

Models and Cuts for the Two-Echelon Vehicle Routing Problem

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Abstract

In Multi-Echelon Vehicle Routing Problems the delivery from one or more depots to the customers is managed by routing and consolidating the freight through intermediate depots, called satellites. This family of problems differs from Multi-Echelon Distribution Systems present in the literature. In the latter the attention is focused on the flow assignment among the levels only, while we also consider the fleet management and the overall distribution system routing.

In this work we consider the Two-Echelon Vehicle Routing Problem (2E-VRP), the basic variant of Multi-Echelon Vehicle Routing Problem with one depot and a fixed number of satellites. First level routing manages depot-to-satellites delivery, while the second level deals with satellites-to-customers delivery. The main goal of this work consists in detecting and defining new classes of valid inequalities for strengthening the continuous linear formulation of the 2E-VRP. Starting from the flow-based model presented in [8], we first extend the model giving a stronger formulation for it. Secondly, several valid inequalities are presented, derived from the TSP and CVRP literature, the network flow formulation, and the connectivity of the network. Extensive computational results on instances with up to 50 customers show an improvement of the best known results between 4% and 15%.

Keywords: *Multi-Echelon VRP, Cuts, Lower Bounds.*

1 Introduction

In Multi-Echelon Vehicle Routing Problems the delivery from one or more depots to the customers is managed by routing and consolidating the freight through intermediate depots, called satellites. This family of problems differs from Multi-Echelon Distribution Systems present in the literature, where the attention is focused on the flow assignment among the levels only. In our case we also consider the fleet management and the overall distribution system routing. The routing in Multi-Echelon Distribution Systems is challenging, both in theory and in practice (e.g., City Logistics problems), where keeping big trucks far from the city, while using efficiently small and environmental friendly vehicles in the historical city centers is one of the main goals [8].

In this work we consider the Two-Echelon Vehicle Routing Problem (2E-VRP), the basic variant of Multi-Echelon Vehicle Routing with one depot and a fixed number of satellites. First level routing manages depot-to-satellites delivery, while the second level deals with the satellites-to-customers delivery. The fleet size is fixed and the trucks are homogeneous inside each level. Vehicles and satellites are capacitated, while neither synchronization between the vehicles in each satellite nor time-windows of the customers are considered. The literature on 2E-VRP, due to the recent introduction of the problem itself,

is limited. A model for the 2E-VRP, able to solve to optimality instances with up to 32 customers, has been presented in [5, 8]. In [8], the authors also derived two math-heuristics able to solve instances up to 50 customers. Concerning larger size instances, a fast cluster-based heuristic method has been proposed by Crainic et al. [2] able to heuristically solve instances up to 150 customers. For an application of the 2E-VRP on freight distribution system we refer the reader to [3], where advanced freight distribution systems are provided, and [4].

The main goal of this work consists in detecting and defining new classes of valid inequalities for strengthening the continuous linear formulation of the 2E-VRP. Starting from the flow-based model presented in [8], we first extend the model giving a stronger formulation for it. Moreover, several valid inequalities are presented, derived from the TSP and CVRP literature, the network flow formulation, and the connectivity of the network.

2 Problem definition and existing cuts

Defined the central depot set $V_0 = \{v_0\}$, a set V_s of intermediate depots called satellites and a customer set V_c , wherein each customer $i \in V_c$ has a positive demand d_i associated, the problem consists in minimizing the total transportation costs, calculated by considering arc costs c_{ij} for shipping goods from one point to the other of the transportation network, while satisfying the demand of all the customers with a limited fleet of vehicles. Differing from classical VRP problems, the freight stored in V_0 must transit through intermediate depots, called satellites, and then be delivered to the customers. The demand of each customer has to be satisfied by only one satellite, and there are no thresholds on minimum and maximum number of customers served by a single satellite. This assumption induces, for each 2E-VRP feasible solution, a partition of V_c set in, at most, $|V_s|$ subsets, each one referring to a different satellite. Customer-satellite assignments are not known in advance, not allowing to solve the problem by decomposition into $|V_s| + 1$ VRPs. Two distinct fleet of vehicles m_1 and m_2 , with different capacity size K^1 and K^2 , are available to serve first and second network level, respectively.

In order to guarantee the feasibility of the 2E-VRP solutions, a *two commodity model* formulation has been introduced. Any 2E-VRP problem is expressed by the following variables:

- first level arc activation variables x_{ij} , $i, j \in V_0 \cup V_s$ and second level ones y_{lm}^k , $l, m \in V_c \cup V_s$, $k \in V_s$, for routing information on directed graph;
- first level flow variables Q_{ij}^1 , $i, j \in V_0 \cup V_s$ and second level ones Q_{lmk}^2 , $l, m \in V_c \cup V_s$, $k \in V_s$, each one related to a direct arc, describing freight quantities running on the two networks;
- customer-satellite assignment variables z_{kj} .

The mathematical formulation is mainly composed by a set of equations, related to the unique assignment of each customer to one satellite, and conservative flow equations, avoiding the presence of subtours in the second network level. Additional arc capacity constraints are introduced, due to the presence of size constraints on the vehicles (see [8] for details).

In [8] two families of valid inequalities are introduced in order to strengthen the 2E-VRP formulation: the *edge cuts*, subtour elimination constraints derived from Traveling Salesman Problem (TSP)

$$\sum_{i,j \in S_c} y_{ij}^k \leq |S_c| - 1, \quad \forall S_c \subset V_c, \quad 2 \leq |S_c| \leq |V_c| - 1, \quad k \in V_s \quad (1)$$

and *flow cuts*

$$Q_{ijk}^2 \leq (K^2 - d_i) y_{ij}^k \quad \forall i, j \in V_c, \forall k \in V_s \quad (2)$$

strengthening the logical constraints linking arc flows to arc usage variables in the 2E-VRP formulation.

3 New families of cuts for the 2E-VRP

Several classes of valid inequalities can be introduced in the 2E-VRP formulation by extending the CVRP literature (for a survey on last advances and trends, see [1]). Under the assumption 2E-VRP feasible solutions, restricted to the second level network, can be seen as solutions of $|V_s|$ VRPs, any valid inequality class for the VRP can be reformulated for the 2E-VRP. Considering the subtour elimination feature concerning edge cuts for the 2E-VRP, the following inequalities restricted to a candidate subset of customer $S \in V_c$

$$\sum_{k \in V_s} \sum_{\substack{i, j \in S \\ i \neq j}} y_{ij}^k \leq |S| - r(S) \quad \forall S \subset V_c \quad 2 \leq |S| \leq |V_c| - 1 \quad (3)$$

are valid for the 2E-VRP, where $r(S)$ is equal to the minimum number of 2nd level vehicles required to serve customers in S .

In the same way, strengthened comb inequalities and multistar inequalities for the VRP ([6], [7]) can be introduced as cutting planes classes also in 2E-VRP formulation.

The existence of the network flows in the mathematical formulation lets us define new classes of valid inequalities, based on the interaction between routing and arc activation variables related to particular sets of arcs. Upper bound variable constraints on arc capacity

$$d_j y_{ij}^k \leq Q_{ijk}^2 \quad \forall i \in V_c \cup V_s, \forall j \in V_c, \forall k \in V_s \quad (4)$$

and special cases of node feasibility inequalities

$$Q_{ijk}^2 - \sum_{\substack{l \in V_c \cup V_s \\ l \neq i}} Q_{jlk}^2 \leq d_j y_{ij}^k \quad \forall i \in V_c \cup V_s, j \in V_c, \forall k \in V_s \quad (5)$$

are cuts for the 2E-VRP and link routing information of LP fractional solution to its arc activation variables.

Other classes of valid inequalities are derived from considering connectivity and feasibility properties of any feasible solution of routing problems, through z_{kj} customer-satellite assignment variables. The following inequality

$$z_{ki} \geq y_{ij}^k + y_{ji}^k \quad \forall i, j \in V_c, i \neq j, \quad \forall k \in V_s \quad (6)$$

stated as constraint on the activation of two arcs incident to the same couple of customer nodes can be seen as a special case of the simple connectivity condition on subroutes not containing satellites in every integer solution.

Concerning route feasibility assumptions, let us define the i -th partition of the customer set V_c defined as follows

$$P_i = \left\{ S_j \subset V_c, j = 1, \dots, m_2 : \bigcup_{j=1}^{m_2} S_j = V_c \wedge S_j \cap S_k = \emptyset, k \neq j \wedge S_j \neq \emptyset \right\}.$$

Then, considered the set \mathcal{P} containing all the possible partitions P_i . An element in \mathcal{P} may not correspond to a feasible solution. A simple rule to exclude a partition P_i from the set of possible solutions is considering if the demand associated to one of its subsets is greater than the capacity size of second level vehicle, i.e., $d(S_j) > K^2$. Another rule is based on considering partitions for which a given subset with a restricted number of customers does exist. If it exists a customer set $S \in V_c$ such that complementary demand is equal to zero, i.e.,

$$m_2 - \left\lceil \frac{d(V_c \setminus S)}{K^2} \right\rceil = 0 \quad (7)$$

then the following inequalities

$$\sum_{j \in S} (y_{jk}^k + y_{kj}^k) + y^k(E(S)) \leq \sum_{j \in S} z_{kj} \quad \forall k \in V_s \quad (8)$$

First Lower Bounds		Previous Bounds		Bounds with cut generation			% Improv. LP	% Improv. LB
Instance	Satellites	LP-relaxation	Prev. LB	New LB	Time	Cuts		
E-n22-k4	6,17	340.74	364.08	407.89	7.00	575	19.71%	12.03%
	8,14	304.19	329.36	376.40	14.00	732	23.74%	14.28%
	9,19	355.65	384.00	440.99	6.00	412	23.99%	14.84%
	10,14	297.47	323.51	363.79	14.00	419	22.29%	12.45%
	12,16	300.41	327.61	377.55	10.00	637	25.68%	15.24%
E-n33-k4	1,9	567.11	602.59	652.77	57.00	984	15.10%	8.33%
	2,13	568.58	598.32	645.07	60.00	1516	13.45%	7.81%
	3,17	565.59	592.43	644.78	58.00	868	14.00%	8.84%
	4,5	588.04	611.54	675.91	44.00	815	14.94%	10.53%
	7,25	585.06	621.99	663.54	52.00	450	13.41%	6.68%
	14,22	606.22	649.36	697.54	53.00	926	15.06%	7.42%
E-n51-k5	2,17	511.57	528.65	550.91	737.00	663	7.69%	4.21%
	4,46	471.90	490.76	516.44	289.00	835	9.44%	5.23%
	6,12	477.58	494.11	519.66	310.00	1451	8.81%	5.17%
	11,19	503.70	522.33	556.58	412.00	686	10.50%	6.56%
	27,47	470.43	488.98	510.07	402.00	542	8.43%	4.31%
	32,37	482.87	502.73	525.38	323.00	920	8.80%	4.50%
E-n51-k5	2,4,17,46	462.85	481.81	510.61	1689.00	684	10.32%	5.98%
	6,12,32,37	458.31	476.40	509.47	2141.00	1089	11.16%	6.94%
	11,19,27,47	459.30	479.37	507.24	1096.00	995	10.44%	5.81%

Table 1: Lower Bounds on 2E-VRP benchmark tests

are valid for the 2E-VRP.

4 Computational results

All the previously mentioned classes of inequalities have been introduced in a Branch and Cut framework and tested on benchmark instance sets from [8]. We consider test instances composed by 21, 32 and 50 customers with 2 and 4 satellites.

The computational results shown in this work are obtained by applying the new valid inequalities to the linear relaxation of the 2E-VRP model from [8].

COIN-OR (*COmputational INfrastructure for Operations Research*) non-commercial software has been chosen for our tests. SYMPHONY package tool, a standard branch and cut solving algorithm for MIPs and LPs, has been used as a solver framework for getting a first LP solution of 2E-VRP instances. Results are performed using a personal computer Intel[©] Premium[©] CPU 1.73 GHz, 1 GB RAM.

Table 1 reports the lower bounds on test instances calculated on LP-relaxation problem. The first and the second columns provide information on the instance, including instance set name and satellite location. Columns 3 and 4 show the value of the objective function of the linear relaxation formulation of 2E-VRP and the lower bound provided by the cuts presented in [8]. Columns 5, 6 and 7 present the details on new lower bound values, the computational time in seconds and the number of active cuts in current LP-fractional solution, respectively. Finally, columns 8 and 9 give the percentage improvement of the new lower bound compared to the LP relaxation and the best results in the literature. According

to the results, the introduction of the new cuts get a relevant improvement on new lower bounds, with an improvement between 4% and 15%. On the other hand, the computational time needed to introduce the new cuts is huge in the largest instances.

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