Optimization Supports Bilateral Frequency Regulation

Fabrizio Rossi* Antonio Sassano⋆ Stefano Smriglio*

*Dipartimento di Informatica, Università di L'Aquila.
Via Vetoio, I-67010 Coppito (AQ)

⋆Fondazione Ugo Bordoni and Dipartimento di Informatica e Sistemistica, Sapienza Università di Roma
Via Ariosto 25, I-00185 Roma

Abstract

We report on the application of Integer Programming to support the frequency coordination among France and Italy in December 2007. The problem is derived from the standard network planning problem by adding two families of complicating constraints. This results in difficult Mixed Integer Linear Programs. We tackle such problems by a tailored MIP heuristic which can be viewed as a variant of the Box Step method for Linear Programming. The heuristic gave excellent practical results, allowing to release 30 regional single frequency networks instead of the 8 guaranteed by the slavish application of international prescriptions.

Keywords: *transmitter, emission power, test-point, Mixed Integer Linear Programming*

1 Introduction

The *International Telecommunication Union* (ITU) is the leading United Nations agency for information and communication technologies. ITU is mandated to manage the frequency spectrum by issuing regulatory procedures for frequency assignment, coordination and registration. A regulatory procedure, hereinafter referred to as *Plan*, is an International Agreement defined by Regional Radiocommunication Conferences.

The first plan for television and sound broadcasting in the *European broadcasting area* dates back to Stockholm, 1961 (ST61). ST61 involved 38 countries and 5,300 transmitting stations (transmitters) in the whole VHF/UFH band. Since 1961 about 75,000 transmitters have been added to the ST61 plan without major difficulty. However, the need for a better spectrum exploitation along with the introduction of digital television (DVB-T) all over Europe motivated a complete redesign of the spectrum for audio/video broadcasting. In particular, the introduction of the digital transmissions requires a transition period where both technologies coexist, giving rise to new needs in spectrum utilization.

The ST61 plan has been revised in the 2006, leading to the Geneva 2006 (GE06) plan. GE06 governs the use of frequencies by broadcasting services in the frequency bands 174-230 MHz and 470-862 MHz in a region that spans from Iceland to Mongolia for a total of 118 countries. It also contains frequency assignments and allotments for the digital broadcasting service (television and sound) along with the plan for analogue television in the transition period.

Although GE06 is the output of a long and complex negotiation process, it is still far from being a ready-to-use solution for planning purposes. In particular, in Italy the frequencies provided by GE06 are largely inadequate to cope with the market demand. Fortunately, GE06 prescriptions are not mandatory:

the Final Agreement explicitly allows the possibility of bilateral agreements between countries that could improve the spectrum utilization.

In this paper we report on the application of Integer Programming to support the bilateral agreement among France and Italy in December 2007. The practical impact of optimization was extraordinary: 30 regional single frequency networks have been set up in Sardinia (plus 13 local ones), instead of the 8 networks guaranteed by the slavish application of GE06 prescriptions. In this way, all requests of existing broadcasters have been fulfilled and further room is left for new operators. In November, 2008 Sardinia was the first large all-digital-region in Europe.

The paper is organized as follow: in §2 a basic description of the system elements is given, in §3 is described the Mixed Integer Linear Programming model supporting the process, while in §4 a sketch of the algorithm along with some computational results are reported.

2 System elements

A *video broadcasting network* consists of a set T of *transmitters* which distribute simultaneously the same *program* over a vast portion of territory, often referred to as *target area*, where receivers are scattered around. In this section we briefly recall the key system features according to the recommendations stated by the International Telecommunication Union (ITU, `www.itu.int`). For an exhaustive description we refer the reader to [4].

The features of the signal transmission are determined by the transmitter configuration, which depends on a set of parameters. In our application, the *emission power* of transmitters represent the decision variables, while the transmission frequencies and the other parameters are fixed. In particular, we are interested in networks that use exactly one frequency (*Single Frequency Network*) and this is assumed in the sequel. The emission power of transmitter i is denoted by P_i.

The models recommended for coverage assessment do not look at individual receivers scattered around, but subdivide the target area into a finite set Z of *test-points*, that is, "small" portions in which the reception features are assumed to be homogeneous. The recommended size for a test-point side ranges from 250 to 2,500 m. To give an idea, when the target area is Italy, this yields to 55,000 to 550,000 test-points. Such a high number of test-points leads to large-scale optimization models. A revenue u_j is defined for each TP $j \in Z$ equal to the number of inhabitants it contains.

The signal emitted by a transmitter propagates according to the terrain orography. the effect of propagation from transmitter i to test-point j is expressed by a fading constant $a_{ij} \in [0, 1]$. Thus, if transmitter i radiates with power P_i, the power density received in j is given by $P_{ij} = a_{ij} \cdot P_i$ $(watt/m^2)$. The set of signals (transmitters) received in j is denoted by $T(j)$. The matrix $[\boldsymbol{A}] = [a_{ij}]_{i \in T, j \in Z}$ is an input for our models and is referred to as *fading matrix.*

A test-point j is *covered* if programs are received clearly. A sophisticated model is recommended for coverage assessment [1]. Among all signals received in j exactly one is selected as reference signal (*reference transmitter*). Then, the other signals are classified as either *useful* or *interfering*. In particular, for each reference transmitter $h \in T(j)$ and testpoint j, a set $W(j, h) \subseteq T(j)$ of useful signals and $I(j, h) \subseteq T(j)$ of interfering signals are identified depending on the technology (see [4] for details). Once signals are classified, an overall useful power density $\mathcal{P}(W(j, h))$ and interfering power density $\mathcal{P}(I(j, h))$ are computed (different combination methods are recommended according to the technology [2]). Finally, testpoint j is regarded as being covered iff there exists a reference signal h such that

$$\frac{\mathcal{P}(W(j, h))}{\mathcal{P}(I(j, h)) + \mathcal{N}} \geq \text{SIR} \tag{1}$$

where $\mathcal{N}$ is the thermal noise and SIR is a value known as *Signal-to-Interference* ratio.

3 The MILP model

The starting point of our development is the standard *Network Planning Problem* already studied in the literature and also applied in practice. This is then enriched by two families of constraints, in order to be applied to bilateral frequency regulation at country boundaries.

The standard planning problem

In the standard setting one major interest of the planner is to establish the emission power P_i of each transmitter so as to maximize the overall population coverage. A MILP model for such a problem has been defined in [4]. Let $w_{jh} \in \{0,1\}$ a binary variable assuming value 1 if j is covered with reference signal h and 0 otherwise; and $z_j \in \{0,1\}$ a binary variable assuming value 1 if test-point j is covered and 0 otherwise. Finally, introduce the *power fading* $p_i \in [0,1]$, that measures power attenuation w.r.t. the maximum feasible power value P_i^{max} (i.e., $P_i = p_i \cdot P_i^{max}$).

The formulation reads as follows:

$$\max \textstyle\sum_{j \in Z} u_j z_j$$
s.t.

$$\sum_{i \in W(j,h)} a_{ij}(h)\, p_i - \sum_{i \in I(j,h)} b_{ij}(h)\, p_i - M w_{jh} \geq \sigma_j - M, \quad \forall j \in Z, \forall h \in T(j)$$

$$z_j \leq \textstyle\sum_{h \in T(j)} w_{hj}, \quad \forall j \in Z$$

$$0 \leq p_i \leq 1, \quad \forall i \in T$$

$$z_j \in \{0,1\}, \quad \forall j \in Z$$

$$w_{jh} \in \{0,1\} \quad \forall j \in Z, \forall h \in T(j)$$

where M is a constant larger than $\sum_{i \in I(j,h)} b_{ij} + \sigma_j$.

Constraints (3) represent the coverage condition (1). The coefficients $a_{ij}(h)$ and $b_{ij}(h)$ are evaluated from the fading matrix $\boldsymbol{A}$ and σ_j depends on the wanted SIR [4]. We refer to such an inequality as *SIR inequality*.

This model turned out to be useful in relevant practical settings [3]. However, two more features have to be considered to design single-frequency-networks in bilateral regulation contexts. The first one deals with looking at a fine antenna shaping, required to have more degrees of freedom and carefully control interference at country boundaries. The second consists in the introduction of restrictions on the power received in the coordination testpoints identified by the neighbour country.

Antenna shaping

Antenna shaping consists in varying the emission power of a transmitter in different directions. This is viable from a technological point of view and greatly enhances planning flexibility, being a key ingredient to increase the coverage in areas with strong interference.

Antenna shaping can be modelled by introducing a finite set of power fading variables p_t^d, where d represents a discrete set of directions (for instance, $d \in \{1, \ldots, 36\}$). Thus, one can rewrite the collection of SIR inequalities for each testpoint $j \in Z$ as follows:

$$\sum_{i \in W(j,h)} a_{ij}(h)\, p_i^{d(i,j)} - \sum_{i \in I(j,h)} b_{ij}(h)\, p_i^{d(i,j)} \geq \sigma_j$$

where $d(i, j)$ is the direction under which testpoint j receives transmitter i. Moreover, the antenna shaping must be feasible, i.e., it must be realizable by an antenna manufacturer. These condition can be ensured by introducing a set of linear relations among power variables. Generically, one requires that:

$$(p_t^1, \ldots, p_t^{36}) \in \mathcal{D}_t \qquad t \in T$$

where $\mathcal{D}_t$ is a polytope, and that:

$$P_t^{d,\min} \leq p_t^d \leq P_t^{d,\max} \qquad d \in \{1, \ldots, 36\}, t \in T$$

Threshold constraints

In the context of the GE06 plan a *bilateral agreement* is an agreement among two neighbouring countries that allows the reciprocal use of forbidden frequencies. The agreement is based on the definition of the following (symmetric) technical condition: *a forbidden frequency can be can be used everywhere in a country if and only if the power received in a set of testpoints chosen by the other country does not exceed an agreed threshold.* Therefore, country A indicates to country B a set of testpoint K along with a threshold value $\theta(k), k \in \{1, \ldots, K\}$ requiring that the total power received in each testpoint cannot exceed such a value. This corresponds to the following set of linear constraints:

$$\sum_{t \in T(k)} c_{tk}\, p_t^{d(t,k)} \leq \theta(k) \qquad k \in K$$

The overall model reads as follows:

$$\max \sum_{j \in Z} u_j z_j$$

subject to

$$\sum_{i \in W(j,h)} a_{ij}(h)\, p_i^{d(i,j)} - \sum_{i \in I(j,h)} b_{ij}(h)\, p_i^{d(i,j)} - M w_{jh} \geq \sigma_j - M \qquad j \in Z, h \in T(j) \qquad (2)$$

$$z_j - \sum_{h \in T(j)} w_{jh} \leq 0 \qquad j \in Z \qquad (3)$$

$$(p_t^1, \ldots, p_t^{36}) \in \mathcal{D}_t \qquad t \in T \qquad (4)$$

$$\frac{P_t^{d,\min}}{P_t^{d,\max}} \cdot r_t \leq p_t^d \leq r_t \qquad t \in T, \quad d \in \{1, \ldots, 36\} \qquad (5)$$

$$\sum_{t \in T(k)} c_{tk}\, p_t^{d(t,k)} \leq \theta(k) \qquad k \in K \qquad (6)$$

$$\boldsymbol{w}, \boldsymbol{z}, \boldsymbol{r} \text{ binary}$$

where r_t is a binary activation variable of transmitter t. Notice that the emission power of each activated transmitter ranges within an interval $[P^{\min}, P^{\max}]$, corresponding to the minimum and maximum emission power in each direction (5).

4 Algorithm and results

It is known that the standard planning problem (3) is already hard to solve by commercial LP solvers [4, 7]. In fact, the coefficient matrix is ill conditioned, the number of SIR inequalities is very large even

Frequency	LP rel.	(%)	Initial sol.	(%)	Final sol.	(%)
36	1621811	99.4	1616917	99.1	1613653	98.9
50	1618548	99.2	1609901	98.7	1605495	98.4
46	1615285	99.0	1608922	98.6	1605320	98.4
47	1616917	99.1	1607780	98.5	1603864	98.3
49	1623443	99.5	1607290	98.5	1602232	98.2
42	1612022	98.8	1607616	98.5	1602102	98.2
53	1616917	99.1	1606311	98.5	1600601	98.1
43	1598969	98.0	1594074	97.7	1584285	97.1
39	1610390	98.7	1602232	98.2	1582653	97.0
32	1616917	99.1	1603864	98.3	1579390	96.8
60	1590811	97.5	1572863	96.4	1558179	95.5
52	1605495	98.4	1574495	96.5	1556547	95.4
55	1585916	97.2	1561442	95.7	1543495	94.6
5	1576127	96.6	1556547	95.4	1540231	94.4
26	1585916	97.2	1559811	95.6	1538600	94.3
57	1581021	96.9	1548389	94.9	1528810	93.7
35	1579390	96.8	1546758	94.8	1522284	93.3
10	1579202	96.8	1536968	94.2	1514126	92.8
65	1567969	96.1	1522284	93.3	1510863	92.6
40	1563074	95.8	1523915	93.4	1502705	92.1
30	1554916	95.3	1528810	93.7	1497810	91.8
64	1561442	95.7	1520652	93.2	1486389	91.1
67	1533705	94.0	1519021	93.1	1485302	91.0
25	1504336	92.2	1496178	91.7	1483125	90.9
22	1496178	91.7	1445598	88.6	1426019	87.4
6	1373808	84.2	1359124	83.3	1341176	82.2
24	1324860	81.2	1311807	80.4	1284070	78.7
9	1333018	81.7	1292228	79.2	1280807	78.5
7	1310176	80.3	1284070	78.7	1266122	77.6
21	1230227	75.4	1164963	71.4	1143752	70.1
56	1114383	68.3	1081751	66.3	1067067	65.4
41	1052383	64.5	1031172	63.2	1009961	61.9
23	1034435	63.4	1026277	62.9	980592	60.1
66	1047488	64.2	1001803	61.4	978961	60.0
29	982224	60.2	974066	59.7	952855	58.4
59	967539	59.3	947960	58.1	928381	56.9
68	943065	57.8	907170	55.6	895749	54.9
62	910433	55.8	900644	55.2	895731	54.9
63	918591	56.3	876170	53.7	894117	54.8
33	868012	53.2	848433	52.0	833748	51.1
38	881065	54.0	837011	51.3	825590	50.6
45	781003	47.9	779905	47.8	763589	46.8
27	784800	48.1	744010	45.6	729326	44.7
11	677114	41.5	611850	37.5	571060	35.0

Table 1: Channels coverage

with a few testpoints and the LP relaxation provides weak upper bounds. To overcome such difficulties a combinatorial heuristic has been devised in [4]. Unfortunately, this cannot be adapted to include threshold constraints (6), since these destroy the structure of the solution neighborhood. Thus, MIP heuristics looks a natural option to be investigated. We first experienced MIP heuristics implemented into state-of-the-art solvers, but these turned out not to perform successfully.

We then devised a tailored MIP heuristic based on two ingredients: a dedicated preprocessing of SIR constraints along with a variant of the `Boxstep` method [5]. First, one can show that if the power range $[P_{\min}, P_{\max}]$ is small enough the coefficients of the SIR inequalities can be suitably reduced and a large number of them can be eliminated, leading to an efficiently solvable problem. Then, starting from a good feasible solution of problem (2)-(5), it is always possible to build a sequence of small boxes $[P^i_{\min}, P^i_{\max}]$ such that the last one contains a feasible solution of (2)-(6).

Table 1 reports computational results for instances corresponding to the whole spectrum in Sardinia. Each row of the table corresponds to a frequency and contains the optimal value of the LP relaxation, value of the (initial) heuristic solution without threshold constraints and the value of the final solution computed by the heuristic. For each such values the corresponding percentage population coverage is also reported. Instances have the following characteristics: $|T| = 25$, $|Z| = 2,577$, $\sum_{j \in Z} u_j = 1,631,601$ (number of inhabitants), $|K| = 574$. The number of SIR Inequalities ranges from $15,000$ to $25,000$ (depending on the channel).

The goal of broadcasters is to reach at least 90 % coverage, but still 80% is considered an attractive network. The MIP heuristic returned 25 out of 44 frequencies with excellent coverage and 5 ranging between 80% and 90%. Using the simple decomposition procedure recommended in GE06 only 8 frequencies, those marked in bold in the table, have been certified to reach high coverages. Thanks to our results 17 out of the excellent frequencies were made available to fill-up the"digital dividend". Moreover, the French administration agreed that such assignments are compatible with the current analog assignments in Corse and Southern France.

References

[1] The Chester 1997 Multilateral Coordination Agreement, Technical Criteria, Coordination Principles and Procedures for the introduction of Terrestrial Digital Video Broadcasting, 25 July 1997.

[2] European Broadcasting Union, Terrestrial Digital Television Planning and Implementation Considerations, BPN 005, second issue, July 1997.

[3] Autorità Nazionale per le Garanzie nelle Comunicazioni, Libro Bianco sulla Televisione Digitale Terrestre, 2000, http://www.agcom.it.

[4] Mannino, C., Rossi, F. and S. Smriglio, The Network Packing Problem in Terrestrial Broadcasting, *Operations Research*, (54/4), July-August 2006, 611–626.

[5] R. E. Marsten, W. W. Hogan, J. W. Blankenship, The Boxstep Method for Large Scale Optimization, *Operations Research*, **23**, 3, May-June 1975, pp. 389–405.

[6] Rappaport, T.S., *Wireless Communications*, Prentice Hall, 2002.

[7] Rossi, F., Sassano, A. and S. Smriglio, Models and Algorithms for Terrestrial Digital Video Broadcasting, *Annals of Operations Research*, **107**, 2001, 267 – 283.

[8] Geneva Regional Radiocommunication Conference 2006, `http://www.itu.int/ITU-R/conferences/rrc/rrc-06/index.asp`.