_f(e) y_f \to \min$$

$$\sum_{i \in L_e} sz(i) x_i^f \leq cap(f) y_f, \quad f \in B_e$$
$$\sum_{f \in B_e} x_i^b = 1, \quad i \in L_e$$
$$\sum_{f \in B_e} y_f \leq M_e \tag{2}$$

Here B_e is the set of facilities of different types, which could be installed on e, $u_f(e)$ and $cap(f)$ are cost and capacity of the facility f; $y_f = 1$ if the facility $f \in B_e$ is installed on e and 0, otherwise; $x_i^f = 1$ if the demand $i \in L_e$ is routed through the facility $f \in B_e$ and 0 otherwise.

If we have share protected demands in the input, but there is no existing share protected demands of the first type, then the problem is the further modification of the bin packing problem, where some items could share space in the bins. We consider this problem in the following reformulation.

Let $H = (V(H), E(H), w)$ be a vertex-weighted graph and φ be its proper k-coloring. The *weight* $w(\varphi)$ *of the coloring* φ is defined as $w(\varphi) = \sum_{i=1}^{k} max\{w(v) : \varphi(v) = i\}$. The *weighted chromatic number* $\chi_w(H)$ is the minimum weight of a proper coloring of the graph H.

Consider the following decomposition problem: given two sequences $(c_1, ..., c_r)$ and $(u_1, ..., u_r)$, find the partition $H = H_1 \cup ... \cup H_k$ into vertex-disjoint subgraphs such that $\chi_w(H_i) \leq c_{j_i}$ and $\sum_{i=1}^{k} u_{j_i}$ is minimal.

Let H be a graph with vertex set L_e, two vertices of H are adjacent if and only if they could not share a channel in facilities installed on e. Then the problem of channel assignment and facilities installation on e is equivalent to the graph decomposition problem above. Note that it could be shown that the converse proposition is also true.

And finally if we have existing share protected demands of the first type in the input then the problem could not be reduced to the modification of the bin packing problem and therefore we developed its straight ILP formulation.

We solve the listed above problems by solving their ILP formulations. For the lack of space in this abstract the formulations are omitted.

4 Algorithm for WDM networks

The branch-and-bound stage of the algorithm for WDM networks is similar to the corresponding stage for TDM network described above. In order to calculate lower bound we relax the channel continuity requirement. For $e \in E^y$ let H_e be the intersection graph of working paths of demands from S_e, z_e and

t_e have the same sense as above. Then $t_e = \sum_{e \in E^y} \left\lceil \frac{|L_e| - |S_e| + \underline{\chi}_e}{N} \right\rceil$ and $z_e = (w(e) + c_t)t_e$. Here $\underline{\chi}_e$ is a lower bound for the chromatic number of H_e, it is calculated as $\underline{\chi}_e = max\{\omega_e, -\lfloor -p/\lfloor \frac{p^2-2q}{p} \rfloor (1 - \{\frac{p^2-2q}{p}\}/(1 + \lfloor \frac{p^2-2q}{p} \rfloor)) \rfloor\}$, where $p = |S_e|$, $q = |E(H_e)|$, $\{x\}$ is the fractional part of the rational number x, ω_e is the size of some maximal clique of H_e, which is calculated by heuristics for maximum clique problem. The second expression above is well-known lower bound for the chromatic number [10].

For the subproblem **IA**) for WDM networks the following ILP model is used.

- For a protected demand $d \in D$, let $Q_d = P_w^d \cup P_p^d$ and for an unprotected demand $d \in D$, let $Q_d = P_w^d$.
- $E_{lo} := \bigcup_{d \in D} E(Q_d)$, $E_{lo}^w := \bigcup_{d \in D} E(P_w^d)$, $E_{lo}^{sh} := \bigcup_{d \in D_{sh}} E(P_p^d)$.
- For a demand $d = (u, v) \in D$, let

$$F_d = \begin{cases} E(Q_d) \setminus \{ux \in E(P_w^d), vs \in E(P_p^d)\}, & \text{if } d \in D_{cl} \cup D_{sh}; \\ E(Q_d) \setminus \{ux \in E(P_p^d)\}, & \text{if } d \in D_{net}; \\ E(Q_d) \setminus \{xv \in E(P_w^d)\}, & \text{if } d \in D_{un}. \end{cases}$$

- $next_d(f)$ is the next after f link of F_d in the direction from u to v.
- For a demand $d \in D_{ex}$ and a link $e \in Q_d$ let $AV_d(e) := \{i, i + N, \ldots, i + \lceil \frac{R_e - i}{N} \rceil N\}$ where i is the number of channel which d should use in e.
- For a demand $d \in D \setminus D_{ex}$ of the first type and a link $e \in Q_d$ let $AV_d(e) := \{1, 2, \ldots, R_e\}$.

  The set $AV_d(e)$ represents the channels available for the demand d.
- For links $e \in E_{lo}^{sh}$ and $f \in E_{lo}^w$, let

$$L_e^f = \{d \in L_e \,:\, d \notin D_{sh} \text{ or } f \in P_w^d \text{ or } (d \in D_{sh} \text{ and } e \in P_w^d)\}.$$

- For links $e \in E_{lo}^{sh}$, $f \in E_{lo}^w$ and integer i, let $L_e^f(i) = \{d \in L_e^f \,:\, i \in AV_d(e)\}$.
- For link $e \in E_{lo} \setminus E_{lo}^{sh}$ and integer i, let $L_e(i) = \{d \in L_e \,:\, i \in AV_d(e)\}$.

We introduce the following variables:

- n_f be the number of multiple facilities on f (i.e. the multiplicity of f), $0 \le n_f \le M_f$;
- $\lambda_d^{i,e}$ equals 1 if the demand d uses the channel $(i-1)\text{mod}N + 1$ on $\lceil \frac{i}{N} \rceil$th multiple facility of e; equals 0 otherwise ($d \in D$, $e \in Q_d$, $i \in AV_d(e)$);
- c_d^e is the auxiliary variable ($d \in D$, $e \in F_d$);

$$\sum_{f \in E_{lo}} (w(f) + c_t) n_f \to \min$$

$$\sum_{i \in AV_d(e)} \lambda_d^{i,e} = 1, \quad d \in D, \quad e \in Q_d; \tag{1}$$

$$\sum_{d \in L_e^f(i)} \lambda_d^{i,e} \leq 1, \quad i \in \bigcup_{d \in L_e^f} AV_d(e), \quad e \in E_{lo}^{sh}, \quad f \in E_{lo}^{w}; \tag{2}$$

$$N n_e - \sum_{i \in AV_d(e)} i \lambda_d^{i,e} \geq 0, \quad e \in E_{lo}, \quad d \in L_e; \tag{3}$$

$$\sum_{i \in AV_d(e)} i \lambda_d^{i,e} - \sum_{i \in AV_d(e)} i \lambda_d^{i,next_d(e)} - c_d^e N = 0, \quad d \in D, \quad e \in F_d; \tag{4}$$

$$\sum_{d \in L_e(i)} \lambda_d^{i,e} \leq 1, \quad i \in \bigcup_{d \in L_e} AV_d(e), \quad e \in E_{lo} \setminus E_{lo}^{sh}. \tag{5}$$

The parameter R_e is calculated by the formula $R_e := min\{N \cdot M_e, \chi\}$, where χ is the number of colors used by some heuristics which finds a coloring of the intersection graph of working and protection paths of demands from $D_{cl} \cup D_{sh}$ and the edge sets Q_d of demands $d \in D_{net} \cup D_{un}$. In our project we used the simple heuristic coloring algorithm from [9].

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