Ranking electoral systems through hierarchical properties ranking

Lorenzo Cioni
Department of Computer Science
University of Pisa
e-mail: lcioni@di.unipi.it

Structure of the presentation

- The mathematical tool
- Theoretical results
- Basic properties (the "wish lists")
- Simple exercises
- More complex exercises
- Ranking the basic methods
- Hierarchy: a real solution or a blind alley?
- Conclusions

The mathematic tool

- *Analysis phase*
  - *definition of a rooted hierarchy*
  - *definition of the matrices of pairwise comparisons*
- *Synthesis phase*
  - *solution of eigenvalues/eigenvector problems*
  - *vector of priorities as a product of matrices*

Definition of a rooted hierarchy

- Made of levels, complete connections between adjacent levels
- level 0 Main Goal (political), level 1 actors, level 2 policies, level 3 criteria, level 4 alternatives
- pairwise comparisons of elements at level i+1 with respect to elements at level i on a 1÷9 ratio scale
- level i m elements and level i+1 n elements: m matrices n x n, m eigenvectors n x 1, n x m eigenvectors matrix
- W (vector of priorities) as a matrix product of eigenvector matrices

Eigen values & eigen vectors

A, positive reciprocal, a_{ij} $i, j = 1, \dots, n$

1. $a_{ii} = 1$
2. $a_{ji} = \frac{1}{a_{ij}}$
3. $a_{ij} = a_{ik} a_{kj}$ with $i, j, k = 1, \dots, n$

A normalized vector w of weights $w_i \in [0, 1]$

$$a_{ij} = \frac{w_i}{w_j}$$

with $i, j = 1, \dots, n$.

$$a_{ij} \frac{w_j}{w_i} = 1$$

$$\sum_{j=1}^{n} a_{ij} w_j = n w_i$$

with $i = 1, \dots, n$.

$$Aw = nw$$

with $w = (w_1, \dots, w_n)$.

$$\sum_{i=1}^{n} \lambda_i = n$$

$$\frac{\lambda_{max} - n}{n - 1}$$

$$C.R. = \frac{C.I.}{R.I.} < 0.10$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	.58	.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Numerical algorithms

1. The less precise÷less complex. We sum the elements of each row and divide such a value with the sum of all the elements of the matrix. The ratio for the i-th row gives the i-th element of the eigenvector w that is normalized by construction.

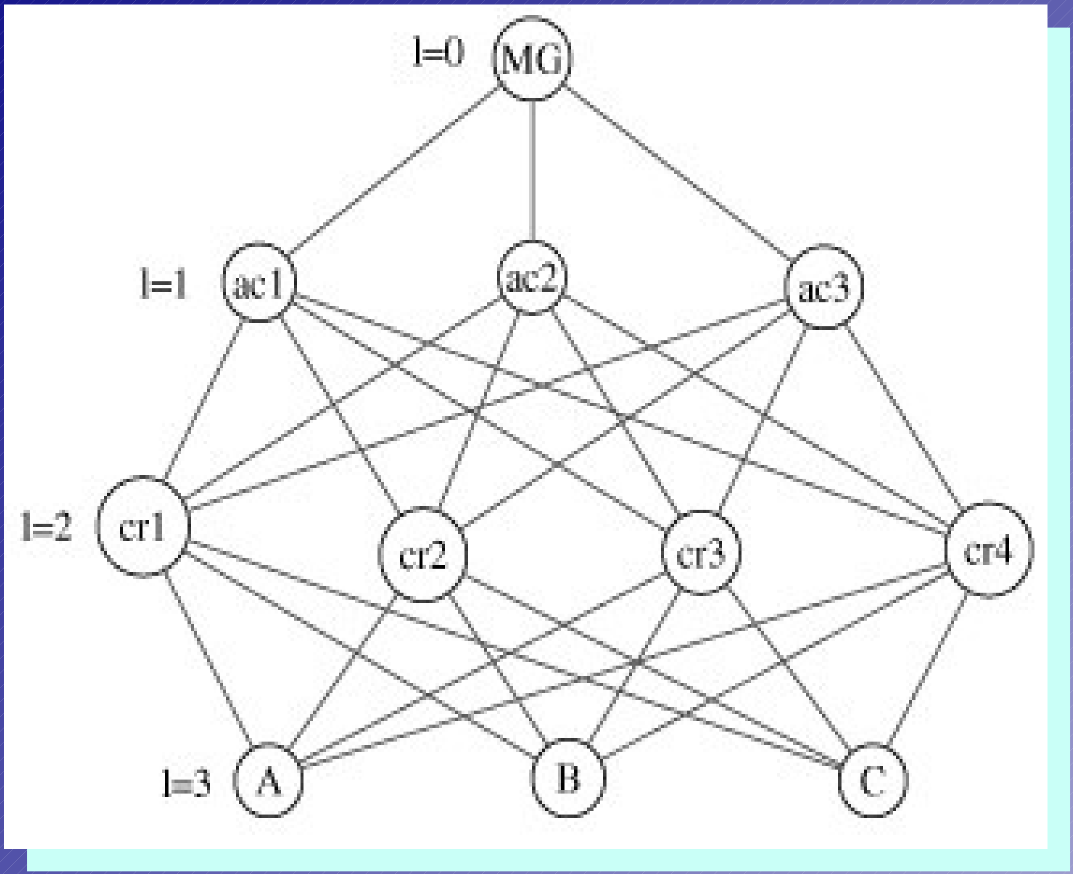
2. Higher precision÷higher complexity. We sum the elements of each column and then we evaluate the reciprocal of each sum. To normalize we divide each reciprocal with the sum of the reciprocals.

3. Good precision÷higher complexity. We evaluate the sum of the elements of each column and divide each element of a column for that sum (we normalize each column) so to obtain a new matrix. At this point we sum the elements on each row of the new matrix and divide the sum for the dimension of the matrix. In this way we evaluate an average over the normalized columns.

4. Good precision÷higher complexity. We multiply the elements of each row among themselves, evaluate the $n-$th root (if n is the dimension of the matrix) of that value and, lastly, normalize each of such values.

5. Exact solution÷highest complexity. We raise the matrix A to an arbitrarily large power and then divide the sum of the elements of each row of the resulting matrix by the sum of the elements of such matrix.

An example of hierarchy



- L_3 eigenvectors matrix at l=2 dimension 3 x 4
- L_2 eigenvectors matrix at l=1 dimension 4 x 3
- L_1 eigenvectors matrix at l=0 dimension 3 x 1
- W priorities of A, B and C with respect to MG
- $W=L_3L_2L_1$ dimension 3X1

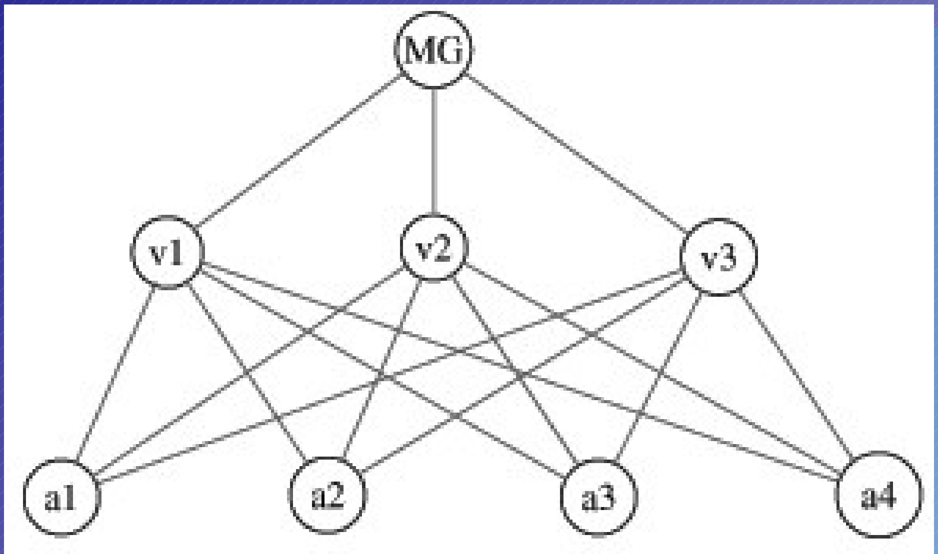
Theoretical [negative] results

- No "perfect" electoral system exists
- Some of the many theoretical impossibility results:
  - *Arrow's [im]possibility theorem (minimal set of properties vs. dictatorship)*
  - *Gibbard-Satterwaite Theorem (strategic voting)*
  - *Sen's Theorem (minimal liberalism)*
- Many tentative solutions, not so many real solutions
- Is AHP a solution? is it "immune" from such "theoretical plagues"?

The “wish lists”

- Basic list: Universal Domain, Transitivity, Pareto condition, Binary Independence;
- Extended list: Anonimity, Neutrality, Separability, Monotonicity, Non-manipulability
- Majoritarian methods list: Condorcet winner, Condorcet Loser, Monotonocity, Pareto principle, WARP, Path Independence
- Proportional methods list: House monotonicity, Quota satisfaction, Population monotonicity, Consistency, stability

A simple exercise



- We pairwise rank a1, a2, a3 and a4 with respect to v1, v2 and v3 (four matrices and four eigenvectors)
- We pairwise rank v1, v2 and v3 with respect to MG (one matrix and one eigenvector)
- $W=L_2L_1$ priorities of a1, a2, a3, a4 with respect to MG
- W ↔ total ordering i.e. priorities of a1, a2, a3 and a4 with respect to MG

A simple exercise: matrices and vectors

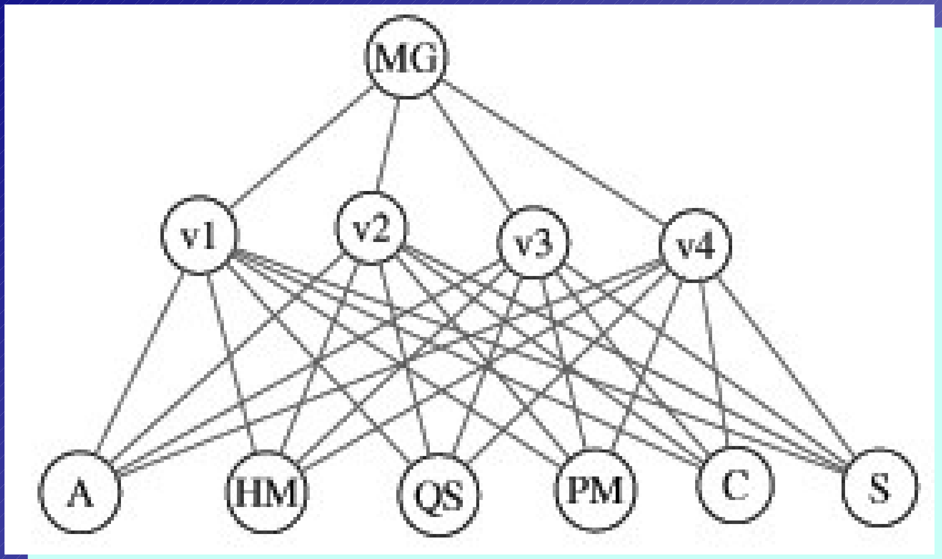
	a1	a2	a3	a4			W1	LAMBDA1	CI	CR
a1	1,00	2,00	5,00	7,00	70,0000	2,8925	0,5488	4,02	0,01	0,01
a2	0,50	1,00	2,00	3,00	3,0000	1,3161	0,2497	4,01		
a3	0,20	0,50	1,00	2,00	0,2000	0,6687	0,1269	4,02		
a4	0,14	0,33	0,50	1,00	0,0238	0,3928	0,0745	4,02		
						5,2701	1,0000	4,02		

	a1	a2	a3	a4			W2	LAMBDA2	CI	CR
a1	1,00	2,00	0,50	0,25	0,2500	0,7071	0,1355	4,03	0,01	0,01
a2	0,50	1,00	0,33	0,17	0,0278	0,4082	0,0782	4,03		
a3	2,00	3,00	1,00	0,33	2,0000	1,1892	0,2279	4,04		
a4	4,00	6,00	3,00	1,00	72,0000	2,9130	0,5583	4,04		
						5,2175	1,0000	4,03		

	a1	a2	a3	a4			W3	LAMBDA3	CI	CR
a1	1,00	1,00	0,20	0,20	0,0400	0,4472	0,0833	4,00	0	0
a2	1,00	1,00	0,20	0,20	0,0400	0,4472	0,0833	4,00		
a3	5,00	5,00	1,00	1,00	25,0000	2,2361	0,4167	4,00		
a4	5,00	5,00	1,00	1,00	25,0000	2,2361	0,4167	4,00		
						5,3666	1,0000	4,00		

W1	W2	W3	W0	W	
0,5488	0,1355	0,0833	0,3333	*0,2559*	*p1*
0,2497	0,0782	0,0833	0,3333	0,1371	p2
0,1269	0,2279	0,4167	0,3333	*0,2571*	*p3*
0,0745	0,5583	0,4167		**0,3498**	**p4**
				0,9999	

A harder exercise



1. $A > HM > QS > PM > C > S$
2. $A \sim HM > QS \sim PM > C > S$
3. $S > C \sim PM > A > HM \sim QS$
4. $QS > HM \sim A > PM \sim C \sim S$

- Proportional methods, properties:
  - Anonimity
  - House monotonicity
  - Quota satisfaction
  - Population monotonicity
  - Consistency
  - Stability
- Ranking of the properties ↔ ranking of the methods

A harder exercise: matrices and vectors

	A	HM	QS	PM	C	S	Prod	Square	W1	LAMBDA	CI	CR
A	1,00	2,00	3,00	4,00	6,00	9,00	1296,00	3,30	0,43	6,02	0,0101	0,0082
HM	0,50	1,00	2,00	2,00	3,00	4,00	24,00	1,70	0,22	6,01		
QS	0,33	0,50	1,00	1,00	2,00	2,00	0,67	0,93	0,12	6,05		
PM	0,25	0,50	1,00	1,00	2,00	2,00	0,50	0,89	0,11	6,04		
C	0,17	0,33	0,50	0,50	1,00	2,00	0,03	0,55	0,07	6,1		
S	0,11	0,25	0,50	0,50	0,50	1,00	0,00	0,39	0,05	6,09		
								7,77	1,00	6,05		

	A	HM	QS	PM	C	S	Prod	Square	W2	LAMBDA	CI	CR
A	1,00	1,00	3,00	3,00	5,00	7,00	315,00	2,61	0,31	6,27	0,0571	0,0461
HM	1,00	1,00	3,00	3,00	5,00	7,00	315,00	2,61	0,31	6,27		
QS	0,33	0,33	1,00	1,00	5,00	7,00	3,89	1,25	0,15	6,28		
PM	0,33	0,33	1,00	1,00	5,00	7,00	3,89	1,25	0,15	6,28		
C	0,20	0,20	0,20	0,20	1,00	2,00	0,00	0,38	0,05	6,29		
S	0,14	0,14	0,14	0,14	0,50	1,00	0,00	0,24	0,03	6,32		
								8,35	1,00	6,29		

	A	HM	QS	PM	C	S	Prod	Square	W3	LAMBDA	CI	CR
A	1,00	2,00	2,00	1,00	1,00	0,20	0,80	0,96	0,13	6,05	0,0049	0,0039
HM	0,50	1,00	1,00	0,50	0,50	0,14	0,02	0,51	0,07	6,00		
QS	0,50	1,00	1,00	0,50	0,50	0,14	0,02	0,51	0,07	6,00		
PM	1,00	2,00	2,00	1,00	1,00	0,33	1,33	1,05	0,14	6,01		
C	1,00	2,00	2,00	1,00	1,00	0,33	1,33	1,05	0,14	6,01		
S	5,00	7,00	7,00	3,00	3,00	1,00	2205,00	3,61	0,47	6,06		
								7,69	1,00	6,02		

	A	HM	QS	PM	C	S	Prod	Square	W4	LAMBDA	CI	CR
A	1,00	1,00	0,33	2,00	2,00	2,00	2,67	1,18	0,15	6,00	0,0008	0,0006
HM	1,00	1,00	0,33	2,00	2,00	2,00	2,67	1,18	0,15	6,00		
QS	3,00	3,00	1,00	7,00	7,00	7,00	3087,00	3,82	0,48	6,01		
PM	0,50	0,50	0,14	1,00	1,00	1,00	0,04	0,57	0,07	6,00		
C	0,50	0,50	0,14	1,00	1,00	1,00	0,04	0,57	0,07	6,00		
S	0,50	0,50	0,14	1,00	1,00	1,00	0,04	0,57	0,07	6,00		
								7,89	1,00	6,00		

W1	W2	W3	W4	W0	W		
0,43	0,31	0,13	0,15	0,2500	0,25	A	1
0,22	0,31	0,07	0,15	0,2500	0,19	HM	3
0,12	0,15	0,07	0,48	0,2500	0,21	QS	2
0,11	0,15	0,14	0,07	0,2500	0,12	PM	5
0,07	0,05	0,14	0,07		0,08	C	6
0,05	0,03	0,47	0,07		0,16	S	4
					1,00	QUOTA METHOD	

A harder exercise: a change in v2

	A	HM	QS	PM	C	S	Prod	Square	W1	LAMBDA	CI	CR
A	1,00	2,00	3,00	4,00	6,00	9,00	1296,00	3,30	0,43	6,02	0,0101	0,0082
HM	0,50	1,00	2,00	2,00	3,00	4,00	24,00	1,70	0,22	6,01		
QS	0,33	0,50	1,00	1,00	2,00	2,00	0,67	0,93	0,12	6,05		
PM	0,25	0,50	1,00	1,00	2,00	2,00	0,50	0,89	0,11	6,04		
C	0,17	0,33	0,50	0,50	1,00	2,00	0,03	0,55	0,07	6,1		
S	0,11	0,25	0,50	0,50	0,50	1,00	0,00	0,39	0,05	6,09		
								7,77	1,00	6,05		

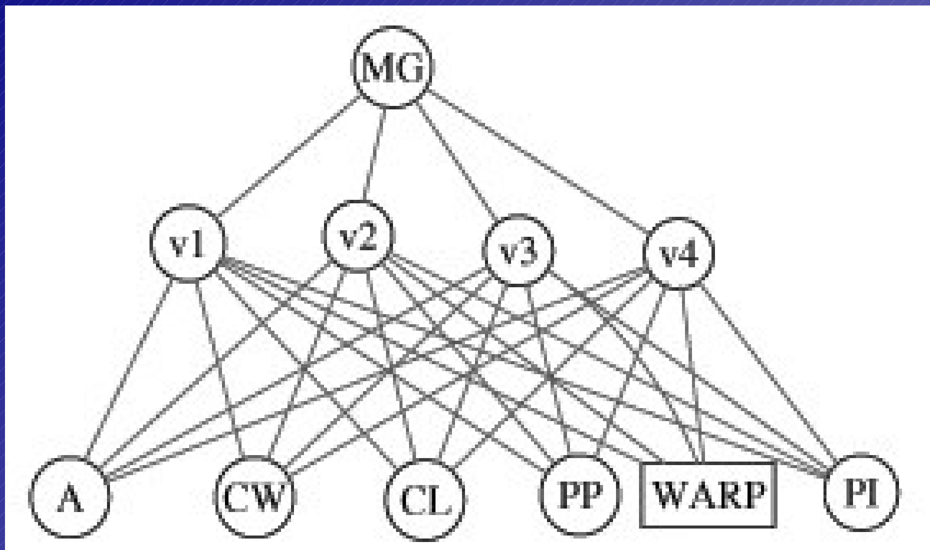
	A	**HM**	**QS**	**PM**	**C**	**S**	**Prod**	**Square**	**W2**	**LAMBDA**	**CI**	**CR**
A	**1,00**	**3,00**	**6,00**	**0,50**	**1,00**	**3,00**	**27,00**	**1,73**	**0,22**	**6,12**	**0,0239**	**0,0192**
HM	**0,33**	**1,00**	**2,00**	**0,20**	**0,33**	**0,50**	**0,02**	**0,53**	**0,07**	**6,03**		
QS	**0,17**	**0,50**	**1,00**	**0,14**	**0,14**	**0,20**	**0,00**	**0,26**	**0,03**	**6,2**		
PM	**2,00**	**5,00**	**7,00**	**1,00**	**2,00**	**4,00**	**560,00**	**2,87**	**0,36**	**6,14**		
C	**1,00**	**3,00**	**7,00**	**0,50**	**1,00**	**2,00**	**21,00**	**1,66**	**0,21**	**6,02**		
S	**0,33**	**2,00**	**5,00**	**0,25**	**0,50**	**1,00**	**0,42**	**0,86**	**0,11**	**6,22**		
								7,92	1,00	6,12		

	A	HM	QS	PM	C	S	Prod	Square	W3	LAMBDA	CI	CR
A	1,00	2,00	2,00	1,00	1,00	0,20	0,80	0,96	0,13	6,05	0,0049	0,0039
HM	0,50	1,00	1,00	0,50	0,50	0,14	0,02	0,51	0,07	6,00		
QS	0,50	1,00	1,00	0,50	0,50	0,14	0,02	0,51	0,07	6,00		
PM	1,00	2,00	2,00	1,00	1,00	0,33	1,33	1,05	0,14	6,01		
C	1,00	2,00	2,00	1,00	1,00	0,33	1,33	1,05	0,14	6,01		
S	5,00	7,00	7,00	3,00	3,00	1,00	2205,00	3,61	0,47	6,06		
								7,69	1,00	6,02		

	A	HM	QS	PM	C	S	Prod	Square	W4	LAMBDA	CI	CR
A	1,00	1,00	0,33	2,00	2,00	2,00	2,67	1,18	0,15	6,00	0,0008	0,0006
HM	1,00	1,00	0,33	2,00	2,00	2,00	2,67	1,18	0,15	6,00		
QS	3,00	3,00	1,00	7,00	7,00	7,00	3087,00	3,82	0,48	6,01		
PM	0,50	0,50	0,14	1,00	1,00	1,00	0,04	0,57	0,07	6,00		
C	0,50	0,50	0,14	1,00	1,00	1,00	0,04	0,57	0,07	6,00		
S	0,50	0,50	0,14	1,00	1,00	1,00	0,04	0,57	0,07	6,00		
								7,89	1,00	6,00		

W1	W2	W3	W4	W0	W		
0,43	0,22	0,13	0,15	0,2500	0,23	A	1
0,22	0,07	0,07	0,15	0,2500	0,13	HM	5
0,12	0,03	0,07	0,48	0,2500	0,18	QS	2
0,11	0,36	0,14	0,07	0,2500	0,17	PM	4
0,07	0,21	0,14	0,07		0,12	C	6
0,05	0,11	0,47	0,07		0,18	S	2
					1,00		LARGEST REMAINDER

Another harder exercise



1. $A > CW > CL > PP > WARP > PI$
2. $CW \sim CL > PP > PI > A \sim WARP$
3. $PP > A > PI > WARP > CW \sim CL$
4. $PP > PI > A \sim WARP > CW > CL$

- Properties of majoritarian methods:
  - Anonimity,
  - Condorcet Winner,
  - Condorcet Loser,
  - Pareto Principle,
  - Weak Axiom of Revealed Preferences
  - Path Independence
- Ranking of the properties $\leftrightarrow$ ranking of the methods

Another harder exercise

	A	CW	CL	PP	WARP	PI	Prod	Square	W1	LAMBDA	CI	CR
A	1,00	3,00	4,00	5,00	7,00	9,00	3780,00	3,95	0,49	6,04	0,01	0,0116
CW	0,33	1,00	1,00	2,00	2,00	3,00	4,00	1,26	0,16	6,11		
CL	0,25	1,00	1,00	1,00	2,00	3,00	1,50	1,07	0,13	6,03		
PP	0,20	0,50	1,00	1,00	2,00	3,00	0,60	0,92	0,11	6,13		
WARP	0,14	0,50	0,50	0,50	1,00	2,00	0,04	0,57	0,07	6,06		
PI	0,11	0,33	0,33	0,33	0,50	1,00	0,00	0,36	0,04	6,07		
								8,13	1,00	6,07		

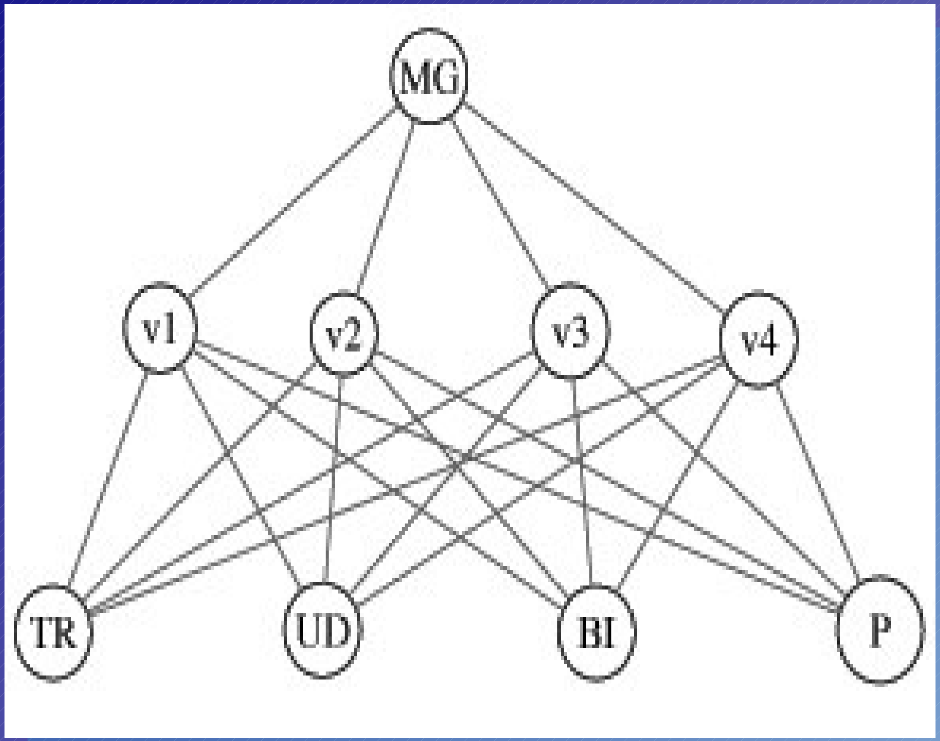
	A	CW	CL	PP	WARP	PI	Prod	Square	W2	LAMBDA	CI	CR
A	1,00	0,14	0,14	0,50	1,00	0,50	0,01	0,41	0,05	6,06	0,01	0,0066
CW	7,00	1,00	1,00	3,00	7,00	5,00	735,00	3,00	0,35	6,01		
CL	7,00	1,00	1,00	3,00	7,00	5,00	735,00	3,00	0,35	6,01		
PP	2,00	0,33	0,33	1,00	3,00	2,00	1,33	1,05	0,12	6,06		
WARP	1,00	0,14	0,14	0,33	1,00	0,50	0,00	0,39	0,05	6,03		
PI	2,00	0,20	0,20	0,50	2,00	1,00	0,08	0,66	0,08	6,08		
								8,52	1,00	6,04		

	A	CW	CL	PP	WARP	PI	Prod	Square	W3	LAMBDA	CI	CR
A	1,00	3,00	3,00	0,50	2,00	2,00	18,00	1,62	0,21	6,13	0,01	0,0107
CW	0,33	1,00	1,00	0,14	0,50	0,33	0,01	0,45	0,06	6,04		
CL	0,33	1,00	1,00	0,14	0,50	0,33	0,01	0,45	0,06	6,04		
PP	2,00	7,00	7,00	1,00	5,00	3,00	1470,00	3,37	0,43	6,02		
WARP	0,50	2,00	2,00	0,20	1,00	0,50	0,20	0,76	0,10	6,06		
PI	0,50	3,00	3,00	0,33	2,00	1,00	3,00	1,20	0,15	6,11		
								7,85	1,00	6,07		

	A	CW	CL	PP	WARP	PI	Prod	Square	W4	LAMBDA	CI	CR
A	1,00	5,00	6,00	0,25	3,00	0,50	11,25	1,50	0,17	6,34	0,06	0,0481
CW	0,20	1,00	2,00	0,17	0,50	0,20	0,01	0,43	0,05	6,19		
CL	0,17	0,50	1,00	0,14	0,33	0,17	0,00	0,30	0,03	6,27		
PP	4,00	6,00	7,00	1,00	4,00	3,00	2016,00	3,55	0,42	6,55		
WARP	0,33	2,00	3,00	0,25	1,00	0,25	0,13	0,71	0,08	6,18		
PI	2,00	5,00	6,00	0,33	4,00	1,00	80,00	2,08	0,24	6,27		
								8,56	1,00	6,3		

W1	W2	W3	W4	W0	W	
0,49	0,05	0,21	0,17	0,2500	0,23	A
0,16	0,35	0,06	0,05	0,2500	0,15	CW
0,13	0,35	0,06	0,03	0,2500	0,14	CL
0,11	0,12	0,43	0,42	0,2500	0,27	PP
0,07	0,05	0,10	0,08		0,07	WARP
0,04	0,08	0,15	0,24		0,13	PI
					1,00	

A ranking exercise



- High level properties:
  - Transitivity
  - Universal Domain
  - Binary Independence
  - Pareto principle

1. $TR > UD \sim BI \sim P$
2. $P > TR > UD > BI$
3. $TR > P \sim UD > BI$
4. $UD > P > TR > BI$

A ranking exercise

	TR	UD	BI	P	Prod	Square	W1	LAMBDA	CI	CR
TR	1,00	3,00	3,00	3,00	27,00	2,28	0,50	4	0,0000	0,0000
UD	0,33	1,00	1,00	1,00	0,33	0,76	0,17	4		
BI	0,33	1,00	1,00	1,00	0,33	0,76	0,17	4		
P	0,33	1,00	1,00	1,00	0,33	0,76	0,17	4		
						4,56	1,00	4		

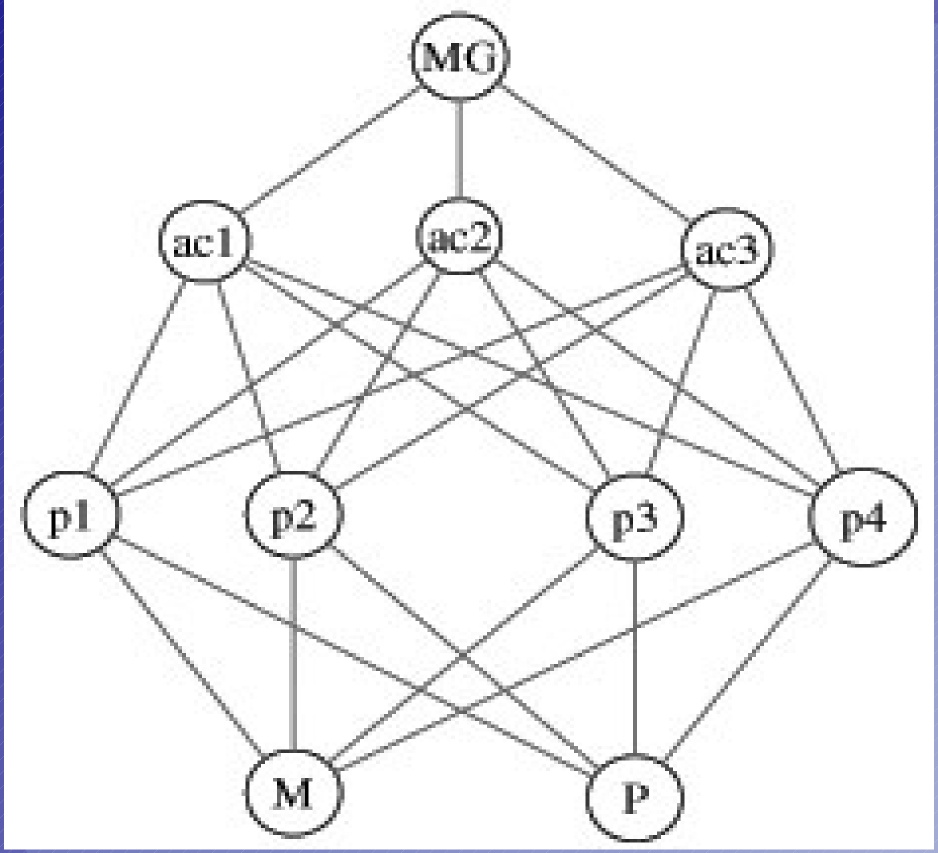
	TR	UD	BI	P	Prod	Square	W2	LAMBDA	CI	CR
TR	1,00	1,00	2,00	0,33	0,67	0,90	0,17	4,02	0,0086	0,0096
UD	1,00	1,00	2,00	0,20	0,40	0,80	0,15	4,03		
BI	0,50	0,50	1,00	0,14	0,04	0,43	0,08	4,01		
P	3,00	5,00	7,00	1,00	105,00	3,20	0,60	4,04		
						5,33	1,00	4,03		

	TR	UD	BI	P	Prod	Square	W3	LAMBDA	CI	CR
TR	1,00	3,00	5,00	3,00	45,00	2,59	0,53	4,01	0,0014	0,0015
UD	0,33	1,00	2,00	1,00	0,67	0,90	0,19	4,00		
BI	0,20	0,50	1,00	0,50	0,05	0,47	0,10	4,01		
P	0,33	1,00	2,00	1,00	0,67	0,90	0,19	4,00		
						4,87	1,00	4		

	TR	UD	BI	P	Prod	Square	W4	LAMBDA	CI	CR
TR	1,00	0,17	3,00	0,25	0,13	0,59	0,10	4,08	0,0220	0,0244
UD	6,00	1,00	9,00	2,00	108,00	3,22	0,54	4,06		
BI	0,33	0,11	1,00	0,14	0,01	0,27	0,04	4,09		
P	4,00	0,50	7,00	1,00	14,00	1,93	0,32	4,04		
						6,02	1,00	4,07		

W1	W2	W3	W4	W0	W		
0,50	0,17	0,53	0,10	0,25	0,32	TR	
0,17	0,15	0,19	0,54	0,25	0,26	UD	
0,17	0,08	0,10	0,04	0,25	0,10	BI	**BORDA COUNT**
0,17	0,60	0,19	0,32	0,25	0,32	P	

Another ranking exercise



- Proportional or Majoritarian?
- Four properties:
  - Electoral Participation
  - Number of Political Parties
  - Electoral Volatility
  - Government stability

Another ranking exercise

EP	M	P	NPP	M	P	EV	M	P	GS	M	P
M	1,00	0,33	M	1,00	0,50	M	1,00	0,20	M	1,00	4,00
P	3,00	1,00	P	2,00	1,00	P	5,00	1,0	P	0,25	1,00

ac1	EP	NPP	EV	GS	Prod	Square	W1	LAMBDA	CI	CR			
EP	1,00	5,00	7,00	7,00	245,00	3,96	0,67	4,02	0,0053	0,0059			
NPP	0,20	1,00	2,00	2,00	0,80	0,95	0,16	4,02					
EV	0,14	0,50	1,00	1,00	0,07	0,52	0,09	4,01					
GS	0,14	0,50	1,00	1,00	0,07	0,52	0,09	4,01					
						5,94	1,00	4,02					
ac2	EP	NPP	EV	GS	Prod	Square	W2	LAMBDA	CI	CR			
EP	1,00	7,00	3,00	7,00	147,00	3,48	0,62	4	0,0010	0,0011			
NPP	0,14	1,00	0,50	1,00	0,07	0,52	0,09	4					
EV	0,33	2,00	1,00	2,00	1,33	1,07	0,19	4					
GS	0,14	1,00	0,50	1,00	0,07	0,52	0,09	4					
						5,59	1,00	4					
ac3	EP	NPP	EV	GS	Prod	Square	W3	LAMBDA	CI	CR			
EP	1,00	0,50	2,00	0,20	0,20	0,67	0,12	4,01	0,0027	0,0029			
NPP	2,00	1,00	3,00	0,33	2,00	1,19	0,21	4,01					
EV	0,50	0,33	1,00	0,11	0,02	0,37	0,07	4,01					
GS	5,00	3,00	9,00	1,00	135,00	3,41	0,60	4,00					
						5,64	1,00	4,01					
W4	W5	W6	W7	W1	W2	W3					W0	W	
0,25	0,33	0,1667	0,8000	0,6665	0,6228	0,1187		0,3039	0,2925	0,5948	0,3333	0,3970	M
0,75	0,67	0,8333	0,2000	0,1593	0,0925	0,2110		0,6961	0,7075	0,4052	0,3333	0,6029	P
				0,0871	0,1922	0,0655					0,3333		
				0,0871	0,0925	0,6049							

Hierarchy: a real solution or a blind alley?

- Open questions(some of the...):
  - is hierarchy enough to escape the "conviction" of theoretical results?
  - is it scalable? (clustering) how do we deal inconsistencies?
  - where does the solution reside? in the hierarchy itself or in the synthesis phase?
- How can we be sure that there is no Arrow-like impossibility theorem "lurking out there"?

Conclusions

- This work represents an attempt to apply AHP to electoral systems
- The aim is to use it for the ranking of social choices and alternatives
- Theoretical and practical/empirical issues require deeper examinations
- The method is suitable for iterative applications and for forward and backward planning
- All this (and much more) will be part of my PhD dissertation

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That's all, folks....

so many things to do and so a short time...

Lorenzo Cioni
Department of Computer Science
University of Pisa
e-mail: lcioni@di.unipi.it