Packet and Message Routing and Integral Flows Over Time

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Abstract

Exchanging messages between nodes of a network is a fundamental issue in network optimization involving routing and scheduling decisions. Application occur, for example, in distributed real-time systems where processes residing at different nodes of the network communicate by passing messages. In order for messages to reach their destinations on time, one has to determine a suitable (short) origin-destination path for each message and resolve conflicts between messages whose paths share a communication link of the network. We provide efficient routing strategies yielding origin-destination paths of bounded dilation and congestion.

Keywords: *packet routing, message routing, network flows over time, real-time systems*

Consider a (directed or undirected) graph $G = (V, E)$, whose edges correspond to the communication links of the network. In the *message routing problem*, each message $M_i = (s_i, t_i, d_i)$ of a given set of messages $\{M_i\}_{i \in I}$ consists of d_i packets of unit size that have to be sent from the origin node $s_i \in V$ to the destination node $t_i \in V$ within a certain time horizon $T > 0$. Usual constraints are (see e.g., [5, 6], or [4, Chapter 37]): (i) it takes one time unit to send a packet on any edge $e \in E$, (ii) at most one packet can traverse an edge per time unit, and (iii) a message has to be completely received by a node before the node can start to transmit it to any other node. The last constraint is due to integrity checks performed by each node and implies that each message M_i has to be sent along a unique path P_i from its origin to its destination node.

In the special case where each message consists of only one packet, message routing reduces to *store-and-forward packet routing*, a fundamental routing problem in interconnection networks (see, e.g., Leighton's survey [3]). Store-and-forward packet routing can be formulated as an *integral dynamic mulit-commodity flow problem* with unit capacities and unit transit times on the edges.

A natural approach for solving the message routing problem is the following two-stage strategy. In the first stage (the *routing stage*), determine the set of paths $\{P_i\}_{i \in I}$. Then, in the second stage (the *scheduling stage*), resolve conflicts between messages sharing an edge. Of course, in order to determine good solutions, the paths chosen in the routing stage must feature certain desirable properties that guarantee the existence of good solutions to the second stage scheduling problem (which turns out to be an acyclic job shop problem).

We describe an algorithm that, given a set of messages $\{M_i\}_{i \in I}$ and a desired dilation Δ, finds a set of paths of dilation at most Δ and congestion smaller than $C^*(\Delta) + \Delta$, where $C^*(\Delta)$ denotes the congestion of an optimal fractional solution with dilation at most Δ. Although our algorithm can be applied for arbitrary message lengths, it even improves upon the performance guarantee of [7] for the special case of store-and-forward packet routing by a multiplicative factor of two. The main difference between our approach and the approach in [7] is our use of a path-based linear programming formulation which turns out to be efficiently solvable as the corresponding separation problem is a special case of the *length-bounded shortest path problem.* A similar idea has been used by Fleischer and Skutella [2] in the general context of dynamic network flow problems. Given an optimal solution to the linear program, we use iterative rounding in order to turn the fractional solution into an integral one, and guarantee that the congestion is not increased by more than Δ. Combining this result with approximation results for the second stage acyclic job shop scheduling problem (see, e.g., [1]) yields new and improved approximation results for message and packet routing.

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