Revenue Management for Transportation Problems

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Abstract

In this paper, we consider the problem of managing a fleet of trucks with different capacity to serve the requests of different customers that arise randomly over time. Customers make a request by demanding the transportation of a given quantity of goods from an origin to a destination, at a given time.

It is well known that these problems arise in the dynamic resource allocation context, where tasks (requests of renting) arriving over time have to be covered by a set of indivisible and reusable resources of different types (trucks with different capacity). The assignment of a resource to a task removes the task from the system, modifies the resources and generates profit. We give a dynamic formulation of the problem. Due to the "curse of dimensionality" the dynamic model cannot be solved to optimality.

Thus to provide a valuable support for the rental decision making process, we develop a linear programming version of the problem and exploit its solution via revenue management techniques.

Keywords: *Dynamic programming, Logistics, Network, Revenue Management*

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1 Introduction

Revenue management methods are very effective to help companies in finding optimal policies to allocate their products in a given planning horizon.

Revenue management is a planning instrument for maximizing revenue by optimally allocating constrained and perishable capacity on differentiated products/services, that are targeted to heterogeneous customer segments and generally sold through advance booking in the face of uncertain levels of demand for service. The fundamental revenue management decision is to either accept or reject an arriving booking request for a specific service and, in the latter case, preserve the availability for probably more valuable demand in subsequent periods.

Today, revenue management plays an important role for service firms in many different industries. While airlines have the longest history of development in revenue management, the techniques also apply to other industries with similar business characteristics, such as hotels, restaurants and car rental, freight transportation and passenger railways, telecommunications and financial services, internet service provision, electric utilities, broadcasting and even manufacturing companies [1].

McGill and van Ryzin [2] give a comprehensive overview of the history of revenue management in transportation, where it has had the greatest impact.

In this paper, we apply revenue management methods and policies to a truck rental problem. We define, on the basis of the arrival process of the requests, the policy for either accept or reject a booking request of rent once it arrives. We address the question of how to coordinate the decisions on fleet management and to treat the randomness in the demand arrivals explicitly by decomposing the problem into time periods and assessing the impact of the current decisions on the future, through the managing of available capacity.

The problem addressed here is a dynamic resource allocation problem, that involves the assignment of a set of reusable resources (vehicles) to tasks (customer demands) that occur over time. The assignment of a resource to a task produces a reward, removes the task from the system, and modifies the state (typically, a geographical location) of the resource [3], [5]. We were confronted with this problem within the context of managing a fleet of trucks rented by a logistic operator to serve customers who request the freight transportation between different nodes in a network. It is a fleet management problem where a vehicle is assigned to a request from one location to another at a given time. The fleet is composed of different type of trucks. At each decision epoch, a certain number of customers arrive in, each requesting a transportation of a certain quantity of goods from a certain origin to a destination. Each customer demand can be satisfy with a truck with a capacity greater or equal to the request.

We give a dynamic formulation of the problem at hand. Dynamic models arise in a great variety of transportation applications as a result of the need to capture the evolution of activities over time. These models allow to find an answer to the following crucial question: '*'Which truck should assign to a customer given the unknown but, probably, more profitable demand that will arrive in the system in the future?*" Due to "the curse of dimensionality", the dynamic programming model cannot be solved optimally. For this reason, in order to provide the decision maker with a tool useful in taking decisions, we develop a linear programming formulation of the problem and apply revenue management techniques to take the best decision.

The present work shares some similarities with that of Topaloglu and Powell [5]. However, the following main differences can be found. First of all, in [5], it is assumed that the customers can ask for different types of vehicles, on the basis of their preferences. In our work, instead, the logistic operator, by evaluating appropriately the convenience, can decide to assign a truck of greater capacity to a certain customer. Indeed, an upgrade can take place. In addition, to address the problem under consideration, we do not follow the approximate dynamic programming approach used in [5]. Indeed, in taking decisions, we adopt revenue management policies, based on booking limits, that require to solve dynamically a linear programming model. A policy that allows the logistic operator to use the same truck to satisfy multiple demands is also deviced. This possibility is not exploited in [5].

The paper is organized as follow. In Section 2 we describe the problem under consideration and give a dynamic formulation of it. Section 3 contains the linear programming formulation of the problem together with the relates revenue management policies. In Section 4, we present and discuss the results of numerical experiments. Finally, we conclude the paper with some final remarks in Section 5.

2 Dynamic Programming Formulation

We consider the problem of a logistic operator that offers a transportation service from a given set of origins to a given set of destinations. The transportation service consists in renting trucks of different capacities to different customers on a given time horizon. Each customer is associated with a certain level of demand. At each time of the planning horizon, the transportation operator has to decide how to manage the overall capacity in the most profitable way, taking into account that complete information about the future demand are not available.

Let $O = \{1, \dots, o\}$ be a given set of origins and $E = \{1, \dots, e\}$ a given set of destinations. It is assumed that $O \cap E \neq \oslash$. The logistic operator transports goods from an origin i, $i \in O$ to a destination j, $j \in E$ by r types of trucks. A truck of type $k \in K = \{1, \dots, r\}$ is associated with a given value of capacity $a(k), k = 1, \dots r$.

Customers can be viewed as partitioned in r different classes. A customer is of class k if he/she requires the transportation of a quantity q_k of goods, such that $a(k-1) < q_k \leq a(k)$, $k = 1, \dots, r$ and $a(0) = 0$. The demand of a class k customer can be satisfied with trucks with capacity $a(k)$ or greater, i.e. an "upgrade" can take place.

In each time period t, the logistic operator has to decide on accepting the request of transferring a given quantity of goods from $i \in O$ to $j \in E$ at time $\bar{t} > t$, with the goal of maximizing the total revenue.

Let $A = [A^1|A^2|\dots|A^r]$, $A \in \mathcal{R}^{r \times u}$, $u = r + (r-1) + \dots + [r - (r-1)] = r^2 + \sum_{l=1}^{r-1} l$ denote a binary matrix, partitioned in r sub-matrices. Each sub-matrix $A^k \in \mathcal{R}^{r \times (r-k+1)}$, $k = 1, \dots, r$ represents the set of possible products to satisfy the demand of a class k customer. In particular, the first column of sub-matrix A^k refers to the truck of minimum capacity $a(k)$ useful to satisfy the demand of class k, whereas the last column corresponds to the truck of maximum capacity $a(r)$ that can be used to satisfy the class k customer.

We indicate each column of matrix A as A_{v_k} $k = 1, \dots, r$, $v_k = v_{min}, \dots, v_{max}$, $v_{min} = (k-1)r - \sum_{s=1}^{k-1}[(k-1)-s]+1$ and $v_{max} = kr - \sum_{s=1}^{k-1}[k-s]$. Each element a_{wv_k}, $w = 1, \dots, r$, $v_k = v_{min}, \dots, v_{max}$ of matrix A is equal to one if truck of capacity $a(w)$ is utilized to satisfy the demand of class k with product v_k and 0 otherwise.

The state of the network is described by a vector $x_i = (x_{i1}, \dots, x_{ir})^\top$ of resource capacities at node i, where x_{ik} represents the number of trucks of type k, $k = 1, \dots, r$ available at node $i \in O$.

Time is discrete, there are T periods indexed by t, which runs forward so that $t = T$ is the departure time.

In each time-period t, at most one request of transportation of class k from node i to all the destination nodes $j \in E$ can arrive. It is assumed that there can be advance information about future demands, that is demand that needs to be served in a future time period $\bar{t}$ may be known in advance in an earlier time period t and we denote with $\lambda_{ijk}^{t\bar{t}}$, $\bar{t} = t \dots T$, $k = 1, \dots, r$ the probability that at time t one transportation request of class k from $i \in O$ to $j \in E$, with departure time $\bar{t} > t$, is made. It holds that $\sum_{\bar{t}=t}^{T} \sum_{k=1}^{r} \sum_{j \in E} \lambda_{ijk}^{t\bar{t}} + \lambda_{i0}^{t} = 1$, where λ_{i0}^{t} is the probability that no request arrives on node i at time t.

We, further, assume that the travel times are random and we indicate with p_{ij}^{τ} the probability that the average travel time from node i, $i \in O$ to node j, $j \in E$ will be $\tau, \tau = 1, \dots, \bar{\tau}$ time units.

Let us introduce boolean variables $u_{ijv_k}^{t\bar{t}}$, with $u_{ijv_k}^{t\bar{t}} = 1$ if and only if the transportation request of class k, from node i to node j with departure time $\bar{t}$, is accepted at time t by using product v_k.

Let $R_{ij}^{\tilde{k}}$ be the revenue obtained by satisfying class k request from i to j with product v_k, $k = 1, \dots, r$, $v_k = v_{min}, \dots, v_{max}$, $\tilde{k} = v_k - \{(k-1)r - \sum_{s=1}^{k-1}[(k-1)-s]+1\} + k$.

The problem can be formulated as a dynamic program by letting $V_t(x_i)$ be the maximum expected revenue obtainable from periods $t, t+1, \dots, T$ given that, at time t, x_i units of capacity are available at node i.

The Bellman equation for $V_t(x_i)$ is reported in what follows:

$$\begin{aligned} V_t(x_i) = \; & \sum_{\bar{t}=t}^{T} \sum_{k=1}^{r} \sum_{j \in E j \neq i} \lambda_{ijk}^{t\bar{t}} \sum_{v_k=v_{min}}^{v_{max}} \max_{u_{ijv_k}^{t\bar{t}} \in \{0,1\}} \Big[R_{ij}^{\tilde{k}} u_{ijv_k}^{t\bar{t}} + V_{t+1}(x_i - A_{v_k} \sum_{\tilde{t}=1}^{\bar{t}} u_{ijv_k}^{\tilde{t}\bar{t}} + \\ & + \sum_{k=1}^{r} \sum_{v_k=v_{min}}^{v_{max}} A_{v_k} \sum_{j \in E j \neq i} \sum_{\tau=1}^{\bar{\tau}} p_{ji}^{\tau} \sum_{\tilde{\tau}=1}^{t-\tau} \sum_{\bar{\tau}=\tilde{\tau}}^{t-\tau} u_{jiv_k}^{\tilde{\tau}\bar{\tau}}) \Big] \\ & + \lambda_{ij0}^{t} V_{t+1} \left(x_i + \sum_{k=1}^{r} \sum_{v_k=v_{min}}^{v_{max}} A_{v_k} \sum_{j \in E j \neq i} \sum_{\tau=1}^{\bar{\tau}} p_{ji}^{\tau} \sum_{\tilde{\tau}=1}^{t-\tau} \sum_{\bar{\tau}=\tilde{\tau}}^{t-\tau} u_{jiv_k}^{\tilde{\tau}\bar{\tau}} \right) \end{aligned} \tag{1}$$

with boundary conditions:

$$V_t(0) = 0, \qquad \forall t;$$
$$V_0(x_i) = 0, \qquad \forall x_i \geq 0;$$
$$V_T(x_{ik}) = -\infty, \qquad \forall x_i \text{ and if for some } k, x_{ik} < 0.$$

It is worth noting that the capacity on node i at a time t is related to following two events.

- The departure of a truck used for the product v_k at time t. The term $A_{v_k} \sum_{\tilde{t}=1}^{\bar{t}} u_{ijv_k}^{\tilde{t}\bar{t}}$ represents this event.
- The arrival to node i of trucks that have left nodes $j, j \in E, j \neq i$ before t and that are expected to be in i at t, considering that the average travel time from j to i is $\tau, \tau = 1, \dots, \bar{\tau}$ time units. The term $\sum_{k=1}^{r} \sum_{v_k=v_{min}}^{v_{max}} A_{v_k} \sum_{j \in E j \neq i} \sum_{\tau=1}^{\bar{\tau}} p_{ji}^{\tau} \sum_{\tilde{\tau}=1}^{t-\tau} \sum_{\bar{\tau}=\tilde{\tau}}^{t-\tau} u_{jiv_k}^{\tilde{\tau}\bar{\tau}}$ is related to this event.

The proposed dynamic programming model is unlikely to be solved optimally due to the curse of dimensionality. In the next section, we present a linear programming formulation of the problem under consideration. The proposed model is static but it is solved in a "dynamic way" by appropriately updating the demand and capacity information at the beginning of each time period. The aim is to support the logistic operator in taking decisions, by adopting appropriate revenue management policies.

3 Linear Programming Formulation

In the linear programming formulation, we replace stochastic quantities by their mean values and assume that capacity and demand are continuous.

Let d be the random vector of cumulative future demand at time t, and $\bar{d}$ its mean; let τ_{ij} be the random variable representing the travel time and $\bar{\tau_{ij}}$ its mean. Furthermore, let:

- $y_{ij}^{\bar{t}v_k}$ = number of products of type $v_k = 1, \ldots, max_{prod}$, where $max_{prod} = r^2 - \sum_{l=1}^{r-1} l$ to be used to satisfy the transportation request from $i,\ i \in O$ to $j,\ j \in E$ at time $\bar{t}$.
- $\bar{d}_{ij}^{\bar{t}k}$ = aggregate demand of transportation from i to j of class k at time $\bar{t}$ from period t to the end of the planning horizon. In particular $\bar{d}_{ij}^{\bar{t}k}$ is the aggregate number of transportation requests from i to j at time $\bar{t}$ belonging to class k.
- $R_{ij}^{v_k}$ is the revenue obtained by satisfying a transportation request of type k from node i to node j by using the product v_k, $k = 1, \ldots, r$, $v_k = v_{min}, \ldots, v_{max}$. In particular, $R_{ij}^{v_k} = R_{ij}^{\tilde{k}}$, where $\tilde{k}$ refers to the type of truck used in the product v_k.
- $\bar{\tau}_{ij}$ is the average travel time from node $i \in O$, to node $j \in E,\ \ i \neq j$.

The total revenue obtainable by the logistic operator at time t when the network capacity is x can be calculated by solving the following optimization problem:

$$R^{LP}(x,t) = \max \sum_{\bar{t}=t}^{T} \sum_{i \in O} \sum_{j \in E j \neq i} \sum_{k=1}^{r} \sum_{v_k=v_{min}}^{v_{max}} R_{ij}^{v_k} y_{ij}^{\bar{t}v_k} \tag{2}$$

$$\sum_{v_k=v_{min}}^{v_{max}} y_{ij}^{\bar{t}v_k} \leq \sum_{j \in E j \neq i} d_{ij}^{\bar{t}k} \quad \forall k, i, j,\ \bar{t} \geq t \tag{3}$$

$$\sum_{j \in E j \neq i} \sum_{v_k=1}^{max_{prod}} a_{kv_k} y_{ij}^{\bar{t}v_k} \leq x_{iw} + \sum_{j \in E j \neq i} \sum_{v_k=1}^{max_{prod}} a_{kv_k} \sum_{\bar{t}=1}^{t-\bar{\tau}_{ij}} y_{ji}^{\bar{t}v_k} \quad \forall k, i,\ \bar{t} \geq t \tag{4}$$

$$y_{ij}^{\bar{t}v_k} \geq 0 \quad \forall \bar{t} \geq t, k, v_k, i, j \tag{5}$$

The objective function (2) represents the total revenue obtainable at time t when the residual capacity on the network is x. Constraints (3) state that the demand of class k can be satisfied with all products of type v_k. Constraints (4) control the availability of a truck of capacity $a(k)$ on node i at time t.

In a revenue management setting, we usually utilize booking limits or bid price controls to accept or reject a request at a certain time [4]. In what follows, we give a description of the proposed acceptation policies, in which the optimal primal variables are used in the booking limits control.

3.1 Revenue-based primal acceptation policy

At a certain point of the planning horizon, decisions about accepting or denying a class $\bar{k}$ request of transportation from i to j with departure time $\bar{t}$, are to be made.

The model is driven by the following event: a request of class $\bar{k}$ from origin node i to destination node j with departure time $\bar{t} > t$ arrives at time t to the transportation operator.

When this event happens, $R^{LP}(x,t)$ is solved and its solution is used to make a decision.

From a primal viewpoint, the strategy to be adopted (referred to as booking limits policy, $\mathcal{BLP}$, for short) assumes the following form.

Step. 1
Solve $R^{LP}(x,t)$. Let $y_{ij}^{\star\bar{t}v_k}$ denote its optimal solution.

FOR $v_k = (\bar{k}-1)r - \sum_{s=1}^{\bar{k}-1}[(\bar{k}-1)-s]+1, \ldots, \bar{k}r - \sum_{s=1}^{\bar{k}-1}(\bar{k}-s)$ do

IF $y_{ij}^{\star\bar{t}v_k} > 0$ THEN

ACCEPT the request with upgrade if $v_k > (\bar{k}-1)r - \sum_{s=1}^{\bar{k}-1}[(\bar{k}-1)-s]+1$;

SET $\tilde{k} = v_k - (k-1)r - \sum_{s=1}^{k-1}[(k-1)-s]+1+\bar{k}$;

SET $y_{ij}^{\bar{t}v_k} = y_{ij}^{\bar{t}v_k} - 1$;

UPDATE appropriately $x_{i\tilde{k}}^{\tilde{t}}$ and $x_{j\tilde{k}}^{\tilde{t}}$;

that is $x_{i\tilde{k}}^{\tilde{t}} = x_{i\tilde{k}}^{\tilde{t}} - 1, \forall \tilde{t} = \bar{t}, \ldots T$, and $x_{j\tilde{k}}^{\tilde{t}} = x_{j\tilde{k}}^{\tilde{t}} + 1$, $\forall \tilde{t} = \bar{t} + \bar{\tau}_{ij}, \ldots, T$;

CALCULATE the revenue obtained from accepting the request;

ELSE

DENY the request.

END IF

END FOR

Step. 2 EVALUATE the next request.

3.2 Revenue-based primal acceptation policy with sharing

It is worth observing that, if more than one request, from the same origin toward the same destination at the same time $\bar{t}$, arrives to the transportation operator, multiple demands can be loaded on the same truck. Indeed, a policy, incorporating the truck sharing (referred to as booking limits policy with sharing, $\mathcal{BLPS}$) can be defined.

Let $Q_{ij}^{\bar{t}} = \{q_{ij}^{1\bar{t}k_1}, \ldots, q_{ij}^{\xi\bar{t}k_\xi}\}$ denote the quantities of goods that are requested to be transported from i to j at time $\bar{t}$ from customers of class $k_1, \ldots, k_\xi$. The main operations executed by the $\mathcal{BLPS}$ can be represented as follows.

Step. 1

Solve $R^{LP}(x,t)$. Let $y_{ij}^{\star\bar{t}v_k}$, $k = 1, \ldots, r$, $v_k = v_{min}, \ldots, v_{max}$ denote its optimal solution.

FOR $\bar{k} = 1, \ldots, r$ do

FOR $v_k = (\bar{k}-1)r - \sum_{s=1}^{\bar{k}-1}[(\bar{k}-1)-s]+1, \ldots, \bar{k}r - \sum_{s=1}^{\bar{k}-1}(\bar{k}-s)$ do

IF $y_{ij}^{\star\bar{t}v_k} > 0$ THEN

SET $\tilde{k} = v_k - (k-1)r - \sum_{s=1}^{k-1}[(k-1)-s]+1+\bar{k}$;

SET $res_{cap} = a(\tilde{k})$,

REPEAT

SELECT a request $q_{ij}^{\xi\bar{t}\bar{k}_\xi}$ from $Q_{ij}^{\bar{t}}$, such that $q_{ij}^{\xi\bar{t}\bar{k}_\xi} \leq res_{cap}$ and $\bar{k}_\xi \leq \bar{k}$;

DELETE $q_{ij}^{\xi\bar{t}\bar{k}_\xi}$ from $Q_{ij}^{\bar{t}}$;

ACCEPT the chosen request;

SET $res_{cap} = res_{cap} - q_{ij}^{\xi\bar{t}\bar{k}_\xi}$

UNTIL $\{Q_{ij}^{\bar{t}} = \oslash$ or $res_{cap} = 0\}$

SET $y_{ij}^{\star\bar{t}v_k} = y_{ij}^{\star\bar{t}v_k} - 1$,

UPDATE appropriately $x_{i\tilde{k}}^{\tilde{t}}$ and $x_{j\tilde{k}}^{\tilde{t}}$,

CALCULATE the revenue obtained from accepting the selected requests;

END IF

END FOR

Problem Class	$\lvert O\rvert = \lvert E\rvert$	Types of Trucks (r)	Time Periods (T)	$[a, b]$
1	5	3	5	$[1, 12]$
2	5	5	5	$[1, 12]$
3	5	3	10	$[1, 12]$
4	5	5	10	$[1, 12]$
5	10	3	5	$[1, 6]$
6	10	5	5	$[1, 6]$
7	10	3	10	$[1, 6]$
8	10	5	10	$[1, 6]$
9	15	3	5	$[1, 3]$
10	15	5	5	$[1, 3]$
11	15	3	10	$[1, 3]$
12	15	5	10	$[1, 3]$

Table 1: Characteristics of the test problems

END FOR

DENY all the requests belonging to $Q_{ij}^{\bar{t}}$ that have not been selected.

Step. 2 EVALUATE the next requests.

4 Computational Experiments

In this section, we present the numerical results obtained by testing the primal policies described in previous section. The main aim is to assess their performance in terms of solution quality.

All the numerical experiments have been carried out in AIMMS 3.7 on a Pentium Intel Core 2 T200 2.0 GHz PC, by considering test examples, defined trying to be quite close to the reality of medium-sized logistic operators.

Different classes of instances have been considered, characterized by an increasing number of origin and destination nodes, number of trucks and time periods, as reported in Table 1. In all the instances, we set $O \equiv E$, that is each node can serve as origin and destination of the transportation request.

For each product v_k and for each origin-destination pair (i, j), $i \in O$ and $j \in E$, the value of the revenue per transportation request $R_{ij}^{v_k}$ was randomly generated into the interval $[200; 1000]$ and considering a revenue increasing with the increase in the capacity and in the distance between the nodes i and j.

For each node $i \in O$ and for each type of truck k, the number of trucks x_{ik} available at node i was randomly chosen from the ranges $[a; b]$, as reported in Table 1. In addition, the average travel time from node i to node j was randomly generated into the interval $[1; T]$. For each test problem, the booking process has been simulated 30 times.

In each simulation run, the transportation requests are randomly generated by applying a two phases procedure. In the first phase, for each origin-destination pair, the number of transportation requests for each class is randomly generated according to a normal distribution, with a given expected demand and a given coefficient of variation. The expected demand and the coefficient of variation have been chosen randomly from the interval $[1; 20]$ and $[0; 1]$, respectively. In the second phase 2, for each origin-destination pair and for each class, booking arrival times in each of the booking periods are randomly generated according to an uniform distribution.

Bookings start at the first time period and demand forecasts are updated every time period of the planning horizon.

In each of 30 simulation runs, all the requests for each test example, generated by applying the strategy described above, are processed based on the two policies, presented in sections 3.1 and 3.2.

The related computational results are reported in Table 2, in which for each class of test problems the average revenue values, obtained by applying both the policies, are given.

As expected, in all the instances the $\mathcal{BLPS}$ provides solutions of quality better that the ones determined by the $\mathcal{BLP}$. Indeed, an average revenue of 119506.56. is obtained with $\mathcal{BLPS}$, whereas the

Problem Class	$\mathcal{BLP}$	$\mathcal{BLPS}$
1	36237.00	52420.19
2	84307.50	102300.78
3	71736.00	82180.89
4	148458.00	187289.44
5	44370.00	73954.07
6	109054.00	133408.48
7	83617.00	115623.54
8	206592.00	251577.43
9	35551.00	44176.04
10	81358.00	103367.54
11	67048.00	91234.86
12	153316.00	196545.43
Average	93470.38	119506.56

Table 2: Average revenue values obtained by applying the proposed booking limits policies

Problem Class	$\mathcal{APE}^{\mathcal{BLP}}$	$\mathcal{APG}^{\mathcal{BLPS}}$
1	0.29%	44.23%
2	0.00%	21.34%
3	0.28%	14.23%
4	0.38%	25.67%
5	0.59%	65.69%
6	0.81%	21.34%
7	2.28%	35.12%
8	1.12%	20.41%
9	0.78%	23.29%
10	0.21%	26.78%
11	3.48%	31.35%
12	3.73%	23.41%
Average	1.16%	29.41%

Table 3: Comparison between the deterministic revenue and the revenue obtained by applying $\mathcal{BLP}$ and $\mathcal{BLPS}$

average revenue obtained by applying $\mathcal{BLP}$ is equal to 93470.38.

In order to assess the quality of the proposed policies, we have compared the revenue determined by applying $\mathcal{BLP}$ and $\mathcal{BLPS}$ with the one obtained by solving the $R^{LP}(x,t)$ model once at the beginning of the time horizon, referred to as deterministic revenue.

In particular, the average percentage error $APE^{\mathcal{BLP}}$ and the average percentage gain $APG^{\mathcal{BLPS}}$ with respect to the deterministic revenue have been determined as follows:

$$APE^{\mathcal{BLP}} = \frac{R_d - R_{\mathcal{BLP}}}{R_d} \times 100;$$

$$APG^{\mathcal{BLPS}} = \frac{R_{\mathcal{BLPS}} - R_d}{R_d} \times 100.$$

The related results are reported in Table 3 and clearly underline that the revenue obtained by applying $\mathcal{BLP}$ is quite close to the revenue achieved in the case of perfect information on the demand. Indeed, the average percentage error is equal to 1.16%. In addition, as expected, by applying a sharing strategy a higher revenue can be obtained. In particular, the average percentage gain is equal to 29.41%.

5 Conclusions

In this paper, we have considered the optimal managing of a fleet of trucks rented by a logistic operator, to serve customers. The logistic operator has to decide whether to accept or reject a request and which type of truck should be used to address it. For this purpose, a dynamic programming formulation and a linear approximation of the problem under consideration have been defined. Based on the proposed linear programming model, borrowing revenue management techniques, two primal acceptation policies have been defined, that use booking limits control. The possibility of loading multiple demands on the same truck (i.e., "truck sharing") has been also exploited.

Given that the linear programming formulation is a heuristic, the proposed policies have been tested in a series of computational experiments, to assess their performance in terms of solution quality. The results collected underlined that the proposed approaches can be considered a valuable tool for the rental decision making process.

It is worth observing that acceptation policies based on bid prices can be devised. In addition, instead to define a policy, incorporating the truck sharing, it is possible to give a programming formulation of the problem under consideration, taking into account not only this aspect, but also the repositioning of empty trucks from nodes, where they are not used, to nodes from which a new transportation request could be satisfied. Revenue management policies can be defined with respect to this new formulation. The aforementioned issues are the subject of current investigation.

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