

A Multi Population Genetic Algorithm for Hop-Constrained Trees in Nonlinear Cost Flow Networks¹

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Abstract

Genetic algorithms and other evolutionary algorithms have been successfully applied to solve constrained minimum spanning tree problems in a variety of communication network design problems. Here we propose a multi population genetic algorithm for yet another communication design problem. The new problem is modeled through a hop-constrained minimum spanning tree also exhibiting the characteristic of flows. In addition to the fixed charge costs, nonlinear flow dependent costs are also considered. This problem is an extension of the well know NP-*hard* hop-constrained Minimum Spanning Tree problem (HMST). The efficiency and effectiveness of the proposed method can be seen from the computational results reported.

Keywords: *Multi population, genetic algorithms, network flows, hop constrained trees, and general nonlinear costs.*

1 Introduction

Communication Network Design has increased significantly in the last decade due to the dramatic growth in the use of internet for business and personal use. A cost-effective structure for a large communication network is a multilevel hierarchical structure consisting of a backbone network (high level) and local access networks (low level) [8]. The Minimum Spanning Tree (MST) problem is one of the best-known network optimization problems used for designing backbone networks, which attempts to find a minimum cost tree network that connects all the nodes in the communication network. There might be other constraints imposed on the design such as the number of nodes in a subtree, degree constraints on nodes (degree constrained), flow and capacity constraints on any arc or node, and type of services available on the arcs or nodes [18, 20]. We consider a problem which is an extension of the Hop-constrained Minimum Spanning Tree problem (HMST), since we also consider flows. All nodes, except for the root node, have a nonnegative flow requirement. Therefore, in addition to finding the arcs that are part of the HMST, we must also find the flow that is to be routed along each of these arcs. Thus, we have named this problem Hop-constrained Minimum cost Flow Spanning Tree, HMFST. The cost, which is to be minimized, is nonlinear and made of two components: arc setup costs, as usual, and nonlinear flow dependent routing costs.

The hop constrained minimum spanning tree problem is frequently encountered in network design problems, for example in computer networks we can find the multicast-routing problem (see, e.g., [4]), where a number of clients and a server are connected by a common communication network. The server wishes to transmit identical information to all clients, and does so by transmitting the data to the nodes it

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directly connects to, and these latter nodes forward incoming data to their respective children in the tree. Assuming that all arcs in the network have roughly the same transmission delay (which is a reasonable assumption in local area networks), it is not hard to see that this problem can be cast in the HMST framework.

The hop constraints are usually used to guarantee a certain quality of service with respect to availability and reliability constraints [17, 21], as well as lower delays [12]. This is expected since availability is the probability that all transmission lines, i.e arcs, in a path from the root node are operational, while reliability is the probability that during a session no arc, in the path being use, will fail. Network delays need to be kept under control, since otherwise they may lead to further problems such as the need for packet reordering. This latter problem as been addressed recently by, e.g., [19].

Woolston and Albin [21] have shown that, by including hop constraints it is possible to generate network designs with a much better quality of service and with only a marginal increase in the total cost. Gouveia [10] presented several node oriented formulations for the hop constrained minimal spanning tree problem based on the Miller-Tucker-Zemlin subtour elimination constraints. The author presented some lower bounding schemes based on Lagrangian relaxation and a fast heuristic for obtaining feasible solutions, which he has subsequently improved as reported in [11]. More recently, Dahl et al. [3] have presented and surveyed several ways of computing lower bounds, including: Lagrangian relaxation, column generation, and model reformulation. Kawatra [15] has addressed a slightly different problem, since he also considers the downtime cost associated with each node that gets disconnected from the central service provider due to arc failure. A Lagrangian relaxation method is developed to find a lower bound of the optimal solution, which is optimized by using subgradient optimization to find Lagrangian multipliers. A branch exchange heuristic is used to obtain a feasible solution from an infeasible Lagrangian lower bound solution. The best feasible solution is retained when the heuristic method stops. The best lower bound given by the Lagrangian relaxation method is used to estimate the quality of the heuristic solution.

Recently, Genetic Algorithm (GA) and other Evolutionary Algorithms (EAs) have been successfully applied to solve constrained spanning tree problems of the real life instances; and also have been used extensively in a wide variety of communication network design problems [8, 9]. For example, some authors have proposed GAs for the capacitated MST problem [1, 16, 5], while others propose GAs for the degree-constrained MST, see for example [13] and [22] for a hybrid GA (with local search).

Here, we proposed a hybrid genetic algorithm to solve the hop-constrained minimum cost flow spanning tree problem (an extension of the hop-constrained MST problem). It should be noticed that in this problem we also have flow decisions to be made. Furthermore, the costs to be minimized include a general nonlinearly flow dependent cost component and fixed cost component. Nonlinear cost functions arise naturally in this type of problems as a consequence of taking into account economic considerations. Set up costs or fixed-charge costs arise, for example, due to the consideration of a new customer or a new route. Economies of scale often exist, and thus an output increase leads to a decrease in the marginal costs. On the other hand, further output increase may lead to an increase in marginal costs, e.g. by implying the need of extra resources. Therefore, discontinuities are observed. These may also arise due to price-discounting. As far as we are aware of, no previous work, except that of [7] has considered HMST with flow requirements and/or having nonlinear costs.

2 Proposed Genetic Algorithm

Since Holland first proposed GAs in the early 1970s as computer programs that mimic the evolutionary processes in nature [14], the GAs have been demonstrating their power by successfully being applied to many practical optimization problems in the last decade.

We propose a multi population Genetic Algorithm (MPGA) that hybridizes the GA described below with a local search method. The GA uses a Tree-Constructor procedure to generate feasible trees and a penalty cost function to force them to satisfy the Hop constraints. Finally, the local search heuristic is applied in order to try to improve the quality of the solutions by searching amongst neighbor feasible

solutions.

The GA is responsible for evolving the chromosomes, which represent the parameters (node priorities) used by the Tree-Constructor. For each chromosome the following are applied, in turn: *Decoding of priorities* that transforms the chromosomes supplied by the GA into node priorities; a *Tree-Constructor* that constructs a feasible tree based on node priorities; and a *Local search* procedure which tries to improve the current solution by means of small perturbations. After a solution is obtained the corresponding quality (cost) is measured and fed back to the GA.

The Hop constraints are handled implicitly by using a penalty function, which is dependent on the constraints violation degree, that is we penalize solutions having more than H arcs in the path from the source node to any node, as follows.

$$\sum_{\text{for all nodes}} \text{Max}(0, \text{patharcs}_i - H) \times M,$$

where patharcs_i represents the number of arcs from the source to node i and M represents the penalty factor. In summary our approach comprises a GA, a Tree-Constructor, a local search and a penalty cost function.

We use a multi population Strategy, where several populations are evolved independently. After a pre-determined number of generations all the populations exchange information. The information exchanged is the chromosomes of good quality. When evaluating possible interchange strategies we noticed that exchanging too much information (exchanging too many chromosomes) leads to the disruption of the evolutionary process. Also, if the populations exchange information with a high frequency they do not have enough time to produce good results because their evolutionary process is disrupted before good solutions can be achieved. Having this information in mind we chose a multi-population strategy that after a pre-determined number of generations (empirically determined) inserts only the best two chromosomes in all populations.

3 Computational results

The algorithm presented in this paper was computationally evaluated by solving a set of randomly generated test problems. The problems considered are amongst the most difficult nonlinear network flow problems since all arcs have cost functions that are neither convex nor concave. The problems used are those proposed in [6], where a thorough description of the generation procedure is provided. Nevertheless they can be downloaded from the OR-Library, see [2].

We consider the following three different cost functions G1-G3:

$$g_{ij}(r) = \begin{cases} 0, & \text{if } r = 0, \\ -a_{ij}r^2 + b_{ij}r + c_{ij} & \text{if } r \leq \bar{R}, \\ a_{ij}r^2 + b_{ij}r + c_{ij} + k & \text{otherwise.} \end{cases}$$

where $a_{ij} = 0$ for G1 and G2, $k = b_{ij}$ for G1, and $k = -b_{ij}$ for G2 and G3.

Cost functions G1 and G2 are variations of the fixed-charge cost function, while cost function G3 is initially concave and then convex. Types G1 and G2 correspond, respectively, to the so called staircase and sawtooth cost functions with two segments.

In table 1 to 3 we summarize the results obtained for uncapacitated problems involving cost functions of types G1, G2, and G3, respectively, with the discontinuity point occurring at 50% of the root node outflow.

Four different hop parameter values have been considered $H = 3, 5, 7, 10$. For $H = 3$ there are 19 problems for which no feasible solution exists and the constraints are effective for almost all problems. For $H = 10$ the hop constraints are no longer effective.

We report the average computational time, in seconds, required by the MPGA. We also report the average deviation from the optimal value. The deviations from the optimal values have been computed using the optimal solutions obtained by the dynamic programming methodology proposed in [7].

For each size, cost function type and H value we have solve 30 problem instances. Thus, overall we have solved 450 problem instances for each hop parameter value H and cost function type.

H	Number of nodes				
	10	12	15	17	19
3	0.000	0.134	0.054	0.082	0.782
5	0.000	0.016	0.000	0.000	0.193
7	0.000	0.000	0.009	0.000	0.011
10	0.000	0.000	0.000	0.000	0.000
Time	12.60	22.50	33.40	50.10	63.00
DP Time	0.03	0.36	9.91	113.13	1972.33

Table 1: Solution quality (% deviation from optimal) and time (s) for cost function type G1.

H	Number of nodes				
	10	12	15	17	19
3	0.143	0.125	0.054	0.082	0.784
5	0.000	0.015	0.000	0.000	0.195
7	0.000	0.000	0.008	0.000	0.011
10	0.000	0.000	0.000	0.000	0.000
Time	12.70	20.40	34.20	50.90	64.10
DP Time	0.04	0.36	9.67	108.89	1864.30

Table 2: Solution quality (% deviation from optimal) and time (s) for cost function type G2.

H	Number of nodes				
	10	12	15	17	19
3	0.000	0.161	0.026	0.161	0.528
5	0.000	0.013	0.000	0.000	0.209
7	0.000	0.000	0.004	0.000	0.009
10	0.000	0.000	0.000	0.000	0.000
Time	15.10	25.30	44.60	64.50	85.00
DP Time	0.04	0.34	9.74	107.89	2053.50

Table 3: Solution quality (% deviation from optimal) and time (s) for cost function type G3.

As can be seen, from the results reported in tables 1 to 3, the multi population genetic algorithm finds an optimal solution for most problems. Exceptions happen for all problem sizes when $H=3$ and for problems with 19 nodes when $H=5$. In order to better understand the results obtained and the conclusions drawn we provide some graphical representations of the results, given in figures 1 and 2. The computational times reported show that the time requirements of the DP algorithm grow very rapidly, however, this trend is not observed in the proposed algorithm, see Figure 2. It should be noticed that each figure shown in the tables and in the graphs is an average obtained after solving 30 problem instances for each combination of problem size, cost function type, and hop parameter value.

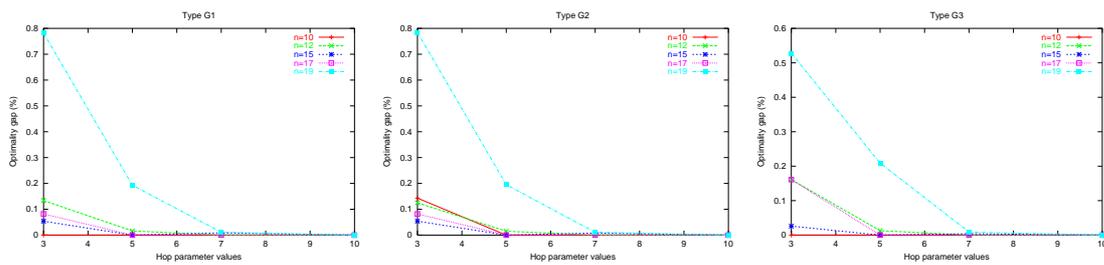


Figure 1: The effect of the hop parameter value on solution quality.

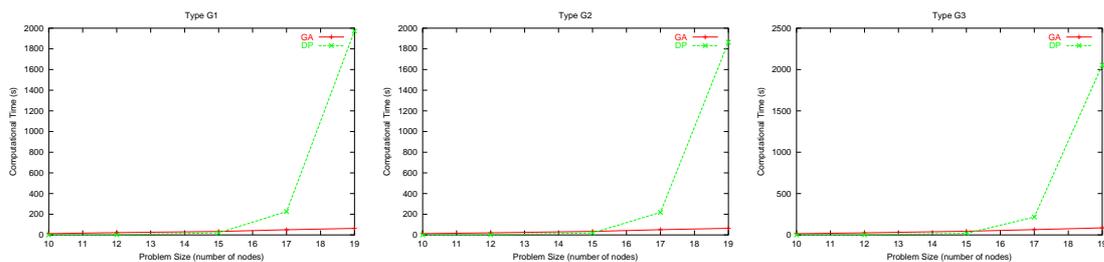


Figure 2: The effect of the hop parameter value on computational time.

4 Conclusions

In this paper we presented a Multi Population Hybrid Genetic Algorithm that finds nearly optimal, actually optimal for most problem instances solved, Hop-Constrained Trees in Nonlinear Cost Flow Networks. The proposed algorithm combines a local search algorithm with a Multi Population genetic algorithm, where several populations are evolved independently.

The results obtained have been compared with existing literature and the comparisons have shown the proposed algorithm to improve upon the efficiency of existing methods [7], since the computational time requirements are modest for all problem sizes (always below two minutes). We have solved 150 problem instances with four different hop-parameter values and three cost function types. When the Hop-constraints are not very tight we were able to find an optimal for almost all problems. Nevertheless, when they are very tight we are still able to find very good solutions.

Thus, the Multi Population Hybrid Genetic Algorithm proposed here is capable of efficiently finding heuristic solutions, close to optimal, for the Hop-Constrained Minimum Cost Flow Spanning Tree Problem, which is NP-hard.

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