Semidefinite Relaxation for Downlink OFDMA Resource Allocation Using Adaptive Modulation

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Abstract

This paper proposes a new binary quadratic formulation for the problem of minimizing power subject to rate and sub-carrier allocation constraints over wireless downlink (DL) Orthogonal Frequency Division Multiple Access (OFDMA) networks when using adaptive modulation. The model allows to decide what modulation and what sub-carriers are going to be used by a particular user in the system depending on its bits rate requirements. Thus, we derive a semidefinite relaxation whereby simulation results, we get a total tightness approximation average gain of 18.65 % to the optimal solution of the problem when compared to the linear programming relaxation obtained by applying Fortet linearization method to the quadratic model.

Keywords: *Orthogonal Division Multiple Access, downlink allocation, adaptive modulation, semidefinite programming.*

1 Introduction

When many users are connected to a Base Station (BS) and a large number of signals are using a wireless channel, greater complexity is generated by the negative phenomena of Multiple Access Interference (MAI) and Multi-path distortions. OFDMA is a suitable technology for combating these negative phenomena and it is currently the type of modulation used in wireless multi-user systems such as IEEE 802.11a/g WLAN, in networks of fixed access such as IEEE 802.16a and also for mobile WiMax deployments networks ensuring high quality of service (QoS) requirements [1]. OFDMA divides the channel into several orthogonal narrow band frequencies forming sub-carriers (sub-channels) giving access to several users simultaneously. In order to solve the resource allocation problem of DL sub-carrier and power over OFDMA systems, several schemes and algorithms have been proposed [2]. In [3] for example, the problem of minimizing total power consumption with constraints on bit error rate (BER) for users requiring different services is formulated while in [4], a sub-carrier allocation algorithm is proposed to increase the total user data rates subject to BER, total transmission power and proportional rate constraints. In this work we deal with the problem of minimizing power subject to assignment sub-carrier constraints depending on the bits required by each user using adaptive modulation, which means varying the number of bits to be sent in the different sub-channels. We state a quadratic formulation of this problem and derive a Semidefinite programming (SDP) relaxation, besides we get an equivalent linear program which is obtained by linearizing the quadratic terms using Fortet linearization [5]. Then, we compare gaps obtained by SDP relaxation and the relaxed solution of the equivalent linear formulation. We use SDP to get tighter bounds due to its proven efficiency in combinatorial optimization [6]. Moreover, the solvers uses interior point algorithms with polynomial time complexity [7]. This paper is organized as follows:

Section 2 provides the general system description. Section 3 details the new quadratic formulation of the general system and its equivalent linear formulation. Section 4 presents and explains the proposed SDP relaxation. Section 5 presents numerical results for the proposed SDP relaxation and those obtained by the equivalent linear model, and finally Section 6 provides some conclusions of this work.

2 System Description

We consider a single cell OFDMA wireless system composed by a base station (BS) and several users. The BS consists of a set of N sub-carriers that have to be assigned to a set of K users using a modulation size of at most M bits in each sub-carrier. Therefore, for each user k we may have a function $f(c_{k,n}, BER_k)$ depending on the amount of bits to be transmitted by the channel pair (k, n) taking into account the BER_k performance of each user. We can use the following formula for user k using subcarrier n and c bits.

$$P^c_{k,n}(t) = \frac{f(c_{k,n}, BER_k)}{\alpha^2_{k,n}(t)} \tag{1}$$

where $\alpha_{k,n}(t)$ represents the time varying channel gain which can be modeled, for example as [8]:

$$\alpha_{k,n}(t) = \sum_{i=1}^{L} w_i \exp^{j(2\pi f_i(t)+\Phi_i)} \tag{2}$$

with L, w_i, f_i and Φ_i being the total number of incident waves, the amplitude, the Doppler frequency and the initial phase of the incident wave, respectively. The main idea is to distribute efficiently sub-carriers of the BS using adaptive modulation while minimizing total power in the system, but also having in mind the R_k bits requirement that each user has. The BS is faced with this NP-hard problem and once the decision is taken, the bits of each user are modulated into an adaptive M-PSK or M-QAM symbol, to be subsequently combined using the inverse fast fourier transform (IFFT) into an OFDMA symbol which is transmitted through a slowly time-varying frequency-selective Rayleigh channel over a bandwidth B.

3 The Quadratic Allocation Scheme

The above problem can be modeled as a quadratic integer programming problem, we call hereby QIP. Using the above power formula (1), the quadratic model can be stated as:

$$\text{QIP}: \qquad \text{Min} \quad \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{c=1}^{M} P^c_{k,n} x_{k,n} y_{k,c} \tag{3}$$

$$\text{st:} \quad \sum_{n=1}^{N}\left[x_{k,n}\sum_{c=1}^{M} c \cdot y_{k,c}\right] = R_k \quad \forall k \tag{4}$$

$$\sum_{k=1}^{K}\left[x_{k,n}\sum_{c=1}^{M} y_{k,c}\right] \leq 1 \quad \forall n \tag{5}$$

$$x_{k,n}, y_{k,c} \in \{0,1\} \tag{6}$$

In this model (3) is the objective function meaning that if a sub-carrier n is assigned to user k using a modulation size of c bits, then $P^c_{k,n}$ has to be minimized. Constraint (4) is the bit rate constraint which uses index c as an integer adaptive modulation parameter to reach the R_k bits needed by each user. Constraint (5) ensures that each sub-carrier must be used by only one user at a time and finally, (6) are binary decision variables. If $x_{k,n} = 1$, then user k is using sub-carrier n and if $y_{k,c} = 1$, then user k uses a size of c bits. Notice that constraint (5) also means that we have a rigid case where each user might use only one size of modulation in its allotted sub-carriers since $\sum_{c=1}^{M} y_{k,c}$ should be equal to one. However, if we introduce index n instead of k for variable $y_{k,c}$, we would have a more flexible situation in which users are allowed to use different sizes of modulations in their assigned sub-carriers. In order to compare our quadratic model with an SDP relaxation, we get the optimal solution of QIP by transforming it into an equivalent Integer Linear Programming model denoted by IP. We introduce linearization variables $\varphi^c_{k,n} = x_{k,n}y_{k,c}$ to get [5]:

$$\text{IP}: \quad \text{Min} \quad \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{c=1}^{M} P^c_{k,n}\varphi^c_{k,n} \tag{7}$$

$$\text{st:} \quad \sum_{n=1}^{N}\sum_{c=1}^{M} c\varphi^c_{k,n} = R_k \quad \forall k \tag{8}$$

$$0 \leq \sum_{k=1}^{K}\sum_{c=1}^{M} \varphi^c_{k,n} \leq 1 \quad \forall n \tag{9}$$

$$x_{k,n} \geq \varphi^c_{k,n} \quad \forall k, n, c \tag{10}$$

$$y_{k,c} \geq \varphi^c_{k,n} \quad \forall k, n, c \tag{11}$$

$$\varphi^c_{k,n} \geq x_{k,n} + y_{k,c} - 1 \quad \forall k, n, c \tag{12}$$

$$x_{k,n}, y_{k,c}, \varphi^c_{k,n} \in \{0, 1\} \tag{13}$$

where constraints (10)-(12) are the linearization Fortet constraints. With this linear IP model, now we derive an SDP relaxation for QIP.

4 The Semidefinite Relaxation

As it is well known, SDP is linear programming over the cone of n square symmetric matrices (S_n) with the condition of satisfying positive semidefiniteness which is defined by the following set $S^+_n = \{Z \in S_n, a \in \mathbb{R}^n / a^T Z a \geq 0\}$ [9]. In this section we derive a semidefinite relaxation of QIP. We define vector z as:

$$z^T = \begin{pmatrix} x_{1,1} & \cdots & x_{1,N} & \cdots & x_{K,N} & y_{1,1} & \cdots & y_{1,M} & \cdots & y_{K,M} \end{pmatrix}$$

Then, let matrix Z be a symmetric positive semidefinite matrix defined by:

$$Z = \begin{pmatrix} W & z \\ z^T & 1 \end{pmatrix} \succeq 0$$

where $W = zz^T$, thus the SDP relaxation can be written as follows:

$$\text{SDP}: \quad \text{Min} \quad \text{Tr}(PZ) \tag{14}$$

$$\text{st:} \quad \text{Tr}(U_k Z) = R_k \quad \forall k \tag{15}$$

$$\text{Tr}(V_n Z) \leq 1 \quad \forall n \tag{16}$$

$$\text{Tr}(\Gamma_{i,j} Z) \geq 0 \quad \forall i < j \tag{17}$$

$$diag(W) = z \tag{18}$$

$$Z \succeq 0 \tag{19}$$

where $\text{Tr}(\cdot)$ denotes the trace operator and P, U_k, V_n are symmetric matrices with entries equal to half the coefficients taken from (3), (4) and (5), respectively. Constraint (18) is a relaxation constraint allowing variables of the last row and column of matrix Z to be between zero and one; ie, relaxing the condition of $z_i^2 = z_i \ \forall i$. The symmetric matrix $\Gamma_{i,j}$ in constraint (17) is used to have positive values in matrix Z and finally constraint (19) imposes the condition of matrix Z to be positive semidefinite.

5 Simulation Results

We solve IP, SDP and LP which is the relaxed solution of IP and simulate one random sample channel varying the number of sub-carriers for different fixed number of users. The sizes of the instances are chosen in such a way we can increase the ratio N/K. This is a reasonable assumption since current wireless systems that utilize OFDMA [12] satisfy these conditions. The maximum modulation size is set to $M = 4$ bits since they are also common values when using M-PSK or M-QAM modulations in OFDMA [11]. The experiments are run for $K = 5$ varying $N = 10$ to $N = 250$, for $K = 10$ with $N = 10$ to $N = 160$, and for $K = 15$ with $N = 20$ to $N = 100$ and the results are shown in tables (1), (2) and (3). Only one channel sample is used due to the high computational effort when computing integer solutions, however some results averaged over 50 channel samples are provided in table (4) using instances of the the same size as in the above experiments. Up to now, we generate random data for powers which

Figure 1: Greedy Heuristic

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Input: x_Sdp, x_Lp, R_k     Output: Feasible Solutions for QIP
for each position
  Pick a uniform Random number r ∈ [0, 1)
  if x_Sdp, x_Lp ≥ r then
    x_Sdp, x_Lp ← 1
  else
    x_Sdp, x_Lp ← 0
end for
if(Not Feasible x_Sdp, x_Lp due to (4),(5) or Not N_k)
  Randomly add or erase subcarriers
end if
for each row of y_Sdp, y_Lp
  Determine the modulation size as: R_k/N_k
    y_Sdp, y_Lp ← 1
end for
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we calculate as $P^c_{k,n} = \frac{c \cdot Rand}{M}$. This assumption is also realistic since higher values are common when using higher modulations. We generate feasible integer solutions using a simple greedy heuristic just to know which one could be better, if LP or SDP. Thus, we compute the greedy heuristic (GH) relative gaps as $Gap_{SDP} = \left[\frac{GH_{SDP}-SDP}{SDP}\right]$ and $Gap_{LP} = \left[\frac{GH_{LP}-LP}{LP}\right]$, respectively. The greedy heuristic is

Table 1: Results for K=5 and M=4

n	IP	LP	GH_{LP}	Gap_{LP}	SDP	GH_{SDP}	Gap_{SDP}
10	0.66	0.51	1.33	1.62	0.66	1.28	0.95
20	0.34	0.24	2.96	11.25	0.28	1.17	3.12
30	0.44	0.31	2.70	7.64	0.37	3.02	7.26
40	0.67	0.55	4.18	6.55	0.64	4.47	5.97
50	1.80	1.35	10.36	6.70	1.79	2.35	0.31
60	0.36	0.21	4.67	21.17	0.28	3.40	11.07
70	1.29	0.82	7.82	8.53	1.08	9.56	7.86
80	3.00	2.09	15.55	6.43	2.91	13.54	3.66
90	†	1.44	13.78	8.59	1.84	12.80	5.96
100	1.55	0.90	9.46	9.51	1.08	9.33	7.65
150	†	2.30	23.65	9.27	2.92	22.00	6.54
200	†	1.79	19.97	10.16	2.25	15.14	5.74
250	†	0.67	21.22	30.72	0.81	22.41	26.60

†: No solution given by Cplex 9.1 due to memory shortage

intended only to find a feasible solution in a fair manner rather than to find the optimal solution of IP. The algorithm for this heuristic is shown in Figure 1 and it simply takes as input the R_k bits needed by each user, the sub-carrier allocation matrices x_{Sdp}, x_{Lp} from SDP and LP, then it does a one randomized

Table 2: Results for K=10 and M=4

n	IP	LP	GH_{LP}	Gap_{LP}	SDP	GH_{SDP}	Gap_{SDP}
10	1.32	0.43	2.61	5.10	0.44	2.34	4.36
20	0.47	0.27	3.77	12.94	0.32	3.06	8.55
30	1.15	0.52	5.67	9.97	0.57	4.91	7.62
40	0.28	0.17	4.60	25.38	0.19	4.74	23.31
50	0.22	0.13	4.66	35.75	0.13	3.69	27.78
60	0.54	0.32	6.23	18.35	0.34	5.45	14.80
70	1.28	0.75	10.55	13.11	0.86	11.13	11.91
80	0.59	0.43	5.23	11.10	0.52	4.50	7.59
90	0.55	0.34	7.42	20.92	0.49	8.07	15.54
100	1.97	1.16	17.46	14.06	1.60	11.44	6.17
120	0.66	0.25	10.41	40.16	0.28	11.04	39.04
140	0.86	0.41	12.09	28.41	0.47	12.45	25.42
160	†	0.91	18,02	18.83	1.17	15.16	11.91

†: No solution given by Cplex 9.1 due to memory shortage

Table 3: Results for K=15 and M=4

n	IP	LP	GH_{LP}	Gap_{LP}	SDP	GH_{SDP}	Gap_{SDP}
20	0.69	0.33	3.71	10.17	0.34	2.90	7.52
25	0.28	0.18	2.97	15.69	0.19	2.72	13.46
30	0.51	0.18	4.36	22.70	0.22	3.67	15.94
35	0.47	0.28	5.41	18.00	0.30	4.43	13.92
40	0.55	0.22	7.78	33.99	0.24	6.84	27.60
45	0.35	0.19	6.06	31.18	0.22	5.94	26.54
50	0.34	0.30	6.70	21.33	0.30	6.37	20.00
55	0.41	0.30	7.56	23.92	0.33	6.45	18.28
60	0.45	0.24	9.60	39.13	0.27	8.77	31.29
70	0.41	0.27	8.02	28.20	0.37	8.42	21.98
80	0.64	0.28	9.09	31.17	0.31	7.90	24.44
90	1.14	0.56	12.06	20.70	0.72	15.71	20.79
100	0.81	0.47	15.46	31.89	0.56	14.32	24.67

rounding iteration on its elements and corrects if there is no feasible solution. To generate these feasible solutions we put the values of $R_k = N_k T$ where $1 \leq T \leq M$ and N_k is a random number of sub-carriers for each user in $1 \leq N_k \leq \lfloor N/K \rfloor$. A Matlab program is developed using Cplex 9.1 and Csdp [10] software

Table 4: Average Results for Some Instances

k	n	Av_{IP}	Av_{LP}	$AvGH_{LP}$	$AvGap_{LP}$	Av_{SDP}	$AvGH_{sdp}$	$AvGap_{SDP}$
5	30	0.83	0.53	5.11	10.67	0.60	3.87	7.17
5	40	0.93	0.59	6.22	13.93	0.70	4.62	9.74
10	50	0.84	0.48	8.76	19.00	0.57	6.10	10.93
10	60	0.77	0.52	10.14	21.07	0.60	7.24	13.25
15	50	0.52	0.30	8.93	32.16	0.33	5.82	18.54
15	60	0.60	0.38	10.77	30.12	0.43	7.71	19.03

for solving SDP, IP and LP. In order to rationalize the benefit introduced by SDP model, we calculate for tables (1), (2), (3) the differences as $\left[\frac{SDP-LP}{LP}\right] \cdot 100$ and we see that for $K = 5, N = 10$ to $N = 250$, we have a tightness average gain of 26%. For $K = 10, N = 10$ to $N = 160$, we have a gain of 17% and for $K = 15, N = 20$ to $N = 100$, a gain of 13%. Notice from these gaps that we get better results when the ratio N/K is bigger. Besides, calculating the gap's differences as $\left[\frac{Gap_{LP}-Gap_{SDP}}{Gap_{LP}}\right] \cdot 100$ we can observe an average gain of 35% in table (1), of 23% in table (2) and 19% in table (3), between the integer solution obtained by SDP and the one given by LP using the greedy heuristic which confirm SDP tightness. We notice again from these gaps that better results are obtained when the ratio N/K is bigger. On the other hand, if we compare each instance of table (4), with their respective one sample channel counterpart in the first three tables, we observe better results in all cases which also suggests that the proposed SDP relaxation is better, in average, than the LP relaxation.

6 Conclusions

In this work, we proposed a new quadratic formulation over DL OFDMA multi-user wireless systems when using adaptive modulation. The model allows to decide what sub-carriers and what modulations have to be used by different users while minimizing power. Thus, we derived a semidefinite relaxation to solve this model and by numerical results we achieved an average total gain in tightness of 18.65 % when compared to the optimal value of the quadratic formulation. Besides, using the SDP and LP relaxations we observed an improvement of 25.49 % for SDP when approximating integer solutions with a simple greedy heuristic. Finally, we conclude that SDP is better than LP and while the bigger the ratio N/K is, the bigger the gain that can be reached.

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