Exact Approaches to the Single Source Network Loading Problem

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Abstract

We consider the problem of deploying a broadband telecommunications system that lays optical fiber cable from a central office to a number of end-customers. We are dealing with a capacitated network design problem that requires an installation of fiber optic cables with sufficient capacity to carry the traffic from the central office to the end-customers. This is the single-source variant of the *network loading problem*.

In this paper we propose a new compact disaggregated mixed integer programming (MIP) formulation for the problem. We project out flow variables by introducing Benders' cuts that are further strengthened by additional inequalities. The whole procedure is incorporated into a branch-and-cut framework.

In our computational experiments we do see improved gaps, when deploying Benders' inequalities, at least in some cases. Reducing the computational cost for separating the Benders' cuts and improving our primal heuristic to help closing the gap faster is the main focus of our ongoing effort.

Keywords: *Network Loading Problem, Branch-and-Cut, Benders Inequalities*

1 Introduction

We consider the problem of deploying a broadband telecommunications system that lays optical fiber cable from a *central office* to a number of *end-customers*. In case of the *fiber to the home* technology, the end-customers represent houses, whereas when deploying *fiber to the curb* technology, the end-customers are usually multiplexor devices. In both cases, we are dealing with a capacitated network design problem that requires an installation of fiber optic cables with sufficient capacity to carry the traffic from the central office to the end-customers. We start with a network without capacities, or with some pre-installed capacities, and search for the installation of at most one cable type per link at minimum total cost.

In this paper we provide a theoretical and computational comparison of several mixed integer programming (MIP) formulations for the problem. We also propose a branch-and-cut approach based on the cut-set inequalities, extended with strengthening Benders' inequalities.

In our computational experiments we do see improved gaps, when deploying Benders' inequalities, at least in some cases. Reducing the computational cost for separating the Benders' cuts and improving our primal heuristic to help closing the gap faster is the main focus of our ongoing effort.

Problem Definition We are given an undirected, connected graph $G^u(V,E)$ with a root node $r \in V$, with edge lengths $l_{ij} \in \mathbb{R}_{>0}$ $\forall \{i,j\} \in E$, and a set of customers $D \subseteq V \setminus \{r\}$. To each customer, a non-negative demand $d_k \in \mathbb{R}_{>0}$ is assigned. Furthermore, on every edge $e \in E$ we are able to install different cable types (modules) $\mathcal{N}_e = \{n_1, n_2, \ldots, n_{|\mathcal{N}_e|}\}$ with capacities $u_{e,n} \in \mathbb{R}_{>0}$, $1 \leq n \leq |\mathcal{N}_e|$ and costs $c_{e,n} \in \mathbb{R}_{>0}$, $1 \leq n \leq |\mathcal{N}_e|$.

The *Network Loading Problem* (NLP) asks for an installation of at most one cable on each edge at minimum total cost, so that the demands can be simultaneously routed from the root to all customers without exceeding installed edge capacities. The problem we are concentrating on is therefore a single-source multiple-sink routing and a link capacity assignment network design problem.

Previous Work Due to its importance in telecommunications, transportation, computer and energy supply networks, NLP has been widely studied in literature. Many authors consider the problem in which a routing from multiple sources to multiple sinks is required. Polyhedral structures of the general NLP and its variants are studied in [13, 10, 2, 9, 14, 1]. Benders' decomposition approaches have been studied as well: for the multiple-source multiple-sink NLP, an exact algorithm was given in [6], whereas in [3] the objective function is extended by flow dependent costs. In [7] the authors look into speeding up Benders' decomposition. The single-source variant has been studied in [12, 11]. In terms of approximation algorithm (NLP) is also known as the single-sink buy-at-bulk network design problem. The best approximation ratio of 76.5 is obtained in [8].

Observe that we allow the flow between the root and some customer to be split apart. Therefore, we are speaking of a non-bifurcated problem formulation.

2 MIP Formulations

Typically, the NLP is modeled using compact flow-based MIP formulations, involving binary design variables and continuous flow variables. When disaggregating flow variables, we face the problem of trading-off between the quality of lower bounds and the size of the underlying linear program.

In this section, we first propose a new disaggregated flow-based formulation. Similar ideas for related problems have been considered in [5, 4]. We then show how to project out flow variables by introducing Benders inequalities. Finally, we recall the cut-set based ILP formulation whose violated inequalities can be separated in polynomial time, as far as single-source NLP is concerned.

It is well known that the MIP formulations of NLP and related problems on bidirected graphs provide better lower bounds than their undirected counterparts. Therefore, we work with directed graphs and transform our input graph $G^u = (V, E)$ into a directed graph $G = (V, A)$ such that $A = \{(i, j) \mid \{i, j\} \in E, j \neq r\}$. The costs of the cable types on the arcs remain symmetric, i.e. $c_{ij,n} = c_{ji,n} = c_{\{i,j\},n}$, $n \in \mathcal{N}_{\{i,j\}}$. It easy to see that no optimal solution of the single-source NLP contains a directed cycle, and therefore, the capacity of every arc will be used in exactly one direction and we can set $u_{ij,n} = u_{ji,n} = u_{\{i,j\},n}$, $n \in \mathcal{N}_{\{i,j\}}$.

2.1 Multi-Cabletype Multi-Commodity Flow Formulation MMCF

To model the problem, we introduce binary variables $x_{ij,n} \in \{0, 1\}$ that are one iff the cable type $n \in \mathcal{N}_{ij}$ is installed on the arc (i, j). To model the feasible routing of the traffic between the root r and any single customer $k \in D$, we use the disaggregated flow variables $f^k_{ij,n}$ that define the *fraction of flow* of commodity $k \in D$, routed on the arc (i, j) using the cable type $n \in \mathcal{N}_{ij}$. The MIP model then reads as follows:

$$\text{(MMCF)} \qquad \min \sum_{(i,j)\in A} l_{ij} \sum_{n\in\mathcal{N}_{ij}} c_{ij,n} x_{ij,n} \tag{1}$$

$$\text{s.t.} \qquad \sum_{(i,j)\in A} \sum_{n\in\mathcal{N}_{ij}} f^k_{ij,n} - \sum_{(j,i)\in A} \sum_{n\in\mathcal{N}_{ji}} f^k_{ji,n} = \begin{cases} -1, & i = k \\ 1, & i = r \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in V,\ \forall k \in D \tag{2}$$

$$\sum_{k\in D} f^k_{ij,n} d_k \leq x_{ij,n} u_{ij,n} \qquad \forall (i,j) \in A\ \forall n \in \mathcal{N}_{ij} \tag{3}$$

$$\sum_{n\in\mathcal{N}_{ij}} x_{ij,n} \leq 1 \qquad \forall (i,j) \in A \tag{4}$$

$$0 \leq f^k_{ij,n} \leq x_{ij,n} \qquad \forall (i,j) \in A\ \forall k \in D\ \forall n \in \mathcal{N}_{ij} \tag{5}$$

$$x_{ij,n} \in \{0,1\} \qquad \forall (i,j) \in A\ \forall n \in \mathcal{N}_{ij} \tag{6}$$

The *flow conservation constraints* (2) ensure the feasibility of the flow sent from r to every customer $k \in D$. The *coupling constraints* (5) take care that, if there is any flow in the cable-type n on the arc (i, j), that cable type has to be in the solution. Inequalities (3) are the *capacity constraints*, i.e. the total flow in cable type n on arc (i, j) must not exceed the capacity of the given cable type n. As cable types are given explicitly, the *disjunction constraints* (4) ensure that on every arc at most one cable type is installed.

This model contains $O(|A| \cdot |\mathcal{N}| \cdot |D|)$ constraints and $O(|A| \cdot |\mathcal{N}| \cdot |D|)$ variables ($|\mathcal{N}| = \max_{e \in E} |\mathcal{N}_e|$), and it is very unlikely that even the most sophisticated MIP solvers may solve instances of moderate size using (MMCF) formulation. Our preliminary computational experiments with (MMCF) also confirmed this. However, (MMCF) provides much stronger lower bounds than its aggregated counterparts. Therefore we propose to project out the flow variables and to introduce Benders inequalities instead, keeping the quality of lower bounds, and even improving them by rounding techniques.

It is easy to see that the explicit cost model can be equivalently modelled by using the *incremental cost* (or *expansion steps*) model (see [13]).

2.2 Benders' Decomposition for MMCF

Let the master problem be the one given by the objective function (1) subject to constraints (4) and (6). A solution x' of the master problem defines a feasible solution for the LP-relaxation of the (MMCF) iff there exist flow variables f satisfying the linear system of inequalities given by (2), (3) and (5), where $x = x'$.

Farkas' lemma states that a linear system of equations $\{Ax \leq b : x \geq 0\}$ has a solution iff $u^T b \geq 0$ for all $u \geq 0$ such that $u^T A \geq 0$. To apply Farkas' lemma to the system (2), (3), (5), we define dual variables $\alpha_i^k, \beta_{ij,n}^k$ and $\gamma_{ij,n}$ associated to (2), (5) and (3) respectively. The polyhedron defined by this system is non-empty iff

$$\sum_{k \in D} (\alpha_r^k - \alpha_k^k) + \sum_{(i,j) \in A} \sum_{n \in N_{ij}} \left(\sum_{k \in D} \beta_{ij,n}^k + u_{ij,n} \gamma_{ij,n} \right) x'_{ij,n} \geq 0. \tag{7}$$

for all (α, β, γ), such that

$$\alpha_i^k - \alpha_j^k + \beta_{ij,n}^k + d_k \gamma_{ij,n} \geq 0 \quad \forall (i,j) \in A,\ \forall k \in D,\ \forall n \in \mathcal{N}_{ij} \tag{8}$$

$$(\alpha, \beta, \gamma) \geq 0 \tag{9}$$

Inequalities (7) are called Benders' cuts. Violated Benders' inequalities can be separated within a branch-and-cut framework as follows. Given a fractional solution x' we minimize the left hand side of (7), s.t. (8)-(9). If this subproblem (SUB) is unbounded, there is an unboundedness direction $(\alpha', \beta', \gamma')$ that defines a violated Benders' cut that can be added to the master:

$$\sum_{(i,j) \in A} \sum_{n \in N_{ij}} \left(\sum_{k \in D} \beta'^k_{ij,n} + u_{ij,n} \gamma'_{ij,n} \right) x_{ij,n} \geq \sum_{k \in D} (\alpha'^k_k - \alpha'^k_r) \tag{10}$$

We can round down the coefficient of variable $x_{ij,n}$ to $\min\left(\sum_{k \in D} \beta'^k_{ij,n} + u_{ij,n}\gamma'_{ij,n}, \sum_{k \in D}(\alpha'^k_k - \alpha'^k_r)\right)$. If the subproblem (SUB) is bounded, it means that we can not find a violated Benders' cut.

2.3 Strenghtening Benders Inequalities by Metric Inequalities

It has been shown in [3] that any Benders' inequality associated to a non-extreme ray can be strengthened to a *metric inequality* as follows. For any fixed $(\alpha', \beta', \gamma')$ satisfying (8)-(9), one can look for $(\alpha(\beta', \gamma'), \beta', \gamma')$ that maximizes the right-hand side of (10):

$$\text{(SPDUAL)} \qquad \max \sum_{k \in D} \alpha_k^k \tag{11}$$

$$\text{s.t.} \quad \beta'^k_{ij,n} + d_k \gamma'_{ij,n} \geq \alpha_j^k - \alpha_i^k \quad \forall (i,j) \in A,\ \forall k \in D,\ \forall n \in \mathcal{N}_{ij} \tag{12}$$

$$\alpha_r^k = 0 \qquad \forall k \in D \tag{13}$$

$$\alpha \geq 0 \tag{14}$$

This problem can be decomposed into the duals of $|D|$ independent shortest path problems. For any $k \in D$, $\alpha_k^k(\beta', \gamma')$ is the length of the shortest (r,k) path corresponding to edge weights $w_{ij}^k = \min_{n \in N_{ij}} \left(\beta'^k_{ij,n} + d_k \gamma'_{ij,n}\right)$.

2.4 Directed Cut-Set-Formulation (CUTSET)

We now recall the cut-set formulation for the single-source NLP on directed graphs. For any subset $S \subset V$, we denote with $\delta^+(S) := \{(i,j) \in A : i \in S, j \in V \setminus S\}$ and $\delta^-(S) := \{(i,j) \in A : i \in V \setminus S, j \in S\}$ outgoing and ingoing cuts respectively. For any $S \subseteq V$ we denote the induced arc set with $A(S) := \{(i,j) \in A : i \in S, j \in S\}$. Projecting out aggregated flow variables $f_{ij} = \sum_{k \in D} \sum_{n \in \mathcal{N}_{ij}} d_k f^k_{ij,n}$ leads to the following cut-set inequalities:

$$\sum_{(i,j) \in \delta^+(S)} \sum_{n \in \mathcal{N}_{ij}} u_{ij,n} x_{ij,n} \geq \sum_{k \in D \setminus S} d_k \quad \forall S \subset V : r \in S, S \cap D \neq D \tag{15}$$

Inequalities (15) can be separated in polynomial time. For a given fractional solution x', we define the directed *support graph* $G' = (V', A')$ where $V' := V \cup \{t\}$ with an additional sink t and $A' := A_1 \cup A_2$ being $A_1 := \{(i,j) \in A : \sum_{n \in N_{ij}} u_{ij,n} x'_{ij,n} > 0\}$ and additional arcs $A_2 := \{(k,t) : k \in D\}$. The arc capacities are set to $\sum_{n \in N_{ij}} u_{ij,n} x'_{ij,n}$ $\forall (i,j) \in A_1$ and to d_k $\forall (k,t) \in A_2$. If the minimum-capacity cut between r and t in G' is less than $\sum_{k \in D} d_k$ it defines a violated inequality (15).

2.5 Cover Inequalities

Given a cutset inequality (15) defined by $S \subset V, r \in S$, define the index set $I(S) = \{(i,j,n) \mid (i,j) \in \delta^+(S), n \in \mathcal{N}_{ij}\}$ and $B = \sum_{k \in D \setminus S} d_k$. Set $M \subset I(S)$ is called a *cover* with respect to $I(S)$ if $\sum_{(i,j,n) \in M} u_{ij,n} < B$ and a *maximal cover* if, in addition, for all $S' \supset M$, $\sum_{(i,j,n) \in S'} u_{ij,n} \geq B$. If M is a maximal cover with respect to $I(S)$, then the following *cover inequalities* are valid:

$$\sum_{(i,j,n) \in I(S) \setminus M} x_{ij,n} \geq 1. \tag{16}$$

In general, the separation of cover inequalities is NP-hard. We show that for the single-source NLP the problem of finding the most violated cover inequality (16) is equivalent to solving the *precedence constrained knapsack problem*. Assume that indices $n \in \mathcal{N}_{ij}$ are sorted according to increasing arc capacities and that demands are integers. To model any cover M with respect to $I(S)$, we introduce the binary variables $z_{ij,n}$ that are equal to one iff $(i,j,n) \in M$. For every arc $(i,j) \in \delta^+(S)$, we define $u_{ij,0} = 0$. The most violated cover inequality can then be found by solving the following MIP problem:

$$\text{(KNAP)} \qquad \max \sum_{(i,j,l) \in I(S)} x'_{ij,n} z_{ij,n} \tag{17}$$

$$\sum_{(i,j,n) \in I(S)} (u_{ij,n} - u_{ij,n-1}) z_{ij,n} \leq B - 1 \tag{18}$$

$$z_{ij,n} \geq z_{ij,n+1}, \quad \forall (i,j,n) \in I(S) \tag{19}$$

$$z_{ij,n} \in \{0,1\}, \quad \forall (i,j,n) \in I(S) \tag{20}$$

Let z' be the optimal solution to (KNAP). The corresponding cover inequality then reads as follows:

$$\sum_{(i,j,n) \in I(S)} (1 - z'_{ij,n}) x_{ij,n} \geq 1.$$

These inequalities are similar to the *band inequalities* for the incremental cost model in [13].

3 Branch-and-cut approach based on Benders' Decomposition

In this section we explain the branch-and-cut scheme based on the (CUTSET) formulation and extended by projected Benders' inequalities introduced in the last section.

Initialization We initialize the LP with capacitated and uncapacitated *in-degree constraints* associated to each customer $k \in D$. Furthermore, for all non-customers $l \in V \setminus D \setminus \{r\}$ we add flow-balance constraints: $\sum_{(i,l) \in A, i \neq j} \sum_{n \in \mathcal{N}_{il}} x_{il,n} \geq \sum_{n \in \mathcal{N}_{lj}} x_{lj,n}, \forall (l,j) \in A$ and $\sum_{(l,i) \in A, i \neq j} \sum_{n \in \mathcal{N}_{li}} x_{li,n} \geq \sum_{n \in \mathcal{N}_{jl}} x_{jl,n}, \forall (j,l) \in A$. These constraints are valid because an optimal solution to the single source NLP is directed-cycle-free.

Separation We separate cutset and connectivity inequalities in a straight-forward way using a maximum-flow procedure. After rounding we add the constraints into the LP. Furthermore, during the separation, we prefer the *minimum-cardinality cuts*: they are obtained during separating connectivity inequalities in a graph in which to every capacity $x_{ij,n}$, an epsilon value is added. By default, up to 100 *nested cut-sets* are added at once, before resolving the LP. Finally, to speed-up the separation, using *back cuts* we also consider reversal flow, from the customer k to the root r.

After no further violated cut-set inequalities can be found, we separate Benders' cuts (10), by solving the linear program (SUB). To speed-up the computation, we add several *disjoint Benders' cuts*: after determination of $(\alpha', \beta', \gamma')$ as described in Sec. 2.2, we add constraints $\beta^k_{ij,n} = 0, \forall \beta'^k_{ij,n} > 0$ and $\gamma_{ij,n} = 0, \forall \gamma'_{ij,n} > 0$ and resolve (SUB).

Primal Heuristic We employ a simple rounding heuristic. Given a feasible fractional solution x, denote the total installed capacity on each arc by $X_{ij} = \sum_{n \in \mathcal{N}_{ij}} u_{ij,n} x_{ij,n}$. Initialize $x' = 0$. Now for every arc (i,j) install the cheapest fitting cable type $x'_{ij,\tilde{n}} = 1 : \tilde{n} = \text{arg-min}_{\{n \in \mathcal{N}_{ij} | u_{ij,n} \geq X_{ij}\}} c_{ij,n}$. The resulting x' is integer feasible. However, it is often too generous and can be reduced quite simply with the help of the minimum cost flow algorithm. We define the arc capacities as X'_{ij} and arc cost of $C'_{ij} = \sum_{n \in \mathcal{N}_{ij}} c_{ij,n} x'_{ij,n}$ and compute the min-cost-flow $f \in \mathbb{R}^{|A|}$. Then we replace the current design x', with a new cheapest fitting $x'' : x''_{ij,\tilde{n}} = 1 : \tilde{n} = \text{arg-min}_{\{n \in \mathcal{N}_{ij} | u_{ij,n} \geq f_{ij}\}} c_{ij,n}$. Since the min-cost-flow algorithm only works for integer capacity and cost X' and C' have to be rounded first.

4 Computational Results

Test Instances We used the street map of the Austrian city Bregenz with 1014 nodes and 1191 edges as underlying network. As customers we considered 4 different sets of nodes with cardinalities 36, 45, 52 and 67, whereas each customer has a demand randomly chosen from $\{4, 8, 12, 16, 20\}$.

CT	N	(capacity c, cost u) ...
NA	2	(30, 2.2) (1020, 146.0)
NB	2	(120, 7.0) (1020, 146.0)
NC	3	(30, 2.2) (60, 4.0) (1020, 146.0)
ND	4	(30, 2.2) (60, 4.0) (120, 7.0) (1020, 146.0)

As cable types we employed 4 different sets uniformly on all edges (see the table above). The resulting networks have been preprocessed by applying degree one- and two-tests.

Experiments We implemented our algorithms using C++ and CPLEX 11.0. An Intel Core 2 computer with 1.8 Ghz and 3.25 GB was used for testing purposes. We tried to solve the test instances with three different MIP models (MMCF), (MCF), i.e. flow aggregated by cable type ($f^k_{ij} = \sum_{n \in \mathcal{N}_{ij}} f^k_{ij,n}$) and (SCF), i.e. flow aggregated by cable type and customer ($f_{ij} = \sum_{k \in D} f^k_{ij}$). We compared these results to three branch-and-cut approaches: (CUTSET), (BEND), which additionally generates Benders' cuts if no cut-sets are found and (SBEND), which uses (SCF) as master problem and generates Benders' cuts in the root node and in every 10-th branch-and-bound node. (MMCF) and (MCF) were unable to produce any reasonable bounds - usually the LP relaxation was not solved within an hour. Table 1 compares the results of the other four approaches.

Conclusion It can easily be shown that (CUTSET) is stronger than (SCF). Similarly, (MMCF), resp. (BEND) is stronger than (CUTSET). Neverthelss, the branch-and-cut approaches could usually not supersede the bounds and gaps achieved with branch-and-bound on the (SCF) model. This is surprising, since the aggregated flow formulation is known to have arbitrarily bad lower bounds. It can probably be explained by very sophisticated mixed integer rounding (MIR) cuts and other advanced procedures integrated into the commercial MIP solver CPLEX 11. Indeed, after turning off all these features, the (SCF) model is not able to produce any reasonable results. (SBEND) attempts to combine the practically successful (SCF) with the computationally more expensive, stronger cuts of (BEND). This works out to some degree, in 5 out of 16 cases (SBEND) produces a better lower bound than (SCF). While the lower bound after the root note is much better for (SBEND) than for (SCF), it seems that the rounding heuristic does not work so well for the branch-and-cut approaches.

Inst						(CUTSET)		(BEND)		(SCF)		(SBEND)	
|D|	|N|	CT	|V|	|E|	UB	Gap	Nodes	Gap	Nodes	Gap	Nodes	Gap	Nodes
33	2	NB	325	490	(b,s) 174491.6	***0.01**	1455	0.58	164	***0.01**	30557	0.36	2070
33	2	NA	325	490	(b) 820199.8	2.94	104	3.65	23	0.91	2482	**0.63**	1130
33	3	NC	325	490	(-) 193829.3	2.42	60	2.66	24	**0.73**	7393	0.75	896
33	4	ND	325	490	(s) 85042.3	4.77	10	3.87	4	**2.56**	2055	2.60	610
41	2	NB	333	498	(c) 206863.2	0.20	3104	0.60	145	***0.01**	7079	***0.01**	1194
41	2	NA	333	498	(c,s,b) 940819.0	10.88	83	11.11	51	8.85	2351	**8.46**	960
41	3	NC	333	498	(s) 230446.0	9.33	10	9.20	5	**1.33**	2684	3.13	420
41	4	ND	333	498	(s) 103786.2	4.72	9	4.22	5	2.42	1557	**2.35**	540
46	2	NB	340	506	(-) 264733.3	0.56	1001	1.27	92	**0.12**	15559	0.38	815
46	2	NA	340	506	(-) 1835563.3	9.71	61	**8.55**	11	9.68	2265	9.36	800
46	3	NC	340	506	(-) 634915.8	3.87	1	3.80	2	**1.10**	1550	2.36	310
46	4	ND	340	506	(-) 150067.4	4.49	19	3.98	7	**2.18**	1961	2.21	610
61	2	NB	351	516	(-) 238483.8	1.22	712	1.76	60	**0.40**	13139	0.81	510
61	2	NA	351	516	(-) 1889229.2	10.22	104	10.59	36	**9.73**	1293	9.90	999
61	3	NC	351	516	(-) 633880.8	6.55	5	6.60	3	**2.76**	1494	3.87	410
61	4	ND	351	516	(-) 141349.6	4.31	9	4.09	7	**2.13**	2101	2.14	700

Table 1: The best known upper bounds are given in **UB** and marked with (c),(b) and (s), when found by (CUTSET), (SBEND) or (SCF). For each approach we show the integrality gap **Gap** $= \frac{UB-LB}{LB} \cdot 100\%$, where LB is the best lower bound computed within the time limit of one hour. **Nodes** denotes the number of branch-and-bound nodes that have been processed. Optimal solutions are denoted by (*).

Future work will concentrate on improving or replacing the primal heuristic and investigating other classes of cuts to strengthen/speedup branch-and-cut.

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