Planning Wireless Networks by Shortest Path

Carlo Mannino[*] Sara Mattia[*] Antonio Sassano[*]

[*]*Dipartimento di Informatica e Sistemistica. Sapienza, Università di Roma*
via Ariosto 25, 00185 Roma (Italy)

Abstract

Transmitters and receivers are the basic elements of wireless networks and are characterized by a number of radio-electrical parameters. A general planning problem consists in establishing suitable values for these parameters so as to optimize some network performance indicator. In this paper we consider a version of the problem called the Power Assignment Problem (PAP), that is the problem of assigning transmission powers to the transmitters of a wireless network so as to maximize the satisfied demand. This problem has relevant practical applications both in radio-broadcasting and in mobile telephony. Typical solution approaches make use of mixed integer linear programs with huge coefficients in the constraint matrix yielding numerical inaccuracy and poor bounds and cannot be exploited to solve large instances of practical interest. In order to overcome these inconveniences, we developed a two-phase heuristic to solve large instances of PAP, namely a constructive heuristic followed by an improving local search. Both phases are based on successive shortest path computations on suitable directed graphs. Computational tests on a number of instances arising in the design of the Italian Digital Video Broadcasting are presented.

Keywords: *wireless network optimization, mixed integer programs, exponential neighborhood search*

1 Introduction

A wireless network consists of a set of radio transmitters distributing services to a set of receivers scattered over a target area. Transmitters and receivers are characterized by their geographical position and by a number of radio-electrical parameters. Due to the very large number of receivers and to the uncertainty on their exact location, several neighboring receivers are typically grouped into a single representative one. A standard aggregation technique consists in subdividing the area of interest (*target area*) into a set of smaller rectangular areas, called *testpoints* (TPs). Each testpoint represents the behaviour of all receivers in the square. The optimization process consists in establishing suitable values for a subset of the radio-electrical parameters associated with the transmitters and the receivers of the network. Different (versions of) wireless network planning problems stem out from different parameters configurations [3]. The problem here addressed, namely the *Power Assignment Problem* (PAP), is the problem of establishing transmission powers so as to maximize the covered population. Natural instances of the PAP arise in the standard planning process of large broadcasting networks. In particular, when re-planning of operating networks are needed in order to satisfy new constraints imposed by network adjustments, new international agreements, or by the introduction of new devices. Indeed, the application which motivated this research is the actual replacement, in broadcasting networks, of the analog technology with the digital one, which is occurring in Italy and all over Europe.

The power emitted by a transmitter in every direction is described by its *antenna diagram* or *radiation pattern*. The radiation pattern is the two-dimensional spatial distribution of radiated energy as a function of the position of the observer along a path or surface of constant radius [1]. The angular dependence of the horizontal radiation patterns is approximated by specifying thirty-six values attached to angles (directions) from 10 degrees to 360° [2]. Consequently, each direction of a transmitter can be regarded as an elementary transmitter, later referred to as a *d-transmitter*. In order to yield feasible antenna diagrams, d-transmitters of a same transmitter t must obey simple technological laws [2]: namely, their ratio cannot exceed specified thresholds. The signal emitted by a transmitter propagates according to its antenna diagram and to territory orography. The power density received a TP r from d-transmitter s is proportional to the emission power p_s of the d-transmitter. The level of the service in a testpoint is considered satisfactory if the ratio between the total contribution of the useful signals and the total contribution of the interfering signals is above a given threshold (SIR inequality). Since there are different potential configurations of the receiving antenna, corresponding to the choice of a different reference signal h (server), for each testpoint we are given a set of SIR inequalities. A testpoint is said to be covered if at least one of the associated SIR inequalities is satisfied. Problem PAP can be readily cast into a Mixed Integer Linear Program [3]. However, it is common experience that the MILP formulations corresponding to instances of some practical interest are far from being solvable by standard Branch&Cut [4], for a number of reasons. First, these instances contain a large number of binary variables, which results in huge search trees. Second, the coefficient matrix is ill conditioned as the entries may differ by several orders of magnitudes. As a consequence, the time to perform standard simplex operations increases and, even worst, the solutions produced by the lp solver are not always reliable. For this reason we decided to resort to an effective two stage heuristic approach. In stage one (cycle-detection) a first feasible solution for the problem is found by heuristically solving a relaxed version of the original problem. In stage two (neighborhood search) we perform a local search in order to improve the quality of the initial solution: the local search is based on the exact solution of a subproblem of the original problem.

2 The model

Let R be the set of testpoints to be covered, let T be the set of transmitters, for all $t \in T$, let $D_t = \{(t,d) : d = 1,\ldots,36\}$ be the set of directions of t and let $D = \cup_{t\in T} D_t$ be the set of all d-transmitters. We introduce a power variable p_s for all $s = (t,d) \in D$, each ranging in the interval $[\epsilon, P_{Max}]$, where $\epsilon > 0$ is a positive small constant. We assume that if $p_s = \epsilon$ for all $s \in D_t$, then t is switched off. Denoting by $R(s) \subseteq R$ the set of testpoints reached by a d-transmitter s, we have $R(s) \cap R(q) = \emptyset$ for all $s, q \in D_t$, $s \neq q$, for every $t \in T$. In other words, at most one out of the 36 d-transmitters associated with a same transmitter $t \in T$ will be received in $r \in R$. For all $r \in R$ we denote by $D(r)$ the set of d-transmitters received in r. We refer to matrix $[\mathbf{A}] = [a_{rs}]_{r\in R, s\in D(r)}$ as the *fading matrix*, which is calculated by means of a suitable propagation model. An assignment $h \in D^{|R|}$ of reference transmitters to all TPs is called *server assignment*. According to the above definitions, we can rephrase PAP as the problem of finding a set $S \subseteq R$ of testpoints and a server assignment $\bar{h} \in D^{|R|}$ such that the following system of inequalites $COV(S,\bar{h})$ is feasible and the population $c(S)$ of S is maximum.

$$COV(S,\bar{h}) \qquad \frac{\sum_{s\in U(r,\bar{h}_r)} a_{rs}\bar{p}_s}{\sum_{s\in I(r,\bar{h}_r)} a_{rs}\bar{p}_s} \geq b_r \qquad r \in S \tag{1}$$

$$\frac{\bar{p}_s}{\bar{p}_q} \leq \Delta_{sq} \qquad t \in T, s, q \in D_t \tag{2}$$

$$\epsilon \leq \bar{p}_s \leq P_{Max} \qquad t \in T, s \in D_t \tag{3}$$

Constant b_r is the receiver *sensitivity*, h_r is the reference transmitter of r, $U(r,h_r) \subseteq D(r)$ is the set of *wanted* signals in r and $I(r,h_r) \subseteq D(r)$ is the set of *interfering* signals in r. Both sets $U(r,h_r)$ and $I(r,h_r)$ depend on the selected server $h_r \in D(r)$. Inequalities (1) are SIR-inequalities, constraints (2) and (3) are technological constraints. In particular, we consider two types of design constraints. Those involving only adjacent directions $s = (t,i), q = (t,i+1)$, for $i = 1,\ldots,36$, with $\Delta_{sq} = \Delta_{adj} = 10^{0.5}$ and those between any pair of directions $s, q \in D_t$, for which $\Delta_{sq} = \Delta = 10^{2.4}$.

3 Cycle detection

Our heuristic approach to the solution of PAP is based on a number of simplifying but quite reasonable technological assumptions. First observe that, due to the wide variability of the fading coefficients and emission powers, the signals received in a testpoint typically differ one from another by order of magnitudes. More specifically, for each $r \in R$, if r is covered with reference signal $h_r \in D(r)$, in most cases there will be only one strongest useful signal, namely h_r, and the contribution of the other useful signals to the numerator of the SIR inequality can be neglected. Similarly, if we assume that for each $r \in R$, $h_r \in D(r)$ there will be only one strongest interfering signal, all other interfering signals can be neglected. However, since emission powers are not known in advance, for each $r \in R$, $h_r \in D(r)$, we split the SIR-inequality into $|I(r, h_r)|$ inequalities obtaining:

$$\frac{a_{rh_r} p_{h_r}}{a_{rt} p_t} \geq b_r \qquad t \in I(r, h_r), h_r \in D(r) \tag{4}$$

and r is covered with reference signal h_r iff all of the constraints (4) associated with r and h_r are satisfied. We can define a problem $COV_2(R)$ which is similar to $COV(R)$, but uses (4) instead of (1). By introducing, for all $t \in T$, $s \in D_t$, a variable $p_s^{dB} = 10 \log_{10} p_s$, and by expressing all constants in dB form, by simple algebra $COV_2(R)$ becomes the problem:

$$COV_2^{dB}(R) \qquad p_{h_r}^{dB} - p_t^{dB} \geq b_r^{dB} + a_{rh_r}^{dB} - a_{rt}^{dB} \qquad r \in R, h_r \in D(r), t \in I(r, h_r) \tag{5}$$

$$p_s^{dB} - p_q^{dB} \leq \Delta_{sq}^{dB} \qquad t \in T, s, q \in D_t \tag{6}$$

$$\epsilon^{dB} \leq p_s^{dB} \leq P_{Max}^{dB} \qquad t \in T, s \in D_t \tag{7}$$

By adding an extra, reference power variable p_0, we can replace each (7) with the pair: $p_s^{dB} - p_0 \leq P_{Max}^{dB}$, $p_0 - p_s^{dB} \leq -\epsilon^{dB}$. Thus, $COV_2^{dB}(R)$ can be rewritten in compact form as:

$$p_j^{dB} - p_i^{dB} \leq l_{ij} \qquad ij \in A. \tag{8}$$

The family of solutions to (8) is the solution set of the dual of a shortest path problem on the weighted graph $G^{dB}(D, R) = (V, A, l)$, with $V = D \cup \{0\}$. Each arc $ij \in A$ corresponds to one of the constraints of COV_2^{dB}: a *testpoint arc* is an arc corresponding to a constraint of type (5). If (r, h_r, t) is a constraint of type (5), we denote by $a(r, h_r, t)$ the corresponding testpoint arc. It is well known that (8) has a solution iff $G^{dB}(D, R)$ does not contain a negative weight directed cycle. Also observe that each negative cycle in $G^{dB}(D, R)$ corresponds to a infeasible subsystem of $COV_2^{dB}(R)$.

The idea to solve $COV_2^{dB}(R)$ is to iteratively identify a negative weight dicycle in $G^{dB}(D, R)$ and remove a suitable subset of testpoint arcs meeting the negative dicycles. In particular, if C is a negative dicycle, then we select a testpoint arc $a(r, h_r, t) \in C$ and remove it from the graph. Intuitively, this corresponds to renounce covering r with reference signal h_r. As a consequence, all testpoint arcs corresponding to the different interferers $I(r, h_r)$ of r and h_r must also be dropped from $G^{dB}(D, R)$. The procedure stops when no negative dicycles are left. The complete procedure is:

1. Build the graph $G^0 = G(R, D) = (V, A, l)$.
2. While G^i contains a negative weight dicycle C^i.
   (a) Choose a testpoint arc $a(r, h_r, t)$ in C^i.
   (b) Build G^{i+1} by deleting all testpoint arcs corresponding to r, h_r, i.e. remove the arc set $A(r, h_r) = \{a(r, h_r, t) \in G^i : t \in I(r, h_r)\}$.
   (c) i := i + 1;
3. Let $q = i$. Compute the shortest path lengths $\tilde{p}_s$ from 0 to $s \in V$ in G^q.

The identification of a negative dicycle C^i in G^i can be performed by applying the Bellman-Ford algorithm, which either finds C^i or returns a feasible solution $\tilde{p}$ to $COV_2^{db}(G^i)$. In order to establish how to select the arc at Step 2a. we tested several criteria. The best one corresponds to selecting the arc which appeared most often in the negative cycles detected so far; ties are broken by selecting the one minimizing the population of the corresponding testpoint.

4 Neighborhood search

The solution $(\tilde{p}, \tilde{h})$ to PAP returned by Procedure *Cycle_Detect* is computed by neglecting the effect of multiple useful and interfering transmitters. In order to increase its quality we developed an efficient exponential neighborhood search which takes into account these contributions to the actual coverage.

Let $(\bar{p}, \bar{h})$ be the current solution. For any $\bar{t} \in T$ define $N_{\bar{t}}(\bar{p}, \bar{h}) = \{(\tilde{p}, \tilde{h})\}$ as the family of solutions obtained by letting $\tilde{p}_s = \bar{p}_s$ for every $s \in D_q$, $q \in T - \{\bar{t}\}$. In other words, $N_{\bar{t}}(\bar{p}, \bar{h})$ is the family of solutions which can be obtained from the original one by changing the power vector $p_{\bar{t}} = (p_{1,\bar{t}}, \ldots, p_{36,\bar{t}})$ associated with the directions $d \in D_t$ of a single transmitter $\bar{t} \in T$, and re-assigning reference signals in all possible ways. We suppose that, for all $t \in T$ and $s \in D_t$, p_s^{dB} belongs to a discrete set of integer dB values $\mathcal{L} = \{L_1, \ldots, L_q\}$. Finally, we define the neighborhood $N(\bar{p}, \bar{h})$ of a solution $(\bar{p}, \bar{h})$ as $N(\bar{p}, \bar{h}) = \bigcup_{t \in T} N_t(\bar{p}, \bar{h})$.

Exploring the neighborhood $N(\bar{p}, \bar{h})$ consists in searching each N_t, for all $t \in T$, and then choosing the best configuration encountered. Searching N_t is equivalent to finding the configuration $(p^*, h^*)_t \in N_t(\bar{p}, \bar{h})$ so that $c((p^*, h^*)_t)$ is maximized. Since powers are fixed for all $z \in T - \{t\}$, we only need to establish the best power vector $p_t^* = (p_{t,1}, \ldots, p_{t,36})^*$ for t and the corresponding new reference signals $h^* \in D^R$. Specifically, p_t^* must be feasible - i.e. satisfy all adjacent and non-adjacent design constraints - and must maximize coverage. Observe that the number of different feasible vectors $h \in N_t$ grows exponential in $|R|$ and $|T|$, being in correspondence with the feasible assignments of TPs to reference signals. However, the optimal solution in N_t can be found in polynomial time (in $|R|$ and $|\mathcal{L}|$). Let $c_\epsilon^t(\bar{p}, \bar{h})$ be the coverage of the solution obtained by $(\bar{p}, \bar{h})$ by switching off $t \in T$, i.e. by letting $(p_{t,1} = \ldots = p_{t,36} = \epsilon)$. Denote by c_{dk} the coverage increase in direction d with respect to $c_\epsilon^t(\bar{p}, \bar{h})$ when $p_{t,d}^{dB} = L_k \in \mathcal{L}$. This coefficient can be efficiently computed and it can assume positive, zero or negative value. Recall that the coverage evaluation procedure also establishes, for each testpoint $r \in R(t, d)$ (the family of testpoints reached by the d-direction of t), the corresponding reference signal $\bar{h}_r$. Finally, recall that $R(t, d_1) \cap R(t, d_2) = \emptyset$ whenever $d_1 \neq d_2$. Now, in order to find the optimum solution in $N_t(\bar{p}, \bar{h})$, we first find the optimum solution when the power of t in its first direction $(t, 1)$ is fixed to some reference value (in L). In other words, We want to find the optimum configuration for t in $N_t(\bar{p}, \bar{h})$ when $p_{(t,1)}^{dB} = L_k \in \mathcal{L}$. We show that this can be done by solving a sequence of shortest path problems in a suitable acyclic directed graph $G_k = G(k, \Delta_{adj}) = (V_k, A_k)$, for $k = 1, \ldots, q$. Each vertex $v_{d,\ell} \in V_k$ of G_k is associated with a direction d and a feasible power level $L_\ell \in \mathcal{L}$. In particular, $v_{1,k} \in V_k$, i.e. there is a vertex associated with direction 1 and power level L_k; then we have a vertex for every other direction and every power level in $\mathcal{L}$, namely $v_{d\ell} \in V_k$, $2 \leq d \leq 36$, $\ell = 1, \ldots, q$. Finally, V_k contains an extra node w. The arcs of G_k are associated with the pair of power levels satisfying the adjacency design constraints $p_s^{dB} - p_u^{dB} \leq \Delta_{adj}^{dB}$ for all adjacent directions s and u. Namely, for $d = 1, \ldots, 36$, $(v_{d,\ell}, v_{d+1,g}) \in A_k$ iff $|L_\ell - L_g| \leq \Delta_{adj}^{dB}$; $(v_{36,\ell}, w) \in A_k$ for all $v_{36,\ell} \in V_k$ such that $|L_\ell - L_k| \leq \Delta_{adj}^{dB}$. Finally, with every arc $(v_{d,\ell}, v_{d+1,g})$ we associate the weight $c_{d\ell}$.

An example of this construction is shown in Fig.1, where, for the sake of simplicity, we have supposed only 5 directions, 7 power levels $\mathcal{L} = \{L_1 = -1, L_2 = 0, L_3 = 1, L_4 = 2, L_5 = 3, L_6 = 4, L_7 = 5\}$, and $\Delta_{adj}^{dB} = 1$. It is easy to see that G_k is a layered graph. Also, it is immediate to verify that any power vector $(p_{t,1}^{dB}, \ldots, p_{t,36}^{dB}) \in \mathcal{L}^{36}$ satisfying $p_{t,1}^{dB} = L_k$ and all the adjacency design constraints, corresponds to the directed path $m = \{v_{1,k}, v_{2,p_{t,2}}, \ldots, v_{36,p_{t,36}}, w\}$. Moreover, the weight of m is precisely the coverage increase with respect to $c_\epsilon^t(\bar{p}, \bar{h})$ when t is assigned powers $p_{t,1}^{dB}, \ldots, p_{t,36}^{dB}$. Analogously, it is easy to see that any directed path $\tilde{m} = (v_{1,k}, v_{2,\ell_2}, \ldots, v_{36,\ell_{36}}, w)$ in G_k corresponds to a power assignment for t, namely $\tilde{p}_{t,1}^{dB} = L_k, \ldots, \tilde{p}_{t,36}^{dB} = L_{\ell_{36}}$, satisfying all adjacency design constraints. Recall

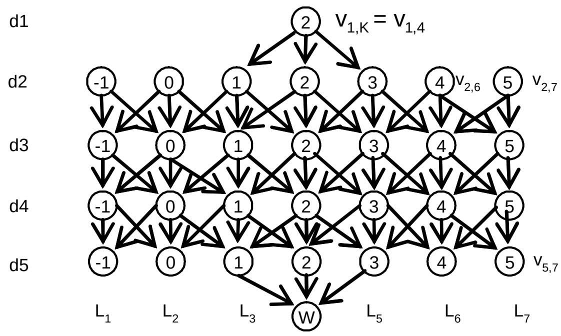

Figure 1: *Example of neighborhood graph*

Input: a PAP instance, $\mathcal{L} = \{L_1, \ldots, L_q\}$, initial solution $(\bar{p}, \bar{h})$
For $t \in T$
  For $k = 1$ to q
    Build graph G_k.
    For $i = 0$ to Δ^{dB}
      Build subgraph G_k^i of G_k
      Find a maximum weighted path m_k^i in G_k^i with associated solution (p_k^i, h_k^i)
    Endfor
  Endfor
Endfor
Return the best path m^* found with associated solution (p^*, h^*)

Table 1: Neighborhood Search procedure

now that feasible power assignments must satisfy both adjacency and non-adjacency design constraints. a maximum path m_k^* in G_k whose corresponding power assignment p_k^* satisfies the non-adjacent design constraints solves the optimization problem in $N_t(\bar{p}, \bar{h})$, when restricted to $p_{t,1}^{dB} = L_k$. Since G_k is layered, G_k is acyclic and m_k^* can be computed by an $O(|A|)$ shortest path algorithm. In order to ensure that also non-adjacent design constraints are satisfied we define, for each G_k, $k = 1, \ldots, q$ a family $G_k^0, \ldots, G_k^{\Delta^{dB}}$ of induced subgraphs with the property that any directed path in G_k^i corresponds to a feasible power assignment and any feasible power assignment corresponds to a path in some G_k^i. Namely, let $\{-\Delta^{dB} + L_k + i, \ldots, L_k + i\}$ be a set of Δ^{dB} contiguous integer power values including L_k, for $i = 0, \ldots, \Delta^{dB}$. Define $\mathcal{L}^i = \mathcal{L} \cap \{-\Delta + L_k + i, \ldots, L_k + i\}$. By definition, there exists $i \in 0, \ldots, \Delta^{dB}$ such that every feasible power vector $\tilde{p}^t$, with $\tilde{p}_{t,1} = L_k$ satisfies $\tilde{p}_{t,d} \in \mathcal{L}^i$, for all d. Thus, we define G_k^i as the subgraph of G_k induced by the vertices corresponding to the power levels in $\mathcal{L}_i$, plus vertex w. Any path m from $v_{1,k}$ to w in G_k^i corresponds to a power assignment satisfying both adjacent and non-adjacent design constraints. Finally, by solving $\Delta^{dB} + 1$ maximum path problems on the graphs $G_k^0, \ldots, G_k^{\Delta^{dB}}$ and choosing the optimum one, we find the optimum solution in $N_t(\bar{p}, \bar{h})$. The Neighborhood Search procedure is summarized in Table 4 and it has been embedded into a standard local search (LS) approach.

5 Computational Results

We tested our heuristics on 56 real-life instances arising in the planning of the new DVB networks in north Italy. Each instance corresponds to a Single Frequency Network, operating at a specific frequency in the UHF band. The instances were generated by Fondazione Ugo Bordoni. Each instance has about 10500 TPs, 15000 SIR inequalities and 250 transmitters. The algorithms were implemented in C++ and run on a Intel Core 2 Duo T7500 / 2.2 GHz, with 4 Gb RAM. We compare our heuristic, which combines the Cycle Detection (CD) procedure with the Local Search (LS) against the commercial solver ILOG-CPLEX version 11.1, with default settings and a one hour time limit, applied to a standard BIG_M formulation to PAP , which slightly extends the one presented in [3] to cope with multiple candidate servers. The results are shown in Table 2, where column *coverage* is the percentage of covered population while *sec.* is the running time (in seconds), Column *UB* is the Upper Bound produced by Cplex, while *LB* is the best coverage found by Cplex. In fact, it strictly dominates both CD and LS in their stand-alone versions. Cplex is not able to solve any of them to optimality within time limit. For most of the instances, Cplex is not able to produce a suitable feasible solution and in 11 cases over 56, it is not even able to solve the lp relaxation. The best approach resulted the one that combines the cycle-detection algorithm and the local search. in 45 cases over 56, it is able to produce solutions which are better than the ones found by cplex (in almost twice the running time).

References

[1] C.A. Balanis, *Antenna Theory: Analysis and Design*, John Wiley & Sons, New York, 1982

[2] R. Beutler, *Frequency Assignment and Network Planning for Digital Terrestrial Broadcasting Systems*, Kluwer Academic Pubishers, 2004

[3] C. Mannino, F. Rossi, S. Smriglio, *The Network Packing Problem in Terrestrial Broadcasting*, to appear in Operations Research

[4] R. Mathar, M. Schmeinck, *Optimisation Models for GSM Radio*, *International Journal of Mobile Network Design and Innovation*, **1**, 1, 2005, 70-75

Name	Cycle Detection (CD)		Local Search (LS)		CD + LS		Cplex	
	coverage %	sec.	coverage %	sec.	coverage %	sec.	UB %	LB %
F5	79,8418%	85,924	95,2727%	2033,27	95,2986%	1415,854	-	0,532149%
F6	41,6027%	94,224	40,4891%	660,707	46,4288%	768,83	52,3237%	47,9199%
F7	73,6214%	94,911	66,0285%	1394,22	84,3445%	982,584	96,864%	0,254587%
F8	57,688%	106,236	54,8027%	1379,88	63,34%	904,503	72,8185%	65,9537%
F9	80,1467%	97,235	91,9451%	1731,12	94,7761%	1607,865	-	0,0559523%
F10	74,7378%	88,468	62,3175%	1280,73	87,2839%	1069,881	98,8187%	1,94629%
F11	61,3924%	126,859	66,7614%	1047,59	74,9865%	1404,389	82,1221%	0,760367%
F21	40,1347%	121,602	36,8315%	747,88	45,3939%	1034,452	48,7%	46,3055%
F22	63,5942%	116,423	70,2906%	1294,07	74,834%	1026,95	78,0683%	0,178576%
F23	66,9851%	114,458	71,3575%	1429,51	79,2129%	1217,798	-	0%
F24	60,7148%	158,824	66,9607%	896,626	70,1393%	1297,454	73,9213%	69,7668%
F25	88,4662%	170,242	95,5985%	1686,94	97,7916%	1641,512	99,6446%	6,27393%
F26	85,8047%	95,253	96,741%	1099,68	98,1487%	1881,453	-	0,173803%
F27	42,6817%	107,546	42,8157%	539,932	48,8265%	955,22	51,6759%	48,229%
F28	66,8339%	114,504	72,4736%	1014,69	79,6364%	1186,474	83,9521%	74,9001%
F29	50,0771%	115,549	41,1089%	730,626	54,1612%	903,13	58,7065%	54,4966%
F30	84,8831%	107,609	95,9258%	1507,24	97,3922%	1458,509	99,8513%	0,0270563%
F31	40,0064%	124,737	37,7534%	730,846	46,3932%	1218,967	49,8142%	47,329%
F32	64,6909%	119,247	67,8447%	1183,59	72,4931%	1317,357	75,5777%	0%
F33	66,6602%	134,036	71,0827%	1107,8	76,4636%	1262,726	78,7034%	2,36324%
F34	71,2243%	118,108	78,0429%	1217,94	84,7092%	1027,416	87,6557%	0,011673%
F35	66,2171%	123,646	72,5449%	1034,61	75,6917%	1383,226	78,5756%	0,0888897%
F36	83,9962%	124,66	96,6734%	1550,08	98,4099%	1694,43	-	0,625398%
F37	87,6432%	157,763	95,6286%	1481,85	97,1552%	1266,893	-	2,59759%
F38	78,5477%	181,88	94,0447%	1729,56	94,5341%	1547,3	97,6486%	91,3435%
F39	49,1384%	125,689	46,657%	769,985	54,3828%	937,731	60,3147%	56,9224%
F40	84,0879%	98,452	95,5456%	1973,28	97,4642%	1474,862	99,8815%	0,118086%
F41	63,5636%	117,405	71,8825%	841,027	76,1615%	1350,755	78,712%	71,0901%
F42	66,4654%	108,81	70,0535%	1374,44	74,4103%	1146,87	77,3379%	0,329673%
F43	68,7493%	109,387	70,1778%	1049,69	80,1439%	1235,507	84,2964%	0%
F44	82,3275%	93,023	95,3856%	1263,99	97,9221%	1636,733	-	0,526409%
F45	72,5564%	102,975	73,1345%	1058,68	84,1291%	1225,945	88,3394%	0%
F46	68,4876%	111,868	70,2297%	1224,68	80,1255%	1051,442	84,5057%	74,0412%
F47	64,9907%	135,939	70,7747%	1280,01	74,424%	1086,915	76,9839%	72,7106%
F48	81,0323%	123,443	95,9466%	1566,34	97,696%	1462,433	99,8464%	0,622013%
F49	83,8589%	80,558	95,4187%	2271,06	96,745%	1689,228	-	0,0115108%
F50	92,2888%	125,596	95,6574%	1400,9	98,1913%	1784,846	-	0,0909464%
F51	40,9997%	120,495	37,3943%	604,313	47,0617%	898,326	49,5654%	47,229%
F52	88,1002%	152,132	96,4587%	1337,47	97,545%	1502,032	-	1,27357%
F53	66,776%	117,312	70,3306%	1119,91	79,7436%	1068,18	84,8421%	0%
F54	67,4301%	113,693	68,1214%	1158,75	73,6662%	1099,036	77,681%	72,2078%
F55	65,4833%	112,866	70,2952%	1375,95	74,1437%	1294,456	76,9225%	0%
F56	67,3372%	104,551	69,5604%	1172,96	74,1539%	1492,501	77,3618%	72,2278%
F57	49,7666%	123,786	41,2418%	805,927	55,558%	958,37	60,8171%	56,3608%
F58	86,8067%	112,242	97,1489%	1529,83	98,042%	1684,702	99,5915%	4,44063%
F59	67,9255%	122,507	72,0547%	1193,74	76,2406%	1021,365	79,0792%	0,686095%
F60	65,2924%	112,944	71,7%	1106,82	74,3476%	887,437	76,7249%	0,0262636%
F61	53,8853%	130,089	42,5093%	947,872	59,0629%	799,86	63,6467%	59,0989%
F62	66,2672%	104,52	71,5899%	1574,53	74,083%	1295,96	77,0095%	72,224%
F63	65,7345%	111,337	71,5625%	1152,86	74,3454%	1465,167	-	0%
F64	68,0427%	163,426	68,713%	1244,44	73,2866%	1254,286	75,7337%	68,9843%
F65	66,7843%	136,359	72,3228%	1402,47	74,902%	1384,469	77,4342%	73,616%
F66	42,7375%	143,583	44,0755%	916,562	49,4907%	1274,303	51,9139%	49,7468%
F67	59,806%	118,981	69,9529%	1540,5	75,5733%	1023,267	78,388%	71,77%
F68	50,1931%	116,625	43,5531%	1026,86	55,9932%	891,151	61,0669%	57,0673%
F69	85,6529%	67,345	97,6251%	1451,49	98,0405%	1043,062	99,8893%	93,3489%

Table 2: Computational Results