Pills of micro economy for the dummies

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Warning: materials contained in the present paper are in no way to be intended as a result of original research work. They are to be viewed as the final (but full of errors and inaccuracies) outcome of the sacking of the books cited in the References. Their aim is to be a support for the studying of microeconomy to their author (and his friends).

1 Introduction

The present short notes contain a bunch of basic materials for absolute beginners in economic theory. Such materials essentially come from [MCWG95] and [Vil04] (though all errors are mine) and include:

1. some remarks on preference relations,
2. an introduction to commodities, consumpion set and competitive budgets,
3. a brief description of the classic demand theory and
4. a short analysis of the Walrasian equilibrium theory in a pure exchange economy.

The main and basic results of calculus and topology are given for granted though some of the main concepts will be reminded whenever they are needed.

2 Basic results

We start with the definition of an abstract set X as te set of possible mutually exclusive alternatives from which consumers must choose and then we define a (preference) relation over such a set and, for sake of convenience, we define a function that represents such a relation and that easier to manipulate and represent since it takes values on $\mathbb{R}$.
The use of a preference relation considers the tastes of the consumers as a primitive characteristic of the individual and represents the approach we will develop in these short notes though it is not the only one, as noted in [MCWG95], pag 5.

2.1 Preference relations

Preference relations (denoted as $\succeq$) summarize the tastes of the consumers and represent binary relations over the set X. The aim of $\succeq$ is to allow the comparison of pairs of alternatives x, $y \in X$. With the notation $x \succeq y$ we mean that x is *weakly preferred* to y or that x is *as least as good as* y.
From $\succeq$ two more relations are easily derived:

1. a *strict preference relation*, denoted as $\succ$
2. an *indifference relation*, denoted as $\sim$

The former is defined as $x \succeq y \iff x \succeq y$ but not $y \succeq x$ and means that x is *preferred* to y.
The latter is defined as $x \succeq y \iff x \succeq y$ and $y \succeq x$ and means that x is *indifferent* to y.
Of such a group of relations the $\succeq$ is the basic one and is assumed to be $rational$ that is *complete* and *transitive*. $Completeness$ means that any time we get two items x and y in X we are able to say if $x \succeq y$ or $y \succeq x$ or both so in no case we can say "we cannot compare x and y".
$Transitivity$, on the other side, means that (anyway we pick x, y and z in X) if we can say that $x \succeq y$ and that $y \succeq z$ we are sure that $x \succeq z$.
$Completeness$ (from which derives $reflexivity$) and *transitivity* embody the rationality of $\succeq$ preference relation. The fact that $\succeq$ is *complete* and *transitive* (and so is *rational*) determines the following properties of the "derived" relations:

1. relation $\succ$ is both $irreflexive$ and $transitive$,
2. relation $\sim$ is $reflexive$ ($x \sim y \; \forall \, x$), *transitive* and *symmetric* (so that if $x \sim y$ then $y \sim x \; \forall \, x, \, y \in X$),
3. if $x \succ y \succeq z \longrightarrow x \succeq z$,
4. if $x \sim y \succeq z \longrightarrow x \succeq z$,
5. $\forall \, x, \, y \in X$ we have either $x \succ y$ or $y \succ x$ or $x \sim y$.

All such properties can be easily derived from the the rationality of $\succeq$. It is easy to see that $\sim$ is an *equivalence relation* over the set X and that both $\succ$ and $\sim$ are transitive.

2.2 Utility functions

Once that preference realtions have been defined and characterized we step further and introduce *utility functions* $u()$ that describe preference relations by assigning numerical values to the elements of X. To be more formal we say that a function

$$u : X \longrightarrow \mathbb{R} \tag{1}$$

is an utility function and represents $\succeq$ if for all x and $y \in X$ we have

$$x \succeq y \iff u(x) \geq u(y) \tag{2}$$

Such a function is not unique since by composing it with any strictly increasing function $f : \mathbb{R} \longrightarrow \mathbb{R}$ we obtain a new utility function that represents

preferences $\succeq$ as $u()$ did. This allows us to say that utility functions are characterized by two types of properties: *ordinal* and *cardinal* ([MCWG95]). *Ordinal properties* are preserved under any strictly increasing transformation whereas *cardinal properties* are not preserved under such transformations.
In order to be able to use utility functions in place of preference relations we need to step further and to establish under which conditions an utility function represents a preference relation.
According to [MCWG95], pag 9, a preference relation can be represented by a utility function only if it is rational. To show that this is true we have to prove that given $u()$ that represents $\succeq$we have that $\succeq$ is rational and so is *complete* and *transitive* ([MCWG95], pag 9). To prove *completeness* we note that since $u()$ is a real valued function over the set X we must have either $u(x) \geq u(y)$ or $u(y) \geq u(x)$ and so, for the aforestated definitions, we must have either $u(x) \succeq u(y)$ or $u(y) \succeq u(x)$ and hence *completeness*.
To show *transitivity* we suppose that $x \succeq y$ and $y \succeq z$ so thatfrom the definition of $u()$ we have $u(x) \geq u(y)$ and $u(y) \geq u(z)$ and hence $u(x) \geq u(z)$ that implies $x \succeq z$. we have so proved that $\succeq$ is rational if it is rapresented with an utility function. Obviously we have x, y and $z \in X$.
The converse is not true: given an arbitrary preference relation we cannot describe it with an utility funcion without assuming that it is *continuos*, further details in forthcoming sections.

3 Basics on consumer choice

We stress the role of the *consumer* in a *market economy* in which the goods and services are available at given and known prices (so that consumers act as *price takers* since they cannot influence prices) or can be exchanged with other goods or services at given and known rates of exchange. We introduce the following concepts at a very low level:

1. *commodity*: commodities are the objects that each consumer can choose,

2. *consumption set*: defines the physical constraints that limit consumer's choices,

3. *Walrasian budget set*: defines the economical constraints that limit consumer's choices and

4. *Walrasian demand functiont* that defines the decisions of a consumer under the aforesaid constraints.

The order of exposition follows faithfully the one adopted in [MCWG95] with some limited diversions in [Vil04].

3.1 The concept of *commodity*

Every consumer has to decide the consumption levels of the various goods and services available on the market. Such goods and services are called *commodities*. It is usually assumed that the number of commodities is finite and equal to L so that we can use an index $l \in \{1, ..., L\}$ to locate a single commodity as x_l. Commodities are collected in the so called *commodity or consumption vector* that contains the list of the amounts of any commodity. We have[1]:

$$x = [x_1, \ldots, x_L] \in \mathbb{R}^L \tag{3}$$

where $\mathbb{R}^L$ is the so called *commodity space*. With a commodity vector we represent the consumption levels of a generic consumer and with a component x_l we mean the amount of the commodity l consumed.

3.2 The concept of *consumption set*

Consumpion vectors usually suffer limitations owed to a number of physical or institutional constraints that prevent the consumption of commodities from assuming negative values or values lower or hifigher than a fixed quantity. Formally we say that we have a *consumption set* as a subset of $\mathbb{R}^L$ (denoted as $X \subset \mathbb{R}^L$) and that *consumption vectors* $x \in X$.
In what follows, on the footsteps of [MCWG95], we define the *consumption set* as the set of the non negative *consumption vectors*, i.e.:

$$X = \mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x_l \geq 0 for\, l = 1, \ldots, L\} \tag{4}$$

If we consider, as an example, the case of two commodities x_1 and x_2 (and so L=2) we can consider the some of the many conceivable combinations (for details see [MCWG95] pages 19-20):

1. both x_1 and x_2 are continuous and non negative and one of the two (or both) may have an upper bound,

2. either x_1 or x_2 can take on discrete non negative values while the other take on continuous non negative values and one of the two (or both) may have an upper bound,

[1] In these notes we try to avoid, whenever possible, the use of superscripts to denote vector transposition so the reader should rely on the context to understand if a given vector is a column or a row vector.

3. both x_1 and x_2 are continuous and non negative and one of the two (or both) may have a lower bound,

4. both x_1 and x_2 are continuous and non negative without any other constraint.

At the end of the present section we note tat the main feature of $X=\mathbb{R}^L$ is the fact that it is a *convex set* so that if we have two consumption vectors x^1 and x^2 in $\mathbb{R}^L$ and we take $\lambda \in [0,1]$ we get that the convex convolution $x=\lambda x^1 + (1-\lambda)x^2 \in \mathbb{R}^L$. The property of convexity of the set X is very important in the general development of the theory and will be maintained throughout the the sections that follows.

3.3 Competitive budgets

At this point we have to face with the the consumption sets that a generic consumer can afford. In order to do that we introduce the concept of price vector

$$p = [p_1, \dots, p_L] \in \mathbb{R}^L \tag{5}$$

with two assumptions:

1. the prices of the L commodities are all publicly known,

2. the consumers are *price takers* in the sense taht they cannot influence the prices since the each consumer's demand of any commodity is a small fraction of the total demand for such a good.

Even if nothing imposes that prices cannot be negative (a negative price simply means that a consumer really pays to consume a commodity) we are going to assume in what follows that $p \gg 0$ so that $p_l > 0 \, \forall \, l \in [1, L]$.
The fact that a consumer can afford a consumption set depends on:

1. the prices $p = [p_1, \dots, p_L]$,

2. the wealth w of the consumer

since a consumption set x is affordable iff the *affordability constraint* is satisfied[2]:

$$px = p_1x_1 + \dots + p_Lx_L \leq w \;\; with X \in \mathbb{R}^L \tag{6}$$

[2]We use no special symbol to denote scalar product of two vectors so any product of two vectors is generally a scalar product and so denotes an element of $\mathbb{R}$.

Such constraints imply that the set of the feasible consumption sets is made up of the elements of the following set, also know as *Walrasian* or *competitive budget set*([MCWG95]):

$$B_{p,w} = \{x \in \mathbb{R}^L_+ : px \leq w\} \tag{7}$$

where p are market prices and w is consumer's wealth. The *consumer's*

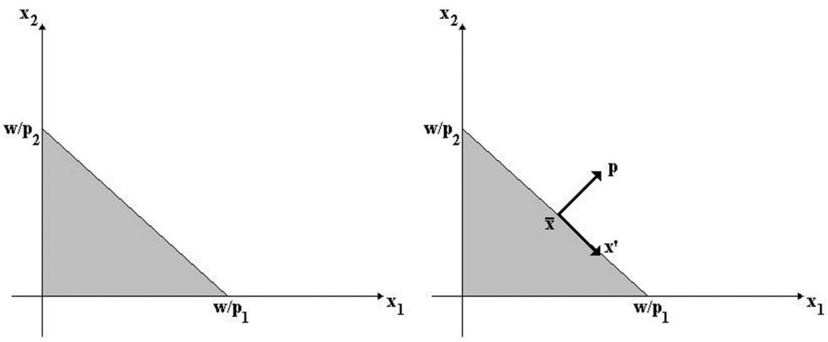

Figure 1: *A Walrasian budget set (shaded regions)*

problem constists in choosing a vector $x \in B_{p,w}$. Figure 1 represents $B_{p,w}$ when $L = 2$. To have a non trivial solution (i.e. $x \neq 0$) we impose $w > 0$. The set $\{x \in \mathbb{R}^L_+ : px = w\}$ is called *budget hyperplane* or *budget line* if $L = 2$ and defines the upper boundary of $B_{p,w}$ that is so a closed and limited set (and therefore is a compact set). It is easy to verify that the slope of the budget line is $-\frac{p_1}{p_2}$ and captures the rate of exchange of the two commodities. The points were the budget line cross the axis have coordinates:

$$(0, \frac{w}{p_2}) \ and \ (\frac{w}{p_1}, 0) \tag{8}$$

so it is easy to understand what happens if prices varies at a constant w. As shown in figure 1 price vector drawn at a generic point $\overline{x}$ of the budget line must be perpendicular to any vectory lying on such a budget line. Indeed if we take another point on the budget line, say x', we have $p\overline{x} = px' = w$ and so $p(\overline{x} - x') = 0$ whence the perpendicularity.
Last but not least we note that budget set $B_{p,w}$ is *convex* so that if we take two consumption sets x' and $x"$ in $B_{p,w}$ and a real value $\lambda \in [0, 1]$ and define a new consumption set as the convex convolution of the others

$$\hat{x} = (1 - \lambda)x' + \lambda x" \tag{9}$$

we have $\hat{x} \in B_{p,w}$, as it is easily verified (simply by using the definition of $B_{p,w}$ and some elementary calculus).
We note that the convexity of the set $B_{p,w}$ depends on that of the set of the consumption vectors X (that we posed equal to $\mathbb{R}^L$). In general we can prove that $B_{p,w}$ is convex as long as X is convex ([MCWG95]). So we have defined a (Walrasian) demand correspondence (since in general it is multi valued) $x(p,w)$ that assigns a set of consumption sets x to each pair (p,w). The reason we have a correspondence is that given a pair (p,w) there can be more than one vector x than can be chosen by the consumer. If we have only one vector we say $x(p,w)$ is a demand function. Such a demand correspondence $x(p,w)$ is supposed to verify the following hypotheses ([MCWG95]):

1. the demand correspondence $x(p,w)$ is *omogeneous of degree zero* if $x(\alpha p, \alpha w)$=$x(p,w)$ for any p,w and $\alpha > 0$,

2. the demand correspondence satisfies Walras' law if $\forall\ p \gg 0$ and $w > 0$ $px = w\ \forall\ x \in x(p,w)$[3].

Homogeneity of degree zero[4] means that if both prices and wealth change in the same proportion then the affordable consumption sets remain unchanged: such a property derives easily from the definition of budget set $B_{p,w}$ since it is easy to see that $B_{p,w}$=$B_{\alpha p,\alpha w}$. On the other hand the fact that the demand correspondence $x(p,w)$ satisfies Walras' law means that the consumer fully uses his wealth (no savings) over his lifetime. The aforesaid properties are more correctly derived from the maximization of the preferences (see next section for details and [MCWG95] for a more serious discussion) and can be proved that they hold under very general circumstances though we have posed them here as assumptions.
As noted in [MCWG95], pag. 24, one immediate consequence of homogeneity of degree zero is that although $x(p,w$ has $L+1$ arguments we can fix the level of one of them so to normalize with respect to it the other L variables: usually normalization can be done either in term of one of the prices p_l for some l or in term of the wealth w by posing it equal to 1. Delving deeper into the subject is out of the scope of the present notes (and also out of the ability of the author).

[3]With the notation $p \gg 0$ we mean that any element of p is strictly greater than 0 whereas with the notation$p \geq 0$ we mean that every component of p is ≥ 0.

[4]We can define homogeneity of any degree as follows: f(x) is homogeneous of degree r if we have $f(tx) = t^r f(x)$.

3.4 Concluding remarks

Demand correspondence $x(p, w)$ represents the choices of the consumer and depends on both the prices p and the consumer's wealth w under the hypotheses:

1. $p \gg 0$
2. $w > 0$.

We can have variations of $x(p, w)$ with w for a fixed vector $\overline{p}$ (wealth effects), variations of $x(p, w)$ with p for a given wealth $\overline{w}$ (price effects) and variations of $x(p, w)$ with both p and w. In the present subsection we give only a few simple remarks about wealth effects (see figure 2). More about the topic, as usually, on [MCWG95].

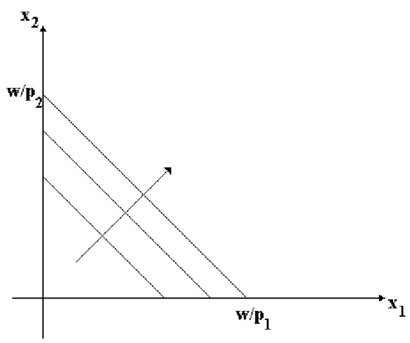

Figure 2: *Wealth effects*

In figure2 we see that as w becomes bigger in the direction of the arrow the budget set becomes bigger and so the consumer can afford bigger consumption sets.
Indeed, given a vector of prices $\overline{p}$ we get that the demand correspondence depends only on the wealth w so we have $x(\overline{p}, w)$ or the so called *consumer's Engel function* whose image in $\mathbb{R}^L_+$ is $\{x(\overline{p}, w) : w > 0\}$ is known as *wealth expansion path*. The important thing to note is that the $\frac{\partial x_l(p,w)}{\partial w}$ gives the wealth effect on the lth commodity: if such a derivative is ≥ 0 (and so a richer consumer can buy a quantity equal or bigger of such a commodity) than the commodity is called *normal* wheras if such a derivative is < 0 the commodity is called $inferior$ so that the richer the consumer is the less he/she buys of such a commodity. Further details, as usually, on [MCWG95].

4 A few notes on ***demand theory***

4.1 Introduction

In the present section we shadow, as usually, [MCWG95], chapter 3 in this case, and start with some notes on *preference relations* and their main *properties* to step, soon after, at introducing *utility functions* so to close the section with something about the consumer's decision problem under the assumption that we have L commodities whose prices are fixed ad cannot be influenced by the consumers.

4.2 Basics on ***preference relations***

The first step ([MCWG95], pag. 41 and followings) is to introduce the concept of consumer's preferences over the commodity vectors x in the consumption set $X \subset \mathbb{R}^L_+$. To describe such preferences we introduce a preference relation $\succeq$ defined on the set X and require that it is *rational* and so that it is:

1. *complete* ,$\forall\ x,\ y \in X$ we have either $x \succeq y$ or $y \succeq x$ or both, and
2. *transitive* ,$\forall\ x,\ y$ and $z \in X$ we have $x \succeq y$ and $y \succeq z \Longrightarrow x \succeq z$.

Besides such properties we introduce both *desiderability* and *convexity* assumptions.
Desiderability assumptions are captured by the concepts of *monotonicity*, *strong monotonicity* and *local nonsatiation* and represent the fact that larger amounts of commodities are preferred to smaller ones. An undrlying assumption is that $\forall x \in X$ if $y \geq x$ then $y \in X$ so any quantity of commodity is available for consumption.
We say that a *preference relation* $\succeq$ on the set X is:

1. *monotone* if given x and $y \in X$ we have $y \gg x \longrightarrow y \succ x$,
2. *strongly monotone* if given x and $y \in X$ we have $y \geq x$ and $y \neq x \longrightarrow y \succ x$,
3. *locally nonsatiated* if $\forall\ x \in X$ and $\forall\ \varepsilon > 0\ \exists y \in X$ such that $\parallel y - x \parallel < \varepsilon$ and $y \succ x$.

Monotonicity requires an increase in all the commodities from x to y so that $y \succ x$ and therefore if some of the commodities do not vary we can have indifference between x and y. On the other hand *strong monotonicity* allows the increase of only a subset of the commodity so to say that $y \succ x$.

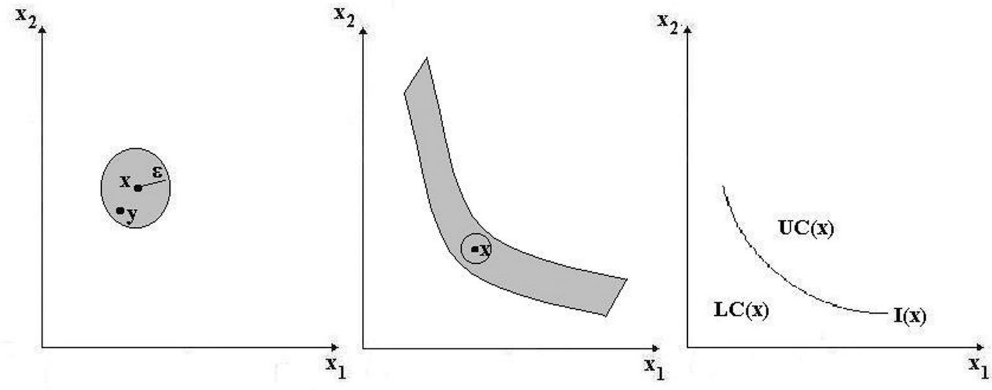

Figure 3: *LNS, UC(x), LC(x) and I(x) sets, see text for details (from [MCWG95]).*

In figure 3 we represent (on the left) an example of local non satiability (LNS): we have a ball with center in x and radius ε and another consumption set $y \in \mathbb{R}^L_+$ that is *(strongly) preferred* to x whereas in the center of figure 3 we show how a thick indifference set violates LNS since if it satisfied LNS there should be a point y preferred to x and contained in the ball shown but this contradicts the fact that all the points within the set are indifferent amongst themselves.
In realtion to the preference relation $\succeq$ we can introduce the three sets on consumption sets shown on the right of figure 3 and namely: the *indifference set* or $I(x)$, the *upper contour set* or $UC(x)$ and the *lower contour set* or $LC(X)$. Such sets are defined as follows:

1. $IC(x) = \{y \in X : y \sim x\}$,
2. $UC(x) = \{y \in X : y \succeq x\}$,
3. $LC(x) = \{y \in X : x \succeq y\}$.

As to the *convexity* of $\succeq$ we state the following definition (from [MCWG95]): a preference relation $\succeq$ on X is *convex* if $\forall\ x \in X$ we have that $UC(x)$ is *convex* (see figure 3, on the right) and os if:

$$y \succeq x \; and \; z \succeq x \longrightarrow \alpha y + (1-\alpha)z \succeq x \tag{10}$$

for any $\alpha \in [0, 1]$.
Convexity conveys the key concept that consumers tend to diversificate their

consumption vectors. The above definition has been stated for a generic set X but, as noted on [MCWG95], can hold only if X is *convex*. Besides *convexity* we use the stronger assumption of *strict convexity*.
A preference relation $\succeq$ satisfies the property of *strict convexity* if:

$$y \succeq x \; and \; z \succeq x \longrightarrow \alpha y + (1 - \alpha)z \succeq x \tag{11}$$

with $y \neq z$ and for any $\alpha \in (0, 1)$.
The present subsection closes with two properties that allow the deduction of a preference relation from a single indifference set. Such proporties are those of *homoteticity* and *quasilinearity*. We simply set out the properties and refer the reader to [MCWG95] for further details.
Homoteticity applies only to *monotone* preference relations and states that a *monotone preference relation* $\succeq$ on $X \in \mathbb{R}^L_+$is *homotetic* if $x \sim y$ then $\alpha x \sim \alpha y \; \forall \alpha \geq 0$.
A preference relation $\succeq$ on $X = (-\infty, \infty)\times \mathbb{R}^{L-1}_+$ is *quasilinear* with respect to commodity 1 (or, in general, respect to commodity l, with the obvious changes) if:

1. all the indifference sets are parallel displacements of each other along the axis of commodity 1 so that if $x \sim y$ then $(x + \alpha e_1) \sim (x + \alpha e_1)$ with $e_1 = (1, 0, \ldots, 0)$ and $\alpha \in \mathbb{R}$,

2. good is desiderable and so $x + \alpha e_1 \succ x \; \forall \; x$ and $\alpha > 0$.

4.3 Preference relations and utility functions

In this section we introduce a function, called *utility function* and denoted as $u()$, tha t can be used to represent preference relations under the assumption that they are continuous. We omit many details (not omitted on [MCWG95]) and go directly to the main point.
The basic property we need is that $\succeq$ is *continuous*. We say that $\succeq$ on X is *continuous* if for any sequence $\{(x^n, y^n\}_n with x^n \succeq y^n \forall n$ with $x = lim_{n\to\infty} x^n$ and $y = lim_{n\to\infty} y^n$ we have $x \succeq y$.
An equivalent way to state continuity of $\succeq$ is to say that $\forall x \; UC(x)$ and $LC(x)$ are both closed. Once established that $\succeq$ is cotinuous we can state that there is a *continuous* $utility \; function$ $u : X \to \mathbb{R}$ that represents such a correspondence so that:

$$x \succeq y \;\longleftrightarrow u(x) \geq u(y) \tag{12}$$

From that definition and both from the properties of $\succeq$ and those of the real numbers it is easy to state that:

$$x \succ y \;\longleftrightarrow u(x) > u(y) \tag{13}$$

and that

$$x \sim y \longleftrightarrow u(x) = u(y) \tag{14}$$

We already noted in past sections that the function $u()$ that represents $\succeq$ is not unique since if we transform it with a strictly increasing function f we again get a utility function that may be no more continuous but in no way we stated that all the utility functions must be continuous but only that if $\succeq$ is continuous then exists a continuous utility function that represents it.
Usually it is also assumed,for analytical convenience, that $u()$ is either *differentiable* or *twice continuously differentiable* so that indifference sets are smooth surfaces.
Since there is a correspondence betwwen $\succeq$ and $u()$ we have that the proprties of the former reverberate on properties of the latter so that:

1. *monotonicity* of $\succeq$ implies that $u()$ is increasing so that $u(x) > u(y)$ if $x \gg y$,

2. *convexity* of $\succeq$ implies that $u()$ is *quasy concave*[5].

Convexity of $\succeq$ does not imply that $u()$ is concave[6]. The last two properties we state with regard to utility functions representing preference relations are the followings[7]:

1. a continuous preference relation $\succeq$ on $X = \mathbb{R}^L_+$ is *homotetic* iff it is represented with a utility function $u()$ that is *homogeneous of degre one*

2. a continuous preference relation $\succeq$ on $X = (-\infty, \infty) \times \mathbb{R}^{L-1}_+$ is quasi-linear with respect to the first commodity if can be represented with a utility function $u() = x_1 + \phi(x_2, \ldots. x_L)$.

4.4 The utility maximization problem

Under the hypotheses that:

1. we have a preference relation that is *rational*, *continuous* and *locally non satiated*,

[5]We say that $u()$ is *quasiconcave* if the set $\{y \in \mathbb{R}^L_+ : u(y) \geq u(x)\}$ is *convex* for all x or, equivalently, if $u(\alpha x + (1-\alpha)y) \geq Min\{u(x), u(y)\}$ $\forall x, y$ and $\forall \alpha \in [0, 1]$.

[6].u() would be concave if $u(\alpha x + (1-\alpha)y) \geq \alpha u(x) + (1-\alpha)u(y)$

[7]We call such properties *cardinal* since are non preserved for an arbitrary increasing transformation of the utility function in contrast with the properties of being increasing and quasiconcave that are preserved under such transformations and that, for such a reason, are called *ordinal*.

2. we have a continuous utlity function () that represents such a preference relation,

3. we have a consumption set X=$\mathbb{R}^L_+$,

4. we have prices $p \gg 0$ and wealth $w > 0$

we can state the consumer's problem of choosing a consumption set within the Walrasian budget set $B_{p,w} = \{x \in \mathbb{R}^L_+ : px \leq w\}$ so to maximize his/her utility level as follows

$$Max_{x \geq 0} u(x) \qquad s.t. \qquad px \leq w \tag{15}$$

and call it *Utility Maximization Problem* or, in short, UMP.
The first thing to state is the existence of at least a solution of such a problem. We are sure that the problem UMP has a solution if $p \gg 0$ and $u()$ are continuous since a continuous function on a compact set always attains a maximum value and budget set $B_{p,w} = \{x \in \mathbb{R}^L_+ : px \leq w\}$ is compact because it is both *bounded* and *closed*.
At this point we examine both the set of optimal consumption sets and the values of the function $u()$ on such sets.
Within UMP there is a rule that assigns x vectors to pairs $(p, w) \gg 0$. such a rule is denoted with $x(p, w) \in \mathbb{R}^L_+$ and is called *Walrasian demand correspondence*. For each pair $(p, w) \gg 0$, $x(p, w)$ may contain even a set of elements (and this is the reason we call it correspondence) and verifies the following properties:

1. is homogenous of degree zero in (p, w) so that $x(\alpha p, \alpha w) = x(p, w)$ $\forall p, w$ and $\forall \alpha \in \mathbb{R}$,

2. satisfies Walras' law so that $px = w \; \forall x \in x(p, w)$,

3. satisfies convexity/uniqueness .

under the assumptions that $u()$ is a continuous utility function repreenting a locally non satiated preference relation defined on $X \in \mathbb{R}^L_+$. $Convexity/uniqueness$ means that if $\succeq$ is convex so that $u()$ is quasiconcave then $x(p, w)$ is a convex set and that if $\succeq$ is strictly convex then $u()$ is strictly quasiconcave and then $x(p, w)$ is single valued. The proof of such properties can be found in [MCWG95], on page 52.
Demand correspondence $x(p, w)$ locates the sets x on which UMP attains its maxima that are described by the *indirect utility function* denoted with $v(p, w) \; in \; \mathbb{R}$ and that for any $(p, w) \gg 0$ gives the utility value of the UMP. As to $v(p, w)$ we simply list its proferties referring the reader to [MCWG95]

for their discussion and proof.
Under the assumptions that $u()$ is a continuous utility function repreenting a locally non satiated preference relation defined on $X \in \mathbb{R}^L_+$ we have that the indirect utility function $v(p, w)$ is:

1. homogeneous of degree zero,
2. strictly increasing in w and non increasing in p_l for any l,
3. quasiconvex so that the set $\{(p, w) : v(p, w) \leq \overline{v}\}$ is convex for any $\overline{v}$,
4. continuous in p and w.

5 Walrasian equilibrium

In this ending section we face the *Walrasian equilibrium model* in its simplest form and therefore in the case of a *pure excahnge economy.* Theory is in itself complex and broad so we must limit to a few hints owing to limitations in both space and science. For a broader treatment of the subject we refer the reader to [MCWG95], chapter 17.
We introduce the notion of *aggregate excess demand function* and try to say something about the existence and uniqueness of Walrasian equilibria as well as about the properties of such equilibria (that generically are finite in number).

5.1 Pure exchange economy

According to [MCWG95] in a *pure exchange economy* the only possible production activities are those of free disposal. This means that the set of producers contains only one member (so $J = 1$ and $Y_1 = -\mathbb{R}^L_+$, wher L is the number of the commodities) while we have I consumers and each consumer ha a consumption set $X_i = \mathbb{R}^L_+$ and an initial endowment vector $\omega_i \in \mathbb{R}^L$. We assume that $\sum_i \omega_i \gg 0$. The preference relation $\succeq_i$ of the consumers is supposed to be:

1. continuous,
2. strictly convex,
3. locally non satiated.

Within such an economy we say that the set $(x^*, y^*) = (x^*_1, \ldots, X^*_I, y^*_1)$ of consumption sets and production at the optimum and a price vector $p \in \mathbb{R}^L$ constitue a Walrasian equilibrium iff the following conditions are satisfied:

1. $y^*_1 \leq 0$, $py^*_1 = 0$ and $p \geq 0$,
2. $x^*_i = x_i(p, p\omega_i)$ $\forall i \in [1, \ldots, I]$,
3. $\sum_i x^*_i - \sum_i \omega_i = y^*_1$

The above conditions allow us to say that in a pure exchange economy in which consumer preferences satisfy the aforementioned conditions a price vector $p \geq 0$ is a walrasiona equilibrium price vector iff:

$$\sum_{i=1}^{L} x_i(p, p\omega_i) - \omega_i \leq 0 \tag{16}$$

Such a condition introduces the vector $x_i(p, p\omega_i) - \omega_i \in \mathbb{R}^L$ as the demand for each good over and above the amount that consumer i possesses in his/her endowment vector ω_i.
Condition (16) suggests a formalization of the excess demand vector for each consumer and of its sum over the set of the I consumers as a function of the prices p.
In this way we define for the generic consumer i the *excess demand function* as

$$z_i(p) = x_i(p, p\omega_i) - \omega_i \leq 0 \tag{17}$$

where $x_i(p, p\omega_i)$ is the Walrasian demand function for the generic consumer i whereas thw *aggregate demand function* is defined by summing for all the consumers so to get the following function:

$$z(p) = \sum_{i=1}^{L} z_i(p) \tag{18}$$

whose domain is the set of non negative price vectors that includes all strictly positive price vectors ([MCWG95]).
The use of equation (18) we can paraphrase equation (16) as follows: the price vector $p \in \mathbb{R}^L_+$ is an equilibrium price vector iff we have $z(p) \leq 0$.
We note that if p is an equilibrium price vector within the economy we have sketched so far we have:

1. $p \geq 0$,
2. $z(p) \leq 0$ and
3. $pz(p) = \sum_i pz_i(p) = \sum_i (px_i(p, p\omega_i) - p\omega_i) = 0$

where we have used (16) and *local non satiation*. This means that for any commodity l we have not only $z_l(p) \leq 0$ but also $z_l(p) = 0$ if $p_l > 0$. Therefore if a commodity has a strictly positive price, at the equilibrium it cannot be in excess supply (so $z_l(p) = 0$) as can happen (so $z_l(p) < 0$), instead, if it is free (and therefore $p_l = 0$).
Following [MCWG95], the next step is assuming that consumer preferences are *strongly monotone* so that from now on we rely on the following assumptions:

1. $X_i = \mathbb{R}^L_+$ for $i \in [1, \ldots, I]$
2. All preference relations $\succeq_i$ are:
    (a) continuous,

(b) strictly convex and

(c) strongly monotone.

Under these assumptions any Walrasian equilibrium must involve a price vector that is strictly positive (i.e. $p \gg 0$) otherwise consumers would ask for infinite quantities of free goods. If preferences are strongly monotone a price vector $p = (p_1, \dots, p_L)$ is a Walrasian equilibrium price vector iff it causes the full consupmtion of all the goods so to *clear the market.* More formally it is a Walrasian equilibrium price vector iff it is the solution of the following system of L equations and L unknowns:

$$z_l(p) = 0 \quad \forall l = 1, \dots, L \tag{19}$$

At this point we introduce the properties of the aggregate excess demand function that are essential in the development of the theory. As usually we only list the properties with a minimum proof referring the reader, again, to [MCWG95], chapter 17.
If we suppose that, for every consumer i, $X_i = \mathbb{R}^L_+$, $\succeq_i$ satisfies the properties we fixed before and $\sum_j \omega_j \gg 0$ then the aggregate excess demand function $z(p)$ defined for price vectors strictly positive ($p \gg 0$) satisfies the following properties:

1. $z()$ is continuous,

2. $z()$ is homogeneous of degree zero so that $z(\alpha p) = z(p) \forall \alpha > 0$,

3. Walras law is satisfied and so $\forall p \in \mathbb{R}^l +$ we have $pz(p) = 0$,

4. $\exists\ s > 0$ such that $z_l(p) > -s$ for every commodity $l \in [1, \dots, L]$ and price vectors p,

5. if $p^n \longrightarrow p$ with $p \neq 0$ but for some $l \in [1, \dots, L]$ we have $p_l = 0$ then $Max\{z_1(p^n), \dots, z_L(p^n)\} \longrightarrow \infty$

Properties form (1) to (4) derive directly from the properties of the demand function $x(p, e)$ (or $x(p, w)$). Moreover to understand property (4) we have to consider that $x_i \in X_i = \mathbb{R}^L_+$ so that a consumer i cannot supply to the market a quantity of a good l that is greater of his/her initial endowment. Last but not least property (5) means that prices cannot be all equal to 0 but if some price go to 0 then the demand of the corresponding good must go to ∞ owing to the fact that preferences are strongly monotone and there is certainly a consumer whose wealth tend to a strictly positive limit since

we have $p \sum_i \omega_i > 0$.
Walras law represents a relation among the price vector, p, and the aggregate excess demand function, $z(p)$. Such a relation gives a way to verify that a price vector $p \gg 0$ clears all markets[8] if it clears all markets but one. As noted in [MCWG95], page 582, if we have $p \gg 0$ and $z_(p) = \cdots = z_{L-1} = 0$ because we have $pz(p) = \sum_l p_l z_l(p) = 0$ and $p_L > 0$ we must have $z_L(p) = 0$. From the previous considerations we have that if we define:

$$\hat{z} = (z_1(p), \ldots, z_{L-1}(p)) \tag{20}$$

as the excess demands vector fo $L-1$ goods but $L-th$ we have that a strictly positive price vector p is a Walrasian equilibrium iff $\hat{z}(p) = 0$.
After having characterized the aggregate excess demand function $z(p)$ we step forward to the problem of the exixstence of a Walrasian equilibrium in a pure exchange economy modelled by means of excess demand function ([MCWG95], page 584).

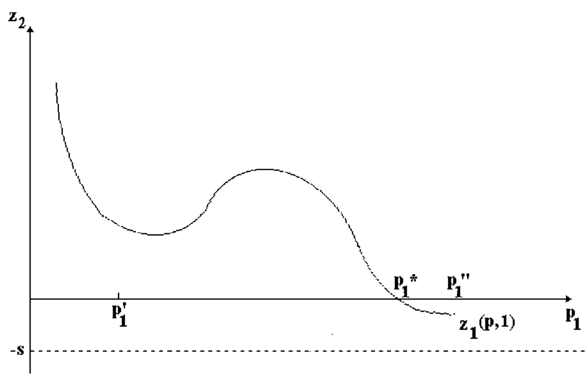

Figure 4: *The existence of an equilibrium, all details in the text (from [MCWG95]).*

We have seen that in case of *continuous*, *strictly convex* and *strongly monotone* preference relations within a pure exchange economy with $\sum_i \omega_i \geq 0$ the excess demand function $z()$ satisfies the aforesaid properties. What we want to prove now is that *any* function $z()$ that satisfies those conditions admits a price vector p such that $z(p) = 0$. The simplest case is shown in figure 4. We use the fact that $z()$ is homogeneous of degree zero to normalize with respect

[8]We note that price vector p clears all markets if $z_l(p) = 0 \ \forall l \in [1, \ldots, L]$.

to the second commodity and put $p_2 = 1$ and look for an equilibrium vector of the form $p = (p_1, 1)$. After normalization we use Walras law to solve the following equation

$$z_1(p_1, 1) = 0 \tag{21}$$

Figure 4 presents such a situation in which we have two values p'_1 where $z_1 > 0$ and $p"_1$ where $z_1 < 0$ and, since $z_1(p_1, 1)$ is continuous, there must be an intermediate value $p^* \in [p'_1, p"_1]$ where $z_1(p^*_1, 1) = 0$ so that an equilibrium vector price must exist. All such considerations in the case $L = 2$ derive fro the properties we have stated for $z(p)$ and their extension to the general case $L > 2$ is not straightforward and will only be sketched in what follows. For further details we refer, as usually, to [MCWG95].
To investigate the general case, followig [MCWG95], we suppose that $z(p)$ is a function defined for $p \in \mathbb{R}^L_{++}$ and satisfies the following conditions:

1. $z()$ is continuous,
2. $z()$ is homogeneous of degree zero so that $z(\alpha p) = z(p)\, \forall\, \alpha > 0$,
3. Walras law is satisfied and so $\forall p \in \mathbb{R}^l+$ we have $pz(p) = 0$,
4. $\exists\ s > 0$ such that $z_l(p) > -s$ for every commodity $l \in [1, \ldots, L]$ and price vectors p,
5. if $p^n \longrightarrow p$ with $p \neq 0$ but for some $l \in [1, \ldots, L]$ we have $p_l = 0$ then $Max\{z_1(p^n), \ldots, z_L(p^n)\} \longrightarrow \infty$.

If all this is verified then the system $z(p) = 0$ has a solution so that a $Walrasian$ equilibrium price vector p exists in a pure exchange economy in which preference relations of the consumers are *continuous*, *strictly convex* and *strongly monotone* and the vector of the aggregate endowments is strictly positive (i.e. $\sum_i \omega_i \gg 0$).
The identification of an equilibrium price requires a proof made up of five steps preceded by a normalization process.
we start with the normalization process that involves a normalization of the prices p. In order to do so and since the functio $z()$ is omogeneous of degree zero we define a unit simplex of the prices as follows:

$$\Delta = \{p \in \mathbb{R}^L_+ : \sum_l p_l = 1\} \tag{22}$$

and look for an equilibrium price vector $p \in \Delta$ though the function $z(p)$ is well defined only if $p \in int\Delta = \{p \in \Delta : p_l > 0 \forall\, l\}$. Summing up the five steps aims at:

1. construct a correspondence from Δ to Δ (the first two steps),

2. argue that any fixed point of the correspondence is a vector price p^* that belongs to the correspondence and is a solution fo $z(p) = 0$ (the third step),

3. prove that the correspondence is *convex valued* and *closed graph* (the fourth step),

4. and finally (in the fifth step) use Kakutani's fixed point theorem to prove that such an equilibrium price vector as a fixed point of the correspondence necessarily exists.

During the first step whenever $p \gg 0$ we define $f(p) = \{q \in \Delta \,:\, z(p)q \geq z(p)q' \,\forall q' \in \Delta\}$ so that given the current set of prices p we define a new price vector q that maximizes the value of the excess demand vector $z(p)$. If $f()$ is a rule that adjusts prices in a direction that eliminates any excess demand the correspondence $f()$ assigns the highest prices to the commodities that are most in excess demand.
So we have

$$f(p) = \{q \in \Delta \,:\, q_l = 0 \, if \; z_l(p) \, < Max\{z_1(p), \ldots, z_L(p)\}\} \tag{23}$$

We note that if $z(p) \neq 0$ for $p \gg 0$ then (from Walras' law) we have $z_l(p) < 0$ for some l and $z_{l'}(p) >=$ for some $l' \neq l$. So, for such a p, if $q \in f(p)$ then there is l such that $q_l = 0$.
If $z(p) \neq 0$ then $f(p) \in \vartheta\Delta$ [9]. After the fixed point correspondence for $p \in int\Delta$, we define we define the correspondence for $\vartheta\Delta$. If $p \in \vartheta\Delta$ we define

$$f(p) = \{q \in \Delta \,:\, pq = 0\} = \{q \in \Delta \,: q_l = 0 if p_l > 0\}\} \tag{24}$$

Since $p_l = 0$ for some l we are sure that $f(p) \neq \emptyset$. Moreover no price on the boundary of Δ can be a fixed point because $pp > 0$ whereas $pq = 0 \; \forall \; q \in f(p)$.
At this point we have that a fixed point of the correspondence $f()$ is an equilibrium. If $p^* \in f(p^*)$ we must have $p^* \notin \vartheta\Delta$ and so $p^* \gg 0$. If $z(p^*) \neq 0$ then $f(p) \subset \vartheta\Delta$ which is incompatible with $p^* \in f(p^*)$ and $p^* \gg 0$ hence if $p^* \in f(p^*)$ it must be $z(p^*)$.
Last two steps aim at proving that the correspondence is:

1. convex-valued so that for any price vector p we have a convex set and

[9] With $\vartheta\Delta$ we denote the boundary of Δ and so $\Delta - int\Delta$.

2. closed graph

and at showing that a fixed point necessariliy exists by applying Kakutani's fixed point theorem. According to such a theorem any convex valued, closed graph correspondence from a nonempty, compact, convex set into itself has a fixed point. Since both Δ and $f()$ satisfy such conditions we are necessarily sure that there exists a price vector p^* that is an equilibrium price vector so that $p^* \in f(p^*)$. The proofs given in the present notes is not complete and at times obscure. A much more plain and clear proof can be found, as usually, on citeMas-Colell:95, page 586.

A Appendix

In the present section we introduce an even simpler version of *Walrasian equilibrium* than that we presented in Section 5. The section is divided in three parts: in the first two we introduce some basic elements and definitions whereas in the last one we enunciate two theorems, one with regard to the existence of the *excess demand correspondence* and the other to the existence of an *equilibrium price vector*. We, moreover, try to highlight any underlying assumption and present the economical meaning (if any) of the symbols we introduce.

A.1 Preliminaries

The very basic concept is that of an *economy* defined as a set of pairs (u_h, e_h) one for each consumers $h \in \{1, \ldots, H\}$. In each pair we have the *utility function* u_h and the so called endowment e_h of each consumer. Each utility function represents a *preference relation* of each consumer.
Once defined the concept of economy we can specialize it in various ways. To do so we now introduce three assumptions, two explicits and one implicit.
The first assumption (*implicit*) is that our model of economy is a *model under certainty* so that the behaviour of the consumers is deterministic. The other two *explicit* assumptions are:

1. economy is a *pure exchange economy* with *finitely many commodities* and *finitely many agents or consumers*,

2. consumers are *price takers* so that cannot influence prices.

This means that we abstract from production and assume that consumers with their initial endowments of the commodities that characterize the economy go to the market where they see prices fixed in some way they cannot influence and exchange their commodities at these prices so to maximize their own utility.
We have an economy with an *equilibrium price* and an *allocation of commodities* where markets clear (so every consumer gets what he/she wants) and everybody optimizes.
To be more formal, we consider an economy with:

$$C\ commodities\ l \in \mathcal{C} = \{1, \ldots, C\} \tag{25}$$

$$H\ consumers\ h \in \mathcal{H} = \{1, \ldots, H\} \tag{26}$$

a vector of prices $p \in \mathbb{R}^C_+$, a family of sets $x_h \in \mathbb{R}^C_+$, that are the consumption sets of the consumers, and a family of sets $e_h \in \mathbb{R}^C_+$ that are their initial

endowments.
At this point we give some definitions so to formalize that each consumer has a basket of consumption goods over which he/she can express a preference relation (represented through a utility function).

Definition A.1 *The consumption set of each consumer is a subset of the commodity space* $\mathbb{R}_+^C$*, denoted by* $X \subset \mathbb{R}_+^C$*, whose elements are the consumption bundles that each consumer can conceivably consume given the physical constraints imposed by his/her environment.*

Definition A.2 *Consumers are supposed to be rationals so that their preference relations* $\succeq_h$ *are both complete and transitive so to allow an ordering of the consumption sets. Continuous preference relations can be represented with continuous functions called* utility functions. *Formally we have that an utility function for consumer* h*,* $u_h : \mathbb{R}_+^C \longrightarrow \mathbb{R}$*, represents preference relation* $\succeq_h$ *if, for all* x_1 *and* $x_2 \in \mathbb{R}_+^C$*, we have*

$$x_1 \succeq_h x_2 \Longleftrightarrow u_h(x_1) \geq u_h(x_2) \tag{27}$$

so that u_h *is a numeric representation of* $\succeq_h$*.*

Before introducing the problem that every consumer faces in our economy and so that of maximizing his/her utility, we have to describe in some way the set within which each consumers can choose his/her consumption sets under two constraints: the prices p he/she sees on the market and the endowment e_h he/she has.
We, therefore, define the *budget correspondence* β as

$$\beta : \mathbb{R}_+^C \times \mathbb{R}_+^C \rightarrow\rightarrow \mathbb{R}_+^C \tag{28}$$

that, for each pair (p, e_h), defines the so called *budget set*:

$$\beta(p, e_h) = \{x_h \in \mathbb{R}_+^C \ : \ px_h \leq pe_h\} \tag{29}$$

Budget sets define, for each consumer, the consumption sets that such a consumer can afford given market prices and personal endowment. The aim of each consumer is the maximization of the personal utility $u_h()$ and so the solution of the following *Utility Maximization Problem*(in short UMP):

$$Max_{x_h \in \mathbb{R}_+^C} \ \ u_h(x_h) \tag{30}$$

subject to

$$px_h \leq pe_h \tag{31}$$

Relations (30) and (31) represent the formal description of UMP and mean that each consumer chooses a consumption set within his/her budget set so to maximize his/her own utility. We are sure that such a problem has at least a solution from Weierstrass theorem since $\beta(p, e_h)$ is a compact set (it is closed and limited) and function $u()$ is supposed continuous.
The way consumers choose their consumption sets is through the demand correspondence $x_h(p, e_h)$ defined as follows:

$$x_h : \mathbb{R}_+^C \to\to \mathbb{R}_+^C \tag{32}$$

such that:

$$x_h(p, e_h) = argmax(UMP) \tag{33}$$

Demand correspondence is characterized in the following closing definition:

Definition A.3 *Demand correspondence $x_h(p, e_h)$ is homogeneous of degree zero so that $x_h(\alpha p, \alpha e) = x_h(p, e)$ for any p, e and $\alpha > 0$.*
Homogeneity of degree zero says that if both prices and endowment change in the same proportion, then the individual's budget set does not change, as can be easily seen from its definition.

A.2 Some maths

We list here, with some comments, three theorems and a definition that prove very useful in the field of Walrasian equilibrium, even in the simplified version we are describing in the present notes: *Weierstrass theorem*, *Maximum theorem*, *Kakutani's fixed point theorem* and *Walras' law*.

Theorem A.1 (Weierstrass theorem) *A continuous function*

$$u_h \;\; : \mathcal{A} \longrightarrow \mathbb{R} \tag{34}$$

on a compact set $\mathcal{A}$ attains a maximum and a minimum value.

We use such a theorem, in the characterization theorem, to prove that $z()$ is a well defined function and, in the existence theorem, to prove that a correspondence we define there is not empty valued.

Theorem A.2 (Maximum Theorem) *Consider a budget correspondence β, an utility function u_h, a demand correspondence x_h and the indirect utility function $v : R_{++}^C \times R_+^C \to R$, $v : (p, e) \to max(UMP)$.*
Assume that β is (non-empty valued), compact valued and continuous, u_h continuous. Then

1. x_h *is (non-empty valued), compact valued, upper hemicontinuous (UHC) and closed;*

2. v *is continuous.*

Theorem A.3 (Kakutani's fixed-point theorem) *Suppose that $A \subset \mathbb{R}^{\mathbb{N}}$ is a nonempty, compact, convex set, and that $\phi : A \rightarrow\rightarrow A$ is a closed correspondence with the property that set $\phi(x) \subset A$ is nonempty and convex for every $x \in A$. Then $\phi(\cdot)$ has a fixed point; that is there is an $x \in A$ such that $x \in \phi(x)$.*

We use Kakutani's fixed-point theorem in the existence theorem to show that the correspondence we have defined there defines an equilibrium price vector p^*. To show that we define a correspondence $\mu : S \rightarrow\rightarrow S$ and than prove that S is *convex*, *compact*, μ is *not empty valued*, *convex valued*, *closed graph* so that we can use the theorem and be sure that $\exists s^* \in S$ such that $s^* \in \mu(s^*)$.

Definition A.4 (Walras' law) *We have it in various versions and we list them here one after the other. We denote with $x_h(p, w_h)$ the demand correspondence of consumer h, $w_h > 0$ the consumer's wealth, e_h the consumer's endowment, $p \gg 0$ the vector of the prices and $z(p)$ the* excess aggregate demand correspondence.

$$pz(p) = 0 \, \forall p \tag{35}$$

$$px_h = w_h \, \forall x_h \in x(p, w_h) \tag{36}$$

$$px_h(p, w_h) = w \, \forall p \, and \, w_h \tag{37}$$

$$px_h = pe_h \, \forall x_h \in x(p, e_h) \tag{38}$$

We can, indeed, describe a consumer either in terms of a monetary wealth w_h or of an endowment of goods e_h. We use the definition as a thesis in the characterization theorem as an hypothesis in the existence theorem.

A.3 Characterization and existence

We have two theorems, one of characterization and one that allows the definition of an equilibrium price vector p. The first theorem, actually, characterizes the *excess aggregate demand correspondence* $z(p)$ in terms of:

1. the *demand correspondence* $x_h(p, e_h)$ [10]

[10] In what follows argument e_h will be disregarded in the notation for $x_h()$.

2. the *endowment* e_h

of each consumer $h \in \{1, \ldots, H\}$. We call $z(p)$ *excess aggregate demand correspondence* because it represents the difference between what consumers have and what they want, summed for all the consumers. More formally, we have:

$$z :\to\to \sum_{h=1}^{H} x_h(p) - e_h \tag{39}$$

The basic hypotheses involve the utility function ($u_h : \mathbb{R}^C + 1$) and the demand correspondence of each consumer: the first must be *continuous* ant *strictly increasing* whereas the latter must take up values in $\mathbb{R}^C_+$ and be *homogeneous of degree zero* so to "absorb" scalings of the parameter p by a factor $\alpha > 0$. Through such a theorem we prove that $z(p)$ is *well defined* (so it is not a pure formalism but has a real meaning), *continuous*, *homogeneous of degree zero*, *bounded from below* and *satisfies boundary conditions* (so that if one of the prices go to 0 the $z()$ is unbounded).
The second theorem is a *theorem of existence* and, by using as hypotheses the conclusions of the first theorem about *excess aggregate demand correspondence*, aims at saying that an equilibrium price vector $p^* \gg 0$ exists such that $z(p^*)$ where with the term *equilibrium price* we mean a price vector such that *demand* equals *supply*. The proof strongly rely on *Kakutani's fixed point theorem* we introduced in the previous subsection.
We note that the theorem proves that an equilibrium price vector exists but in no way gives tools for its determination or says something about its uniqueness. We stop here, referring the reader,as usually, to [MCWG95].

References

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