Routing in a partially selfish network[1]

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Over the past years, the impact of the behavior of selfish, uncoordinated users in congested networks has been investigated intensively in the computer science and operations research literature. In this context, *network routing games* have proved to be an appropriate means of modeling selfish behavior in networks. The basic idea is to model the interaction between the selfish network users as a noncooperative game. We are given a directed graph with latency functions on the arcs and a set of commodities, each with an associated demand. Every demand represents an infinite population of players, each controlling an infinitesimal fraction of flow. The latency that a player experiences to traverse an arc is given by a non-decreasing function of the total flow on that arc. Moreover, every player acts selfishly and routes his flow along a minimum-latency path from its origin to the destination; this corresponds to a common solution concept for noncooperative games, that of a *Nash equilibrium*. In such a state no player can improve his own latency by unilaterally switching to another path.

It is well known that Nash equilibria can be *inefficient* in the sense that they need not achieve socially desirable objectives. A Nash equilibrium in general does not minimize the total cost; or said differently, selfish behavior may cause a performance degradation in the network. The *price of anarchy* is defined as the worst-case ratio of the cost of a Nash equilibrium over the cost of a system optimum. In recent years, considerable progress has been made in quantifying the increase in the price of anarchy caused by the selfish behavior of noncooperative network users. Due to a large efficiency loss, researchers have proposed different approaches to reduce the price of anarchy in network routing games. We discuss the impact of *Stackelberg routing* to reduce the price of anarchy in network routing games. In this setting, some fraction $\alpha \in [0, 1]$ of the entire demand is first routed centrally according to a predefined algorithm and the remaining demand is then routed by the selfish players.

Although several advances have been made recently in proving that Stackelberg routing can in fact significantly reduce the price of anarchy for certain network topologies, the central question of whether this holds true in general is still open. We show that the answer is negative, by proving that the price of anarchy achievable via Stackelberg routing can be unbounded even for single-commodity networks.

In light of this negative result, we consider bicriteria bounds. We develop an efficiently computable Stackelberg strategy that induces a flow whose cost is at most the cost of an optimal flow with respect to demands scaled by a factor of $1 + \sqrt{1 - \alpha}$. Thus, we derive a smooth trade-off that interpolates between the absence of centralized control (doubling the demands is sufficient) and completely centralized control (no scaling is necessary). We also discuss the effectiveness of an easy-to-implement Stackelberg strategy, called SCALE, for which we prove bounds for a general class of latency functions that includes polynomial latency functions as a special case.

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