An optimization model for path diversity protection in IP-over-WDM networks

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Abstract

The paper addresses an optimization problem related to flow and link capacity design in resilient two-layer networks. The considered problem assumes that each link established in the upper layer is supported by a single path in the lower layer, and that traffic demands are protected by path diversification in the upper layer. Two mixed-integer programming formulations of this problem are presented and discussed. Since direct resolving of these formulations requires pre-selection of "good" candidate paths in the upper layer, the paper presents an alternative approach which is based on decomposing the resolution process into two phases, resolved iteratively. The first phase subproblem is related to designing lower layer path-flows that provide the capacities for the (logical) links of the upper layer. The second phase is designing the flow patterns in the upper layer protected through path diversification. In this phase we take into account multiple failures of the logical links (so called *shared risk link groups*) that result from single failures of the lower layer links. The effectiveness of the proposed two-phase method is illustrated with numerical examples.

Keywords*: network design, two-layer network optimization, resilient routing, path diversity*

1 Introduction

A multi-layer network requires a complex network model with a layered structure of resources, operated according to layer-depended network protocols [6]. Resources of a layered network form a hierarchical structure with each layer constituting a proper network in itself. The paper addresses optimization of traffic flows within a two-layer IP-over-WDM network. In the considered network model the lower layer corresponds to a WDM network composed of the WDM cross-connects interconnected by WDM links (called optical links, as they are realized on optical cables). The upper layer corresponds to an IP network composed of the IP routers interconnected by IP links (called logical links), see [1]. In such an architecture, the IP links are established by means of the lower layer WDM paths, composed of the optical links.

The paper addresses an optimization problem related to flow and link capacity design in the resilient two-layer IP-over-WDM networks. We consider a particular version of the problem where each link established in the upper layer is supported by a single path in the lower layer, and where the traffic demands are protected by path diversification in the upper layer. The assumed objective is to minimize the total dimensioning cost of the optical links. Since in our model each IP link is supported by one (unique) path in the WDM layer, the unit dimensioning cost for a given realization of a particular upper layer link is calculated as the length of its unique supporting path—the sum of the unit capacity costs of the optical links composing the considered path. Thus, for a given set of traffic requirements (demands) the considered problem consists in determining: (i) a set of IP layer path flows (IP path flows), (ii) the resulting capacities of the IP links, (iii) their realizations by means of single path flows (WDM path flows) in the WDM layer, and (iv) the resulting capacities of the optical links for which the total dimensioning cost is minimized.

As WDM constitutes a physical layer, the WDM optical links (a WDM link can be treated as a set of optical cables) are subject to failures. We make a typical (and realistic) assumption that during a failure exactly one of the optical links becomes unavailable (for example, as a result of a cable cut), and all the IP links supported by paths traversing the affected optical link become unavailable too. Therefore, a failure of a single optical link may cause unavailability of several IP links, and this is seen, in practice, as a multiple link failure in the upper layer. In the literature such a failure model is called *shared risk resource group* (see [8]).

Suppose that the upper-layer flows assigned to a specific traffic demand can be bifurcated and diversified (recall that the non-bifurcated flow assumption refers only to the lower layer). Diversification of the flows constitutes a means for protecting traffic in the IP-over-WDM network against failures of the optical cables in the WDM layer. We refer to such a protection mechanism as *path diversity* (see [2, 6]).

In the two-layer network model the set of lower layer nodes is typically different than the set of the upper layer nodes, and in fact the former is a proper superset of the latter. When all the node sites comprise both WDM and IP devices, the two-layer network design problems can in most cases (also in our case) be reduced to an equivalent one-layer optimization problem. The reduction consists in establishing one-to-one equivalence among the IP links and WDM links (each IP link is supported by a path composed of a single WDM link). Still, when only a subset of the sites comprise both WDM cross-connects and IP routers, such a reduction is not feasible—this case is studied in the balance of the paper.

The paper is organized as follows. In Section 2 we formulate the considered problem as two mixed-integer programs (MIPs). Since both formulations require identifying all possible paths in the upper layer, resolving these using general MIP solvers is inefficient. Hence, in Section 3 we present a dedicated method based on decomposing the resolution process into two iteratively invoked phases. The numerical results illustrating the efficiency of the method are presented and discussed in Section 4. The paper is summarized in Section 5.

2 Problem formulation

Let $\mathcal{V}$ and $\mathcal{W}$ (with $\mathcal{V} \subset \mathcal{W}$) be the sets of the IP and WDM nodes, and let $\mathcal{E}$ and $\mathcal{F}$ be the sets of the upper- and lower layer links, respectively. Then, the two network graphs are defined as $\mathcal{G}(\mathcal{V},\mathcal{E})$ – the upper layer graph, and $\mathcal{H}(\mathcal{W},\mathcal{F})$ – the lower layer graph. Demands $d \in \mathcal{D}$ are represented by the pairs of the upper layer nodes. Each demand $d \in \mathcal{D}$ requires demand volume (bandwidth) h_d between its end nodes which is realized by means of flows assigned to the candidate paths from set $\mathcal{P}_d$; all the paths in $\mathcal{P}_d$ which traverse link $e \in \mathcal{E}$ are denoted by $\mathcal{P}_{ed}$ ($\mathcal{P}_{ed} \subseteq \mathcal{P}_d$). In the considered model, each IP link is supported by one WDM path. For link $e \in \mathcal{E}$ such a unique path is selected from the set of predefined candidate WDM paths $\mathcal{Q}_e$. The set $\mathcal{Q}_{fe}$ ($\mathcal{Q}_{fe} \subseteq \mathcal{Q}_e$) denotes the set of all paths in $\mathcal{Q}_e$ containing the lower layer link $f \in \mathcal{F}$.

It is assumed that the lower layer links are subject to failures, and only one lower layer link can fail at a time. As not necessarily all links are subject to failures, the set of all failures (referred to as failure states), denoted by $\mathcal{S}$, is in fact a subset of the set of optical links $\mathcal{F}$. Hence, $\mathcal{S} \subseteq \mathcal{F}$ and each failure state $s \in \mathcal{S}$ corresponds to a failure of a link $f \in \mathcal{F}$; this link will be denoted by $f(s)$.

In the considered problem the state-dependent variables x_{dps} express the (upper layer) flow of demand $d \in \mathcal{D}$ assigned to path $p \in \mathcal{P}_d$ in state $s \in \mathcal{S}$. Variable x_{dp0} determines the maximum of x_{dps} over all $s \in \mathcal{S}$; these variables are used to specify the variable y_e ($e \in \mathcal{E}$) expressing the load (capacity) of the upper layer link $e \in \mathcal{E}$. The link flows $y_e,\ e \in \mathcal{E}$ specify the requirement imposed on the lower layer paths. For each upper layer link $e \in \mathcal{E}$ such a link flow is realized by a non-bifurcated lower layer flow specified by path flows variables $z_{eq},\ q \in \mathcal{Q}_e$. Since the flow realizing link e must be non-bifurcated, it is additionally characterized by binary variables u_{eq} where $u_{eq} = 1$ identifies the path $q \in \mathcal{Q}_e$ selected to realize flow of link y_e. The capacity of each lower layer link $f \in \mathcal{F}$ is denoted by variable Y_f; the cost of realizing one unit of capacity on link f is denoted by ξ_f. Then, binary variables $w_{es},\ e \in \mathcal{E}$, called the upper layer link failure coefficients, determine the set of the upper layer links affected by failure state $s \in \mathcal{S}$. Finally, the upper layer path failure coefficients are represented by binary variables r_{dps} which determine whether the IP path $p \in \mathcal{P}_d$ is affected by failure $s \in \mathcal{S}$ (due to unavailability of the WDM link $f(s)$ used to realize the links of path p). The objective of the optimization problem is to minimize the total capacity cost of the lower layer links. Its MIP formulation is as follows:

$$\textbf{\textit{minimize}} \quad \textstyle\sum_{f\in\mathcal{F}} \xi_f Y_f \tag{1a}$$
$$\textstyle\sum_{p\in\mathcal{P}_d} x_{dps} \geq h_d \qquad d \in \mathcal{D},\ s \in \mathcal{S} \tag{1b}$$
$$x_{dps} \leq x_{dp0} \qquad d \in \mathcal{D},\ p \in \mathcal{P}_d,\ s \in \mathcal{S} \tag{1c}$$
$$x_{dps} \leq (1 - r_{dps}) h_d \qquad d \in \mathcal{D},\ p \in \mathcal{P}_d,\ s \in \mathcal{S} \tag{1d}$$
$$\textstyle\sum_{d\in\mathcal{D}} \sum_{p\in\mathcal{P}_{ed}} x_{dp0} \leq y_e \qquad e \in \mathcal{E} \tag{1e}$$
$$\textstyle\sum_{q\in\mathcal{Q}_e} z_{eq} = y_e \qquad e \in \mathcal{E} \tag{1f}$$
$$\textstyle\sum_{q\in\mathcal{Q}_e} u_{eq} \leq 1 \qquad e \in \mathcal{E} \tag{1g}$$
$$z_{eq} \leq M u_{eq} \qquad e \in \mathcal{E},\ q \in \mathcal{Q}_e \tag{1h}$$
$$\textstyle\sum_{e\in\mathcal{E}} \sum_{q\in\mathcal{Q}_{fe}} z_{eq} \leq Y_f \qquad f \in \mathcal{F} \tag{1i}$$

$$\textstyle\sum_{q\in\mathcal{Q}_{f(s)e}} u_{eq} \le w_{es} \qquad e\in\mathcal{E},\ s\in\mathcal{S} \tag{1j}$$
$$w_{es} \le r_{dps} \qquad e\in\mathcal{E},\ d\in\mathcal{D},\ p\in\mathcal{P}_{ed},\ s\in\mathcal{S}. \tag{1k}$$

The above formulation assumes non-negativity of all the continuous variables. The three constraints (1b)–(1d) assure that at least h_d amount of bandwidth is allocated to demand d in each state $s \in \mathcal{S}$, according to the path diversity protection mechanism. Constraint (1e) is a conventional capacity constraint determining the values of the IP link flows y_e. Constraint (1f) assures that the flow y_e of each IP link $e \in \mathcal{E}$ is supported by the WDM path flows which are non-bifurcated due to constraints (1g) and (1h) (where M is a large constant). Constraint (1i) is a capacity constraint for an optical link. Constraint (1k) makes sure that $r_{dps} = 1$ when at least one link of path $p \in \mathcal{P}_d,\ d \in \mathcal{D}$ is affected by failure $s \in \mathcal{S}$. The fact that link $e \in \mathcal{E}$ is affected by failure s is expressed as $w_{es} = 1$, and this is assured by constraint (1j) ($w_{es} = 1$ is forced when the link which fails in state s, i.e., link $f(s)$, is in the unique path supporting link e).

Formulation (1) is the so called link-path formulation based on path flows. It requires that the sets of candidate paths (in both layers) are given in advance. This poses a severe problem when we wish to take into account all possible paths because the number of paths in a graph grows exponentially with the number of nodes. To partially alleviate this issue we notice that formulation (1) can be transformed to a node-link formulation for the lower layer. Such a node-link formulation takes (implicitly) all possible paths into account so that predefinition of the candidate paths for the lower layer is not necessary.

Let $f \in \delta^+(v)$ and $f \in \delta^-(v)$ be the sets of links outgoing from, and incoming to node $v \in \mathcal{W}$, and let Δ_{ve} be a constant equal to 1 if v is the starting node of e, to -1 if v is terminating node of e (and 0, otherwise). Let z_{fe} ($f \in \mathcal{F},\ e \in \mathcal{E}$) be a variable specifying the flow realizing upper layer link e on the lower layer link f. Besides, we define binary variables u_{fe} which are called realizations of the upper layer links and determine whether link f is traversed by the path selected to establish link e. The modified formulation is as follows:

$$\textbf{\textit{minimize}} \quad \textstyle\sum_{f\in\mathcal{F}} \xi_f Y_f \tag{2a}$$

(1b) - (1e), (1k)

$$\textstyle\sum_{f\in\delta^+(v)} z_{fe} - \sum_{f\in\delta^-(v)} z_{fe} = \Delta_{ve} y_e \qquad v\in\mathcal{V},\ e\in\mathcal{E} \tag{2b}$$
$$\textstyle\sum_{f\in\delta^+(v)} u_{fe} \le 1 \qquad v\in\mathcal{V},\ e\in\mathcal{E} \tag{2c}$$
$$z_{fe} \le M u_{fe} \qquad e\in\mathcal{E},\ f\in\mathcal{F} \tag{2d}$$
$$\textstyle\sum_{e\in\mathcal{E}} z_{fe} \le Y_f \qquad f\in\mathcal{F} \tag{2e}$$
$$u_{f(s)e} = w_{es} \qquad e\in\mathcal{E},\ s\in\mathcal{S}. \tag{2f}$$

Constraint (2b) expresses the flow conservation law characteristic for the node-link notation. The non-bifurcated routing of the link flows y_e in the lower layer is assured by binary variables u_{fe} and constraint (2c). Due to (2d) only the flows of the selected links can be positive (M is in this case the maximal capacity of an IP link). Finally, (2e) is the capacity constraint for the lower layer links, and (2f) is a counterpart of (1j).

As demonstrated in [9], the network optimization problems related to the path diversity protection assuming multiple-link failures are $\mathcal{NP}$-hard. Therefore, as discussed in [4], they cannot be formulated as linear programs using the compact node-link notation, and must use the non-compact link-path notation, as in formulation (1).

3 Resolution approach

MIP formulations (1) and (2) represent an $\mathcal{NP}$-hard problem. Therefore, it is not surprising that available MIP optimization solvers are able to resolve them only for small network instances. Hence, we have developed a method which decomposes the resolution process into two subsequently invoked phases. The proposed method allows to identify the set of the necessary candidate paths in the upper layer using the column generation technique (see [6]), and to design the IP links realizations by resolving an appropriate MIP of the size significantly smaller than the size of (1).

The proposed approach assumes that realizations (i.e., paths in the lower layer) of the upper layer links are known during the first phase. The implication of this assumption is two-fold. First, it means that the values of variables r_{dps} are fixed and given, and second, that the unit costs of the upper layer links are also fixed and given. Hence, we can solve the upper layer optimization problem through the path generation technique described in [5]. The resulting problem, denoted by $\mathbf{M}$, is as follows.

$$\textbf{\textit{minimize}} \quad \sum_{e\in\mathcal{E}} \zeta_e y_e \tag{3a}$$
$$\sum_{p\in\mathcal{P}_d} r_{dps} x_{dp0} \geq h_d \qquad d\in\mathcal{D}, s\in\mathcal{S} \tag{3b}$$
$$\sum_{d\in\mathcal{D}}\sum_{p\in\mathcal{P}_{ed}} x_{dp0} = y_e \qquad e\in\mathcal{E}, \tag{3c}$$

where ζ_e is the unit capacity cost of the upper layer link $e \in \mathcal{E}$ resulting from the assumed realization of e in the lower layer: ζ_e is calculated as the sum of ξ_f along the path selected to realize link e. Problem $\mathbf{M}$ is referred to as the master problem in the context of path generation. Below we describe the path generation algorithm for (for details see [5]).

Consider the master problem with fixed sets of candidate paths $\mathcal{P}_d,\ d \in \mathcal{D}$. Let $(\lambda_d^s)^*\ (d \in \mathcal{D},\ s \in \mathcal{S})$ be optimal dual variables corresponding to constraints (3b), and let $\Lambda_d^* = \sum_{s\in\mathcal{S}}(\lambda_d^s)^*\ (d \in \mathcal{D})$. At each iteration we are interested in finding, for each demand $d \in \mathcal{D}$, a shortest path p with respect to the generalized length $\langle p\rangle = \sum_{e\in p}\xi_e + \sum_{s\in\bar{\mathcal{S}}_p}(\lambda_d^s)^*$ (where $\bar{\mathcal{S}}_p$ is the set of all states $s \in \mathcal{S}$ in which path p fails). If this length is smaller than Λ_d^* then the path is added to $\mathcal{P}_d$ since we can expect that this will improve the current optimal solution of the master problem. The iterations stop when for no demand such a path exists—then the sets of candidate paths contain all necessary paths that assure the optimality of the master problem with respect to all possible paths. While the pricing problem stated above (i.e., shortest path generation) is solvable in polynomial time (using, for example, the Dijkstra algorithm) in the case of single-link failures, it becomes difficult in the case of multiple-link failures. Then, however, the pricing problem can be approached in a way described in [7]. The basic idea is to compute the dual length $\langle p\rangle = \sum_{e\in p}\xi_e + \sum_{s\in\bar{\mathcal{S}}_p}(\lambda_d^s)^*$ of each path p. The efficiency of the procedure is improved by skipping the paths for which some domination rules as proposed in [7] can be applied.. The set of all the non-dominated paths can be generated by means of a label-setting algorithm for *shortest-path problems with resource constraints* (SPPRC) [3], where the resources are the failure states.

As an extension of the SPPRC algorithm, we have also introduced path length limitation—an important contribution to the reduction of the size of the set of non-dominated paths (introduced for another problem in [2]). The extension is based on the observation that excessively long paths are useless as they cannot improve the current solution, or the solutions they represent are known to be worse than some already known solutions. For example, a simple path length restriction may be expressed as follows: $\sum_{e\in p}\xi_e < \Lambda_d^*$. Also, the knowledge about some path p' representing a feasible solution can help to tighten the path length restriction. In such a case we are only interested in finding a path p satisfying $\sum_{e\in p}\xi_e < \sum_{e\in p'}\xi_e + \sum_{s\in\bar{\mathcal{S}}_{p'}}(\lambda_d^s)^*$. Applying path length limitation results in significant reduction of the pricing time.

The proposed two-phase method for resolving the considered problem is given as Algorithm 1.

Algorithm 1 The decomposed iterative procedure

Step 1. For each upper layer link $e \in \mathcal{E}$ find its cheapest realization, i.e., a lower layer path cheapest with respect to the lower layer unit link costs ξ_f. Denote the resulting upper layer link failure coefficient vector by $\boldsymbol{w}^0$, and calculate the upper link unit capacity costs $\boldsymbol{\zeta}_e, e \in \mathcal{E}$.

Step 2. Resolve problem $\mathbf{M}$ using path generation for the fixed vector $\boldsymbol{w} = \boldsymbol{w}^0$, and denote the obtained upper link load vector by $\boldsymbol{y}^0$.

Step 3. Resolve problem $\mathbf{R}$ for the fixed vector $\boldsymbol{y} = \boldsymbol{y}^0$, and denote the resulting upper layer link failure coefficient vector by $\boldsymbol{w}^0$. If the the cost $\sum_{e\in\mathcal{E}} y_e(\sum_{f\in\mathcal{F}}\ \xi_f u_{fe})$ has not decreased, stop. Otherwise, calculate the upper link unit capacity costs $\boldsymbol{\zeta}_e,\ e \in \mathcal{E}$ and go to Step 2.

The subproblem of the second phase (Step 3 of Algorithm 3), denoted by $\mathbf{R}$, is an optimization problem of identifying the realization of the given upper layer links, i.e., the appropriate lower layer paths. Derivation of problem $\mathbf{R}$ is a bit complicated. Let us start with a simplified version of problem $\mathbf{R}$ used in Step 1. It is specified by constraints (2b) (with $y_e \equiv 1$), (2c), (2f), and by the following objective function:

$$\textbf{\textit{minimize}} \quad \sum_{e\in\mathcal{E}}\sum_{f\in\mathcal{F}}\xi_f u_{fe} \tag{4}$$

(of course this problem can be solved by the shortest-path algorithm, for example by the Dijkstra algorithm). The resulting vector of the upper link realizations $\boldsymbol{u} = (u_{fe}:\ \ f \in \mathcal{F},\ e \in \mathcal{E})$ defines the unit link costs $\zeta_e,\ e \in \mathcal{E}$ and the link failure coefficients $\boldsymbol{w} = (w_{es} : \ e \in \mathcal{E}, \ s \in \mathcal{S})$ which in turn determine new values of the path failure coefficients $\boldsymbol{r} = (r_{dps} : \ d \in \mathcal{D}, \ p \in \mathcal{P}_d, \ s \in \mathcal{S})$ for the next iteration of the first phase (Step 2) of the method, i.e., for problem (3).

Note that the above version of problem **R** does not depend on the output of problem **M** and therefore will stop not lead to any iterations. Because of that we redefine problem **R**.

Now, observe that problem **M** can equivalently be formulated as a system of constraints (1b)–(1e) with objective function (3a). Regarding the dual associated with this formulation, let $\boldsymbol{\lambda} = (\lambda_{dps} : \ d \in \mathcal{D}, \ p \in \mathcal{P}_d, \ s \in \mathcal{S})$, $\boldsymbol{\beta} = (\beta_{dps} : \ d \in \mathcal{D}, \ p \in \mathcal{P}_d, \ s \in \mathcal{S})$, $\boldsymbol{\gamma} = (\gamma_{dps} : \ d \in \mathcal{D}, \ p \in \mathcal{P}_d, \ s \in \mathcal{S})$, and $\boldsymbol{\pi} = (\pi_e : \ e \in \mathcal{E})$ be the vectors of Lagrangean multipliers associated with constraints (1b), (1c), (1d), and (1e), respectively. Using theses multipliers we write down the dual:

$$\textbf{\textit{maximize}} \ \textstyle\sum_{d \in \mathcal{D}} \sum_{s \in \mathcal{S}} h_d \lambda_{ds} - \sum_{d \in \mathcal{D}} \sum_{p \in \mathcal{P}_d} \sum_{s \in \mathcal{S}} (1 - r^*_{dps}) h_d \beta_{dps} \tag{5a}$$

$$\textstyle\sum_{s \in \mathcal{S}} \alpha_{dps} \leq \sum_{e \in \mathcal{E}_p} \pi_e \qquad d \in \mathcal{D}, \ p \in \mathcal{P}_d \tag{5b}$$

$$\lambda_{ds} \leq \alpha_{dps} + \beta_{dps} \qquad d \in \mathcal{D}, \ p \in \mathcal{P}_d, \ s \in \mathcal{S} \tag{5c}$$

$$\pi_e \leq \xi_e \qquad e \in \mathcal{E}. \tag{5d}$$

Let $(\boldsymbol{\lambda}^0, \boldsymbol{\alpha}^0, \boldsymbol{\beta}^0, \boldsymbol{\pi}^0)$ denote the optimal solution of (5), and consider path p and state s for which $r^*_{dps} = 1$ and $\beta^0_{dps} > 0$. Suppose that the value of r^*_{dps} is forced to be zero, and (5) is re-optimized. It can be shown that new value of λ_{dps} can be smaller than λ^0_{dps} because corresponding β_{dps} is now minimized due to its negative coefficient in (5a). Thus, setting r_{dps} to zero in the next step of the procedure can potentially decrease the optimal value of the primal objective function, i.e., the dimensioning cost.

Similarly, setting r^*_{dps} to zero for path p and state s for which $\alpha^0_{dps} > 0$ can also lead to decreasing the optimal value of objective function (5a). Due to these observations we consider an alternative objective function of **R** which takes into account the output of **M**. Let $\mathcal{I}_a$ and $\mathcal{I}_b$ be the sets of triplets (d, p, s) for which $\alpha^0_{dps} > 0$ and $\beta^0_{dps} > 0$, i.e., $\mathcal{I}_a = \{(d, p, s) : \ \alpha^0_{dps} > 0\}$ and $\mathcal{I}_b = \{(d, p, s) : \ \beta^0_{dps} > 0\}$. The proposed objective function reads:

$$\textbf{\textit{maximize}} \ \textstyle\sum_{(d,p,s) \in \mathcal{I}_b} \beta^0_{dps} (1 - r_{dps}) + \sum_{(d,p,s) \in \mathcal{I}_a} \alpha^0_{dps} (1 - r_{dps}). \tag{6}$$

Clearly, when (6) is used, formulation of **R** must also involve an appropriate set of constraints (1k) related to triples (d, p, s) contained in $\mathcal{I}_a \cup \mathcal{I}_b$. On the other hand, the subsequent solutions visited by the procedure could, in practice, be very distant. Thus, it can be advantageous to use an objective function which combines both: (4) and (6).

$$\textbf{\textit{minimize}} \ \varepsilon (\textstyle\sum_{e \in \mathcal{E}} \sum_{f \in \mathcal{F}} \xi_f u_{fe}) + (1 - \varepsilon)(\sum_{(d,p,s) \in \mathcal{I}_b} \beta^0_{dps} r_{dps} + \sum_{(d,p,s) \in \mathcal{I}_a} \alpha^0_{dps} r_{dps}), \tag{7}$$

where $0 \leq \varepsilon \leq 1$ is an optimization parameter. The first component of (7) minimizes the total length, with respect to ξ_f, of all the selected paths. It is aimed at forbidding usage of relatively long paths even if α^0 and β^0 associated with these paths are promising.

Notice that for given realization of the upper layer links problem **M** can be infeasible due to empty set of allowable candidate paths. Using specific inequalities we can exclude the infeasible link realizations from the solution space of problem **R** without solving problem **M**. The basic form of these inequalities refers to a cut-set in graph $\mathcal{H}(\mathcal{W}, \mathcal{F})$. Let $\delta(\mathcal{W}')$ be a cut-set associated with a subset of nodes $\mathcal{W}' \subseteq \mathcal{W}$.

$$\textstyle\sum_{e \in \delta(\mathcal{W}')} w_{es} \geq 1 \qquad s \in \mathcal{S}. \tag{8}$$

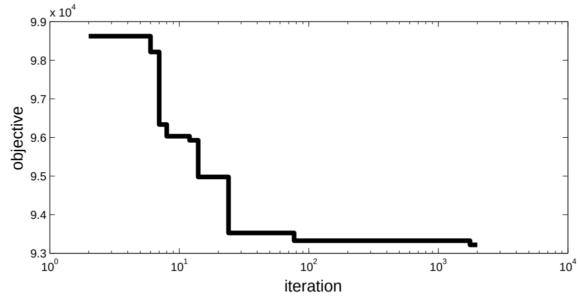
Inequality (8) assures that at least one IP link must be available for a given cut-set $\delta(\mathcal{W}')$ in $\mathcal{H}(\mathcal{W}, \mathcal{F})$. In particular, (8) is valid for the set of links outgoing from one specific node, i.e., $\mathcal{W}' = \delta^+(v)$. Because the number of potential cut sets grows exponentially with the number of nodes, we assume that only specific cut sets, related to one, two or three nodes could be examined in the practical implementations.

Even if appropriate inequalities (8) are introduced to the formulation of problem **R**, it may still appear that the feasible solution space of problem **M** is empty (inequality (8) states necessary but not sufficient condition for feasibility of realizations) for a specific realization of the IP links. In such a case we exclude the current solution from the feasible solution space of problem **R** using simple inequality. Let $\mathcal{U}_0$ and $\mathcal{U}_1$ be the sets of (e, f) pairs for which u_{ef}'s are equal to zero and one in the excluded realization, respectively. The discussed inequality reads:

$$\textstyle\sum_{(e,f) \in \mathcal{U}_0} u_{ef} + \sum_{(e,f) \in \mathcal{U}_1} (1 - u_{ef}) \geq 1. \tag{9}$$

Appropriate inequality (9) is introduced into the formulation of problem **R**, each time problem **M** is infeasible, and a cut-set which assures feasibility of **M** cannot be identified. This inequality simply forbids using the same configuration $\boldsymbol{u}^0$ of lower layer paths again.

Figure 1: Two phase algorithm: improvement of the objective function (1a) for *pdh*, $\varepsilon = 0.9$, and $\kappa = 1.0$.



4 Numerical results

In our computational experiments we assessed the correctness of the introduced two-phase method, evaluated the influence of parameter ε in (7) on the algorithm efficiency, and checked what is the rate of improvement of the generated solutions in the consecutive iterations. To do this we implemented Algorithm 1 using optimization package CPLEX 10.0 to resolve problems **R** and **M**. The computations were conducted on a PC equipped with P4 Quad Core processor and 4 GB memory.

In the experiments we used two network instances from Survivable Network Design Library (SNDlib, see http://sndlib.zib.de): *pdh* (11 nodes, 34 links, 24 demands) and *newyork* (16 nodes, 49 links, 240 demands). The network topologies defined the lower layer graph. The sets of the IP nodes were defined as subsets of the WDM nodes. Whether a particular WDM node was enriched with the IP functionality (and in effect became also an upper layer node) was decided randomly with probability κ. In the experiments, parameter κ took the values 1.0, 0.8, and 0.5. In the first case all the nodes of the lower layer were also the the nodes of the upper layer. For $\kappa = 0.8$, 80% of the lower layer nodes were also the nodes of the upper layer, and so on. The topologies of the upper layer were in all cases fully connected. The sets of the demands contained all the demands related to the selected IP nodes.

The major goal of the experiments was to investigate the influence of the value of parameter ε on the cost of the obtained solutions. For this purpose we run Algorithm 1 using different values of this parameter. In the computations we considered all single WDM link failures. The time limit was set to 2 hours. In tables 1 and 2 we present the values of the objective function (1a) of the best solution found within the assumed time limit. Column *LB* defines a lower bound on the objective function. It was computed as the optimal solution of the single-layer path diversity design problem (taking into account all single-link failures) with the network topology and the unit capacity link costs of the lower layer, and the demand matrix from the upper layer.

Table 1: Two phase algorithm: objective values for different values of ε and κ for the *pdh* network.

	objective							
κ	LB	$\varepsilon \in \langle 0.0 - 0.8 \rangle$	$\varepsilon = 0.85$	$\varepsilon = 0.88$	$\varepsilon = 0.9$	$\varepsilon = 0.92$	$\varepsilon = 0.95$	$\varepsilon = 1.0$
1.0	93217.6	98622.7	93323.8	93323.8	93217.6	93389.7	93389.7	93632.4
0.8	79521.3	91118	88553.3	88661.7	88637.9	88637.9	88652.3	88652.3
0.5	38804.8	57378.7	57378.7	56550.8	56121.2	56121.2	56121.2	56121.2

Table 2: Two phase algorithm: objective values for different values of ε and κ for the *newyork* network.

	objective				
κ	LB	$\varepsilon \in \langle 0.0 - 0.9 \rangle$	$\varepsilon = 0.92$	$\varepsilon = 0.95$	$\varepsilon = 1.0$
1.0	24191.4	26012.9	25533.5	25758.5	25474.6
0.8	18457	21981.8	21501.6	21211.8	21379
0.5	17673.3	21197.1	21197.1	20400.7	20426.2

According to the results presented in tables 1 and 2 we may conclude that parameter ε in the objective function (7) (used when resolving the lower layer problem **R**) strongly influences the efficiency of the two-phase algorithm. It appeared that the approach based on weighting both components of (7) is more efficient in terms of the resulting cost function that the dual based indicators or the shortest path lengths when used stand-alone ($\varepsilon = 1$ or $\varepsilon = 0$). Analyzing Figure 1, illustrating the algorithm convergence, we conclude that for a proper value of ε the algorithm quickly finds a good quality solution in several tens of iterations, which is only slightly improved in the further iterations.

5 Concluding remarks

In the paper we have investigated an optimization problem related to designing a resilient two-layer IP-over-WDM network. In the considered problem the capacities of the WDM links must be large enough to accommodate flows associated with selected realizations of the IP links. Since the WDM links are subject to failures and the network is supposed to be robust to failures, we have assumed that demand flows are protected through path diversification. The related optimization problem has been formulated in terms of a mixed-integer programs (in two versions). Because the problem is in general *NP*-hard, the MIP optimization solvers, are capable to solve it only for small network instances. Thus, in the paper we have proposed a dedicated method for resolving it. The method is based on iterative resolving of two subproblems, each related to optimizing flows in a distinct network layer. In our numerical experiments we have tested the effectiveness of the method for different settings of weighting parameter ε in objective function (7). The experiments revealed that neither dual based indicators, nor shortest path lengths when used stand-alone could provide the solutions of the best quality. As the experiments have shown, the method is capable of providing good-quality solutions in a moderate time.

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