Game Theory and Environmental Issues

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Contents

List of Figures

List of Tables

1 Game Theory and Environmental Issues

In this paper we briefly examine some of the basic concepts about **co-operative game theory** (CGT in what follows). Such games, unlike what happens in **non co-operative game theory** (or $NCGT$) where games are based on the strategic interactions among rational players, are based on the concept of **coalition** as a group of players that signed some binding agreement.
Our aim is to present the main features of such games, through a very short primer, and describe the concepts of coalition and environmental games.
In an ad hoc section we also briefly present how $NCGT$ can be used to describe potential solutions to environmental problems both in single shot and in repeated games.

1.1 The general framework

With the general term of **externality** we mean the whole of the effects of any activity on a third party outside the usual market's transactions. When such effects are positive (as in case of technological development) we speak of **positive externalities** whereas, if they are negative (as in case of pollution), we speak of **negative externalities**.
Whenever a player (for instance a country), with its economical activities, causes damages to itself and to some other (even all the other) countries we speak of **international environmental externalities** that can be even reciprocal in nature since if player's i activities damage all the others (and itself), i can also be damaged by player j's activities. In this case damages to the environment can be seen as an **international public bad** whereas any reduction of such damages (through positive actions) can be seen as an **international public good**.
In absence of any international authority that can impose correct behaviours to the players or the payment of penalties and the execution of compensatory actions to the damaging players one possible solution is the stipulation of agreements among the players (the so called multilateral or international agreements).
Agreements can involve any number of players and can be analysed both within the CGT and within $NCGT$. The main difference between the two paradigms is that, in the former case, we speak of **stable** and **self enforcing** agreements among players whereas in the latter we have players pursue their own interests and occasionally co-operate, if this allows each of them to maximise his/her own expected utility.

1.2 Public goods

In economic theory we define the concept of **good** as any thing that can be subject to economic exchange since it can satisfy a need, it is available in limited quantity, inferior to the demand, and is available or accessible for being used.
For our purposes goods can be of two types:

1. **private goods**;
2. **public goods**.

A **private good** must satisfy the **principle of competition**: the quantity of the good consumed by an agent cannot be consumed by another agent. In case of a **public good** this never happens so that the use of such goods by an agent does not prevent the concurrent use by other agents since there is no destruction caused by use.
Goods are also characterised by the following properties:

1. **rivalry/non-rivalry**,
2. **excludability/non-excludability**.

We say that a good is non-rival if its use by an agent does not diminish the availability for other agents otherwise it is called rival. On the other hand, we say that a good is non-excludible if its use cannot be prevented or limited otherwise it is called excludible.
The two properties are independent from each other so that we can define the following four categories of goods:

1. rival and excludible;
2. non-rival and excludible;
3. rival and non-excludible;
4. non-rival and non-excludible.

A **public good** is both **non-rival** and **non-excludible**.

1.3 Digressions around agreements and problems

In this section we give some definitions of some terms we use in what follows. Our aim is to clarify the concepts of **International Environmental Agreement** (or IEA in short) and of of **International Environmental**

Problem, or IEP in short.
An **agreement**([Cob88]) represents a decision that two or more people have reached together and can be see also as the act of reaching a decision that is acceptable to everyone involved. Implicit in this definition is the concept of unanimity to mean that a group of people agree about something ([Cob88]).
A **problem** ([Cob88]) represents a situation that is unsatisfactory and causes difficulties to a group of independent involved entities (people, groups, nations and so on). Moreover a problem involves the use of some rational skill to be solved (if a solution exists). In the context of Game Theory the entities involved (i. e. the players) are supposed to be rational and act rationally though this hardly happens in practise where countries (and their leaders) very often undertake demagogic, myopic and shortsighted decisions. In what follows, however, we rely on the classical assumption of rational players.
International ([Cob88]) means something that involves different countries i. e. a set of political independent entities. The **international dimension** is very important, in the context of IEA, because it prevents the possibility of the existence of an enforcing authority that can compel countries to respect the agreements they sign and give effective penalties to agreements violating countries. Since there is no enforcing supranational authority to guarantee for the respect of the environmental agreements the only solution is to try to design self-enforcing agreements or agreements that are too costly for every signer country to violate.
The last word which we have to define is **environmental**. According to ([Cob88]) environmental means concerned with or relating to the natural world and so includes public goods but also many private goods that, however, are excluded from the area of application of IEA since they are governed by the rules of private property laws.
As every classification also the one we adopted here presents a number of drawbacks. The main of such drawbacks are due to the use of a sort of contraposition between the terms **agreement** and **problem** and the dual meaning of the term **international**.
It should be evident that an agreement presuppose the existence of a problem that the signers wishes either to solve or at least to submit to a set of shared rules through the signing of that agreement.
On the other hand, the existence of a problem involving at least two parties in no way implies the signing of an agreement among either a subset or the full set of those parties, since it can happen that:

1. the problem is claimed to be either non-existent or of lesser importance that some more urgent problem (such as economical development or low unemployment rate);

2. the problem is left unsolved and all parties behave as usual;

3. the problem is solved by one of more of the parties without any co-operation from the remaining parties and without any co-ordination among the "willing parties" (free-riding);

4. a solution is discussed but every party wishes it is implemented by the others since it claims to have no responsibility in the problem;

and so on.
As to the word "international" we give it two slightly different meanings and hope that the context is enough to solve any ambiguity.
According to the first meaning, used in the case of IEAs, with "international agreement" we want to denote that it involves a vast group of players/countries, potentially including all the countries of the world, or, in any way, o big subset of all the countries of the world.
According to the second meaning, used in the case of IEPs, with "international problem" we want to denote that it involves at least two players/countries that suffer a common problem about the environment or any other public good.
Anyway we refer to the next section for a set of hopefully clarifier examples.

1.4 Examples of $I[E]A$s and IEPs

We now present two examples of **International Agreements** IA and then skip to some examples of both $I[E]A$s and IEPs.
The first example of IA ([You94]) is represented by the agreement for the allocation of the spectrum of broadcasting frequencies for radio and satellite communication governed by an article of the **International Telecommunication Convention** of 1965. With this agreement it is recognised that both radio frequencies and the geostationary orbit are limited natural resources that must be used efficiently and economically so that all countries may have access to both, according to their needs and to the technical facilities at their disposal. In this case the agreement tries to regulate the access to rival but non excludible public goods.
The second example of IA ([You94]) concerns the commercial mining of deep ocean bed that is rich of mineral resources. The allocation of the mining rights of the deep ocean bed (or seabed) was one of the main topics of the **Law of the Sea Conference** convened by the United Nations in 1973. The seabed is a rival and excludible public good but was declared, on that occasion, a common heritage of mankind (a "global common") on which all countries have a stake. According to this principle seabed mining should

have been undertaken on behalf of the international community.
In that occasion world countries were divided in two groups:

1. developed countries (such as United States, Germany and Japan) with the know-how and capital to mine the seabed;

2. the less developed countries without such capabilities.

The conference participants decided to form two new agencies for the supervision of the exploitation of deep sea bed:

1. an **International Seabed Authority** with the task of licensing all the mining activities;

2. an international mining entity called "Enterprise" to mine the seabed on behalf of less developed countries.

To guarantee a fair exploitation of the deep sea bed resources a **divider-chooser procedure** was implemented in the **Law of the Sea Treaty** so to avoid that the commercial interests in the developed countries would drive them to seize the best mining sites to the detriment of the less developed countries. Every time a mining company applies to the International Seabed Authority to get the permission to mine in a given area of the seabed, such company (acting as the divider) must propose two parallel sites from which the Enterprise (acting as the chooser) chooses one. In this way the Enterprise is guaranteed to get, in its estimation, at least the half of all the best seabed mining sites.
At this point one could object that both the aforesaid examples involve, in some way or the other, the environment. This is undoubtedly true but it must be noted that, in general, we speak of IEAs essentially in case of "bads" more than of "goods".
From this point of view we can list some of the IEAs that have been signed during the years ([FR01]):

1. the Oslo Protocol on sulfur reduction in Europe in 1994;

2. the Montreal Protocol on the depletion of the ozone layer in 1987;

3. the Kyoto Protocol on the reduction of greenhouse gases in 1997.

In addition we can find applications of this kind of agreements in the following contexts ([CEF05]):

1. global warming;

2. acid rains;

3. high sea fisheries;

4. water management.

The main difference among these IEA and the aforesaid IE is that the former aim at reducing a "global bad" without the intervention of a supranational authority whereas the latter aim at the management of a "global good" even with the intervention of a supranational authority.
This difference reflects on the nature of the agreements that must be self-enforcing so to incite all the signers to comply with every clause of an agreement they ratified.
As to the IEPs we could mention every case where two or more contiguous countries must face an environmental problem. We are going to discuss some of these cases in section 3.4 where we adopt a $NCGT$ approach and try to describe the interactions among the countries mainly using games in strategic form.

1.5 A short primer on Co-operative Game Theory

In this section we try to give the basic ideas and concepts of Co-operative Game Theory (CGT) mainly using [FoSS99]. We list some basic results and properties without any formal proof and privileging an intuitive and informal approach.
In case of Co-operative Games (in short CGs) players are allowed to join in groups (called **coalitions**) and to act jointly. In this case joint actions are preferred over players acting alone.
The main issues of analysis in CGT are:

1. the formation of coalitions (and their characterisation);

2. the distribution of the fruits (both credits and burdens) of the co-operation;

under reasonable players' rationality assumptions.
As to the second issue a crucial point is the availability of a linearly transferable commodity, such as money, that allows the compensation of the efforts that must be done to pursue a common goal. If such a commodity exists we speak of **Transferable Utility** (TU) games (or games with side payments). If such a commodity does not exist we speak of **Non Transferable Utility** (NTU) games. In this section we are going to examine only TU games.

1.5.1 Representation of games in CGT

Within the *CGT* the most used representation of games uses the concept of **characteristic function**:

$$v \,:\, 2^N \longrightarrow \mathbb{R} \tag{1}$$

where $N = \{1, \ldots, n\}$ is a finite set of players and 2^N is the power set of N and includes all the possible coalitions of the players (including the empty set). The only required property of function v is the following:

$$v(\emptyset) = 0 \tag{2}$$

If we use this form of representation we denote a game Γ as:

$$\Gamma = \{N, v\} \tag{3}$$

We now give an interpretation of function v. If $S \subset N$ is any coalition then $v(S)$ is the maximum utility that the members of S can attain without the co-operation of the members of $N \setminus S$. According to this interpretation $v(N)$ is the maximum utility that can be attained by all the players. N is called the **grand coalition**.
Given a game $\Gamma = \{N, v\}$ we list some of its properties. In what follows we have $S, T \subset N$.

1. A game $\Gamma = \{N, v\}$ is termed **superadditive** if for all disjoint coalitions S and T we have:

   $$v(S \cup T) \geq v(S) + v(T) \tag{4}$$

   We can extend such a definition to an arbitrary number of disjoint coalitions as follows:

   $$v(\cup_k S_k) \geq \sum_k v(s_k) \tag{5}$$

   with:

   (a) $S_k \subset N$ for all k,

   (b) $S_k \cap S_l = \emptyset$ for all $k \neq l$.

2. A game $\Gamma = \{N, v\}$ is termed **weakly superadditive** if, for any coalition $S \subset N$, we have:

   $$v(N) \geq v(S) + \sum_{i \in N \setminus S} v(\{i\}) \tag{6}$$

   In this case, if we have a coalition S, we have that the grand coalition gets at least what is obtained by that coalition and all the remaining players acting as monads or singleton sets.

3. A game $\Gamma = \{N, v\}$ is termed **monotone** if the condition $S \supset T$ implies $v(S) \geq v(T)$ so that larger coalitions cannot achieve less.

4. Given a player $i \in N$ we define the **marginal contribution of player** i as a function:
$$d_i \,:\, 2^N \longrightarrow \mathbb{R} \tag{7}$$
such that:
$$d_i = \begin{cases} v(S \cup \{i\}) - v(S)\, if\ i \notin S \\ \\ v(S) - v(S \setminus \{i\})\, if\ i \in S \end{cases} \tag{8}$$
As a consequence we define a game $\Gamma = \{N, v\}$ to be **convex** if, for each player $i \in N$, we have:
$$d_i(S) \leq d_i(T) \tag{9}$$
for any coalitions $S \subset T$. With the concept of convexity we want to represent the fact that the value of the coalitions increase more rapidly as coalitions become bigger.

5. A game $\Gamma = \{N, v\}$ is termed to be with **constant sum** if for any coalition $S \subset N$ we have:
$$v(S) + v(N \setminus S) = v(N) \tag{10}$$

6. A game $\Gamma = \{N, v\}$ is termed **rational** if the grand coalition N is such that:
$$v(N) \geq \sum_{i \in N} v(\{i\}) \tag{11}$$
For **inessential rational games** we have:
$$v(N) = \sum_{i \in N} v(\{i\}) \tag{12}$$
whereas for **essential rational games** we have:
$$v(N) > \sum_{i \in N} v(\{i\}) \tag{13}$$
If the game is inessential the players have no real incentive to form a grand coalition (since they get no better utility than that they get by acting alone) whereas, in case of essential games, they have an incentive to join in the grand coalition since at least one of them is better off by joining and the others are no worse off.

At this point, we introduce a definition that allows us to verify that two games $\Gamma = \{N, v\}$ and $\Gamma' = \{N, v'\}$ are strategically equivalent (or SE). We say that $\Gamma = \{N, v\}$ and $\Gamma' = \{N, v'\}$ are SE if we have:

1. $\alpha \in \mathbb{R}_{++}$,
2. $\beta_1, \ldots, \beta_n \in \mathbb{R}$,

such that between the characteristic functions of the two games we have an affine transform:

$$v(S) = \alpha v'(S) + \sum_{i \in S} \beta_i \tag{14}$$

Relation (14) is an equivalence relation since it can be proved to be:

1. reflexive;
2. symmetric;
3. transitive.

At this point we introduce the concept of $(0,1)-$**normalised game** that we use in a theorem we state without proof and that defines a property of the strategic equivalence of games.
A game $\Gamma = \{N, v\}$ is a $(0,1)-$**normalised game** if:

$$\begin{cases} v(\{i\}) = 0 \; fr \; all \; i \notin S \\ \\ v(N) = 1 \end{cases} \tag{15}$$

Theorem 1.1 *Given a game $\Gamma = \{N, v\}$, among the games that are SE to it we have one and only one $(0,1)-$**normalised game**.*

We introduce now a class of games $\Gamma = \{N, v\}$ that are:

1. superadditive,
2. $(0,1)-$normalised,

and call them **simple** if, for any coalition S, we have:

1. $v(S) = 0$ and S is a losing coalition;
2. $v(S) = 1$ and S is a winning coalition.

A game $\Gamma = \{N, v\}$ is said to be **symmetric** if $v(S)$ depends only on $\mid S \mid$.
At this point we need at least one method to derive function v for a game $\Gamma = \{N, v\}$. In this section we mention only a method that represents a link between $NCGT$ (of games in normal or strategic form) and CGT (of games in characteristic function form).
If we have a game G in normal form:

$$G = \{S_1, \dots, S_n; f_1, \dots, f_n\} \tag{16}$$

with strategy sets S_i and continuous payoffs functions f_i for each of the n players, we can define a coalition:

$$S = \{i_1, \dots, i_k\} \subset N \tag{17}$$

and its complement:

$$N \setminus S \tag{18}$$

and define $v(S)$ as the **security level** of coalition S in a two players zero sum game where the players are the coalitions S and $N \setminus S$. In other words, and more formally, $v(S)$ is the max over x_S (the strategies of the members of S) of the min over $y_{N \setminus S}$ (the strategies of the members of $N \setminus S$) of the sum of the payoffs of the members of S.
Moreover, we have:

1. $v(\emptyset) = 0$,
2. $v(N) = \sum_{s_1 \in S_1, \dots, s_n \in S_n} \sum_{i=1}^{n} f_i(s_1, \dots, s_n)$

In this way we have defined $v(S)$ as a function of the only players of S so to define $\Gamma = \{N, v\}$ as a co-operative game in characteristic function form.
From this process of construction of v we have:

1. $\Gamma = \{N, v\}$ is superadditive;
2. $\Gamma = \{N, v\}$ is with transferable utilities since we sum the payoffs of the members of the coalition S.

If we start from games G in normal form we always get $TU-$games in characteristic function form but the converse is not true, in general. If $\Gamma = \{N, v\}$ is superadditive can be thought of as a game derived from a normal-form non co-operative game through a **max min** construction (cf. [FoSS99] for further details).

1.5.2 Allocations, imputations and dominance

At this point, after having defined the characteristic function v for a game, we have to solve:

1. the problem of how the profits (or the costs) from a partial ($S \subset N$) or total (N) co-operation are achieved by the [grand] coalition;

2. the problem of how the profits (or the costs) from a partial ($S \subset N$) or total (N) co-operation of the players are divided among the players themselves.

The first problem will not be discussed here so that we give $v(S)$ for granted, for any coalition S.
The second problem can be dealt with according to this route:

1. we introduce some sets whose definition is necessary to characterise the allocations of $v(S)$ (also termed **the worth** of the coalition S) among the members of S;

2. we give some further definitions;

3. we list the main solution tools of the allocation problem.

If $N = \{1, \ldots, n\}$ is the whole set of the players of the game Γ and if $S \subset N$ is a coalition, we can define for any $x \in \mathbb{R}^n$:

$$x(S) = \sum_{i \in S} x_i \tag{19}$$

so that:

$$x(\emptyset) = 0 \tag{20}$$

For any coalition S, x(S) represents the sum of the individual payoffs of the members of the coalition. For a game $\Gamma = \{N, v\}$ we introduce the following sets.

1. The set of all **feasible payoffs**:

$$I^{**}(N, v) = \{x \in \mathbb{R}^n \ \mid \ x(N) \leq v(N)\} \tag{21}$$

as the set of the payoff vectors x whose sum is at the most equal to the worth of the coalition S.

2. The set of all **efficient payoffs**:

$$I^*(N,v) = \{x \in \mathbb{R}^n \mid x(N) = v(N)\} \tag{22}$$

as the set of the payoff vectors x whose sum is equal to the worth of the coalition S. The payoff vector of I^* are also called *preimputations*.

From the aforesaid definitions we have the set:

$$I^{**} \setminus I^* \neq \emptyset \tag{23}$$

whose elements can be improved by giving more to some players without hurting the remainders, in case the grand coalition forms.
With regard to a payoff vector x we introduce the concept of **individual rationality** and say that x is individually rational if, for any $i \in N$, we have:

$$x_i \geq v(\{i\}) \tag{24}$$

so that any player can get from joining the grand coalition at least what would get by acting as a singleton set. If we join this definition with that of preimputations we get a new set:

$$I(N,v) = \{x \in \mathbb{R}^n \mid x(N) = v(N)\, and\, x_i \geq v(\{i\})\, for\ all\, i \in N\} \tag{25}$$

We call the members of $I(N,v)$ **imputations**.
We note that:

1. individual rationality is a basic feature of any rational behaviour;

2. most solution concepts use only subsets of I as the possible payoffs;

3. the set I is never empty in rational games.

Now we introduce the concept of **solution** on a set of games $\mathscr{G}$.

Definition 1.1 (Concept of solution) *A solution on $\mathscr{G}$ is a function σ that associates to each game (N,v) of $\mathscr{G}$ a (possibly empty) subset $\sigma(N,v)$ of $I^*(N,v)$.*

Both on $I^{**}(N,v)$, on $I^*(N,v)$ and on $I(N,v)$ we can define a relation that allows a comparison of the payoffs vectors x. We define it on I as follows.

Definition 1.2 (Concept of dominance) *Given two imputations $x \in I$ and $y \in I$ and a coalition S we say that x dominates y through S and write:*

$$x \succ^S y \tag{26}$$

to denote that:

$$x_i > y_i \tag{27}$$

for all $i \in S$ and:

$$x(S) \leq v(S) \tag{28}$$

According to relation (27) every player of S would prefer imputation x to imputation y whereas relation (28) says that coalition S can obtain at least $x(S)$ that is a feasible payoff.
In general we say that x dominates y and write:

$$x \succ y \tag{29}$$

is there is at least one coalition S such that:

$$x \succ^S y \tag{30}$$

1.5.3 The solutions concepts

Given a game $\Gamma = \{N, v\}$ with TU we have the following classical solution concepts:

1. the **core**;
2. the **stable set**;
3. the **kernel**;
4. the **nucleolus**;
5. the **Shapley value**.

All such methods aim at the definition of an imputation that satisfies certain properties.
A close examination of such methods if far beyond the scope of these notes so that we are going to give here only some short hints, mainly about the first two and the last methods.

1.5.4 The core in a nutshell

The **core** ([FoSS99]) of a game $\Gamma = \{N, v\}$ is a set of imputations $x \in \mathbb{R}^n$ (with $n = \mid N \mid$) that give to each coalition S as much as the coalition could obtain without the support of the players in $N \setminus S$.

Formally the core is composed of the vectors x that are solutions of the following linear inequalities:

$$\begin{cases} x(S) \geq v(S) \, \forall S \subset N \\ \\ x(N) = v(N) \end{cases} \tag{31}$$

with:

$$x(S) = \sum_{i \in S} x_i \tag{32}$$

The core of the game $\Gamma = \{N, v\}$ is denoted as $C(\Gamma)$ and can be either empty (so that no imputation exists) or too wide to be of practical utility. Anyway we can say that the imputations of the core are both individually and coalitionally rational.
As it is in the spirit of these notes, we now list the main features of the core, for further details cf. [FoSS99].

1. For weakly superadditive games the core coincides with the set of undominated imputations.

2. If an imputation x belongs to $C(\Gamma)$ then it is **stable** since no coalition could have motivation and power to change the outcome of the game.

3. The core, by definition, is a convex polyhedron and, as we have already noted, can be empty. This event occurs for essential constant sum games whose core is empty. On the other hand we can define conditions that assure that $C(\Gamma) \neq \emptyset$. If we define for any coalition $S \subset N$ the following indicator function:

$$w_S \,:\, N \longrightarrow \{0, 1\} \tag{33}$$

as:

$$w_S(i) = \begin{cases} 1 \, if \; i \in S \\ \\ 0 \, if \; i \notin S \end{cases} \tag{34}$$

we can define a coalition structure:

$$\mathscr{C} = \{S_1, \ldots, S_m\} \tag{35}$$

of distinct and non empty coalitions as a **balanced set** if we have $\gamma_1, \ldots, \gamma_m \in \mathbb{R}_+$ (called weights that are said to balance $\mathscr{C}$) such that, for all $i \in N$ we have:

$$\sum_{i=1}^{m} \gamma_i w_{S_j}(i) = 1 \tag{36}$$

or:

$$\sum_{j:i \in S_j} \gamma_i = 1 \tag{37}$$

If we have $\gamma_i = 1$ for all $i \in N$ then $\mathscr{C}$ is simply a partition of N so that a balanced collection of coalitions can be seen as a generalised partition. We use these concepts in our context and define a game $\Gamma = \{N, v\}$ as a **balanced game** if, for every balanced collection $\mathscr{C}$ and a set of weights $\gamma_i = 1$ for all $i \in N$, we have:

$$\sum_{j=1}^{m} \gamma_j v(S_j) \leq v(N) \tag{38}$$

We use the concept of balanced game to state that:

(a) if a game $\Gamma = \{N, v\}$ is such that $C(\Gamma) \neq \emptyset$ then Γ is balanced;

(b) if a game $\Gamma = \{N, v\}$ is balanced then $C(\Gamma) \neq \emptyset$;

(c) a game $\Gamma = \{N, v\}$ is balanced iff $C(\Gamma) \neq \emptyset$.

Another interesting concept is that of **veto player** that applies to simple games, with winning and losing coalitions. Given a simple game $\Gamma = \{N, v\}$ and given a player $i \in N$ we define it as a **veto player** if:

$$v(N \setminus \{i\}) = 0 \tag{39}$$

In a simple game there can be more than one veto player so that we can define the following set:

$$J^v = \{i \in N \mid v(N \setminus \{i\}) = 0\} \tag{40}$$

or the set of the veto players. The main result deriving from the definition of this concept is the following: for a simple game $\Gamma = \{N, v\}$ we have $C(\Gamma) \neq \emptyset$ iff $J^v \neq \emptyset$. For the moment, for a general game, the only way we have to verify that $C(\Gamma) \neq \emptyset$ is to check for balancedness. Since this check can be very difficult in some cases, we need some more easy to use tool such as convexity.
We can, indeed, state that if a game game $\Gamma = \{N, v\}$ is convex then $C(\Gamma) \neq \emptyset$. The proof can be found, as usual for the topics of the present section, in [FoSS99].

1.5.5 The stable set in a nutshell

1.5.6 The kernel in a nutshell

1.5.7 The nucleolus in a nutshell

1.5.8 The Shapley value in a nutshell

1.6 Some notes on coalitions

A co-operative game $\Gamma = \{N, v\}$ is characterised by a set of players N of n elements indexed as $i = 1, \ldots, n$. Any subset $S \subset N$ defines a group of players that are supposed to act co-operatively or, more formally, defines a coalition. The set N, from this point of view, defines the so called **grand coalition** that includes all the players
The set N can be thought as partitioned in many ways.
The simplest way is the following: some of the n players join in a coalition S whereas all the others are in the complementary coalition $N \setminus S$.
In this case we have a partition of N. If the game is superadditive we have:

$$v(N) \geq v(S) + v(N \setminus S) \tag{41}$$

Usually, however, we cannot define $N \setminus S$ as a coalition since, if we define the members of S as signatories of an IEA, the members of $N \setminus S$ are called non signatories and behave as singletons so that, more correctly, we should write:

1. $S \subset N$;
2. $\{i\}$ for all $i \in N \setminus S$

In this way we have:

$$N = S \cup (\cup_{i \in N \setminus S} \{i\}) \tag{42}$$

This is a situation where we have:

1. a single coalition S;
2. a group of selfish players.

A coalition may, therefore, widen by "attracting" singletons whereas can shrink if any country leaves the coalition and acts again as a singleton. These features are captured by the concepts of:

1. internal stability;
2. external stability.

A coalition is **internally stable** if none of its members has incentives to leave it whereas is **externally stable** if none of its non members has incentives to join the coalition.
If we, however, consider a singleton as a coalition, we have:

1. it is externally stable, if we bind it to remain a singleton;
2. it is internally stable, if we bind it to remain non empty.

In general ([CMO03] and [FR01]) over the set N we can define a **coalition structure** $c = \{c_1, \ldots, c_M\}$ such that it is a partition of the player set N or:

1. $\cup_{i=1}^{M} c_i = N$,
2. for every $i, j = 1, \ldots, M$, $i \neq j$ we have $c_i \cap c_j = \emptyset$.

The coalition structure so defined contains M coalitions c_i each of which contains a certain number of countries $\mid c_i \mid$. We have:

1. $\sum_{i=1}^{M} \mid c_i \mid= n$;
2. $1 \leq M \leq n$;
3. the biggest coalition is the grand coalition with $\mid c_1 \mid= n$ so that $M = 1$ and the set of the singletons is empty;
4. if $M = n$ we are in a situation of no co-operation so that all players act as singletons and there is no non trivial coalition (with the term **trivial coalition** we mean any subset $S \subset N$ such that $\mid S \mid= 1$).

With C we denote the set of all coalition structures over the set N[1].
Given any coalition structure c we can assign it an equilibrium evaluation:

$$\pi(c) = \{\pi_1(c_1, c), \ldots, \pi_M(c_M, c)\} \tag{43}$$

(with $\pi_i(c_i, c)$ referred to coalition $c_i \in c$) with:

$$\pi(c) \in \Pi(C) \tag{44}$$

as a set of payoffs resulting form a maximisation of the players according to a given rule on the ground of:

1. a coalition structure;

[1]The set of players can be also denoted as I with cardinality n. In what follows we try to summarise the content of a bunch of papers that do not present a notational homogeneity. We apologise for any error and inconsistency.

2. a rule of gains sharing from co-operation among the members of a coalition.

The first assumption we can make on coalitions is that **all the players are ex-ante identical**. This means that ([CMO03]) each player has the same strategy space in the stage of the game where he has to make a choice. In accordance with this hypothesis ([FR01]) we can identify the members of a coalition structure with their sizes. In this way we have:

$$c = (c_1, \ldots, c_M) \tag{45}$$

where:

1. c_i now represents the number of members of a coalition within coalition structure c;

2. coalitions are ordered, according to their sizes in descending order:

$$c_1 \geq c_2 \geq \cdots \geq c_M \tag{46}$$

   A coalition structure c (with M members) is not static so that it can be modified in a new structure c' (with M' members) with the following operations ([FR01]).

   (a) **Coarsening** if we switch from c to c' by merging two coalitions of c in one coalition of c'.

   (b) **Concentration** if we switch from c to c' if one member moves from one coalition of c to another coalition of equal or larger size.

   Both operations involve a sequence of steps so the switching from c to c' occurs through a sequence of coalition structures. This implies that $M' \leq M$. We note that given c we can apply to it both operations and obtain a new structure c' but that, given a pair c and c' they are not always comparable either under coarsening or under concentration.

2 Environmental games

In this section we try to outline the main characteristics of IEAs ([FR01]). In all models of IEAs the fundamental assumption is that such agreements must be self-enforcing since there is no supranational authority/agency that can establish binding agreements and punish the signers who violate them. The main problem of such agreements is free-riding, mainly in case of international pollution control, though such a problem may be detected also in other kind of treaties (for instance in case of treaties that ban some kind of arms such as anti-person mines). We can define two kinds of free-riding:

1. outer free-riding as the incentive of a country to remain out of an agreement so to benefit from the efforts made by the signatories;

2. inner free-riding as the incentive of a country who signed an agreement to violate its spirit since a violation both affects only limitedly the whole result and is generally hard to detect owing to problems in monitoring and, especially in case of environmental control, to the difficulties of ascribe individual responsibilities.

The central issue of this section is the description of a typical environmental game within the general framework of $ReducedSetGames$ (or RSG). The concurrent framework called **Dynamic Game Models** (or DGM in short) will not be examined here. Within DGM we use models with infinitely repeated games in which countries agree on some contract at the first stage and then enforce it in subsequent stages using credible threats.
Within RSG we can find models based on two approaches, namely:

1. models that use the concepts of **internal stability** and **external stability**,

2. models that use the concept of **core**,

to determine an equilibrium coalition structure.
In both cases we have a two stage game (with a possible preliminary stage [CMO03]) where:

1. at the optional preliminary stage, countries may define unanimously a set of constraints on either the agreement (such as a minimum participation constraints and the size of such a participation) or on the following stages;

2. at the first stage the different countries decide on coalition formation;

3. at the second stage the countries choose the emission levels and how they distribute the gains from co-operation.

In models that use **internal/external stability**, at the first stage a country can choose:

1. to join the coalition of the supporters of an IEA and become a signatory;

2. to stay out as a non-signatory and act as a singleton, no competing coalition is in order.

At the second stage the members of the coalition act as a single player and jointly maximise the coalition's welfare whereas each of the singletons maximise his own welfare.
The players/countries can choose either simultaneously or sequentially, usually the singletons chooses first and after the coalition chooses.
Another point is how the benefits are distributed among the members of a coalition. In [FR01] the following manners are discussed:

1. no distribution, since all signatory countries are ex-ante symmetric so they receive the same payoff and no distribution is needed;

2. distribution according to the Nash bargaining solution;

3. distribution according to the Shapley value.

The payoffs are distributed at the end of the second stage and the rules of the game, such as:

1. rules for coalition formation;

2. choice of emission levels;

3. distribution of payoff;

are exogenously fixed.
After an equilibrium coalition is formed we have that it is stable if it is both:

1. **internally stable**, so that none of the signatories has an incentive to leave the coalition;

2. **externally stable**, so that none of the non-signatories has an incentive to join the coalition.

The other approach makes use of the **core**. In this case we have:

1. in the first stage we verify if there is any incentive for a group of countries (or even for a country alone) to deviate from a coalition structure;

2. in the second stage coalitions choose an emission vector (so to maximise coalition's aggregate payoff) and, in case of asymmetric countries, a transfer scheme is defined.

In case one or more countries deviate the coalition breaks up and all the countries act as singletons.

3 An environmental game

3.1 Introduction

Following [FR01] and [CMO03] we present here a simple example of environmental game within the frame of *RGS* models.
The game is composed of three stages:

1. a **constitutional stage** ([CMO03]);
2. a **coalition stage**;
3. **policy stage**.

In the first optional stage the countries can choose, non co-operatively and unanimously[2] some general rule such as the **minimum participation level** or the minimum number of countries that must sign a treaty in order for it to enter into effect. If such a common rules are absent or are supposed exogenously fixed this preliminary stage is missing.
In the second stage the countries essentially decide if they join or not to a coalition. All this can be modelled through a set of rules. Usually this stage is called **coalition stage** and its structure varies depending on the membership rule that is adopted.
In the second stage ([CMO03]) there can be a **a binary choice game** by which each country can only choose either to be a member of the coalition or to act as a (outer) free rider[3]. In this case the only possible coalition structure is:

$$c = (c_k, 1_{n-k}) \tag{47}$$

or:

1. a coalition of k countries of n;
2. $n-k$ countries not within the coalition and acting as (outer) free riders.

[2]We note here that the requisite of unanimity may favour the "worse solution". If all countries must agree on a decision because it takes effect it can happen that a too high or a too low level is fixed: the former case may be about the minimum participation level whereas the latter may be about the global target of abatement level.

[3]We can define two types of free rider: outer and inner. The outer free rider is a country/player that does not sign an agreement but benefits from its positive spillovers. This behaviour is due to the fact that environmental spillovers cannot be made exclusive. The inner free rider is a country/player that signs an agreement but that does not comply with its obligations. This behaviour is favoured by the fact that the practical effects of the agreements are usually hard to monitor and that many violations are hard both to detect and to ascribe to a culprit.

In the third stage the basic assumptions are:

1. coalitions are formed;
2. all coalitions act simultaneously (Nash-Cournot strategy).

Under these assumptions all the members of a coalition tend to maximise aggregate payoff of the coalition.

3.2 The constitutional stage

In two following subsections we briefly examine the second and third stage of the game. As to the first stage we note that, whenever it is the present, it produces a set of parameters that represents the frame of the agreement the countries are going to sign. In the case of **minimum participation stage** the main product is ([CMO03]) a parameter:

$$\alpha = \frac{s}{n} \tag{48}$$

where $s \in [0, n]$ is the number of signatory countries so that $\alpha \in [0, 1]$ represents the share of the n negotiating countries that must sign (and ratify) an agreement so that it can enter into effect[4]. We an have, for example:

1. $\alpha = 0$ so that an agreement enters into effect whichever is the number of the signer countries;
2. $\alpha = 1$ so that an agreement can enter into effect only if all the negotiating countries sign (and ratify) it;
3. $\alpha = 0.55$, as in the case of Kyoto Protocol, so that, if all countries are considered symmetric[5], at least 55% of the negotiating countries must sign (and ratify) the agreement so that it can enter into effect.

[4] Usually the sign of an international agreement from a country is a two stage process:

1. the agreement is signed as a preliminary act;
2. the agreement is ratified (maybe even from an authority distinct from that who signed) as a formal and definitive act.

[5] Countries are, in general **asymmetric** in the following senses:

(a) since they suffer in different ways from the damages of the environmental problem to which the agreement tries to provide a remedy;

(b) since they benefit in different ways from the positive effects of the agreement;

(c) since their economical and/or social and /or political structures differ very much;

(d) since they posses technologies inadequate to comply with the obligations of the

3.3 The coalition stage

During this stage (also called **coalition formation game**) each country decides either to join a coalition or to remain a singleton country.Such a decision depends on:

1. the rules that govern this stage;
2. the value of the per-membership partition function determined in the policy stage.

The key point of the coalition stage is represented by the various ways in which a coalition can be formed. To describe such a formation process we can use one of the following games ([FR01]):

1. **cartel formation** game;
2. **open membership** game;
3. **exclusive membership** $\Delta-$game;
4. **exclusive membership** $\Gamma-$game;
5. **sequential move unanimity** game;
6. **equilibrium binding agreement** game.

3.4 The policy stage

According to [FR01], during the coalition stage we have a global pollution (or global emission) game in the following form:

$$\pi_i = \beta(e_i) - \phi(\sum_{j=1}^{N} e_j) \tag{49}$$

where we have, under classical hypotheses.

1. $0 < e_i < e_i^{max}$, emission level of country i, with a lower (0) and an upper (e_i^{max}) bounds;
2. β, benefit function of each countries (identical for all countries owing to the supposed symmetry), strictly concave or such as $\beta' > 0$ and $\beta'' < 0$;

agreement but they accept to sign it since they hope to receive such technologies in exchange for their adhesion.

3. ϕ, damage function of country i (identical for all countries owing to the supposed symmetry), convex or such as $\phi' > 0$ and $\phi'' \geq 0$.

Equation (49) represents a general form that allows the derivation of a small number of conclusions. Other used forms, more specialised and more easy to manage, are the followings:

$$\pi_i = b(de_i - \frac{1}{2}e_i^2) - c(\sum_{j=1}^{N} e_j) \tag{50}$$

where:

$$\beta(e_i) = b(de_i - \frac{1}{2}e_i^2) \tag{51}$$

and:

$$\phi(e_i) = c(\sum_{j=1}^{N} e_j \tag{52}$$

$$\pi_i = b(de_i - \frac{1}{2}e_i^2) - \frac{c}{2}(\sum_{j=1}^{N} e_j)^2 \tag{53}$$

where, again:

$$\beta(e_i) = b(de_i - \frac{1}{2}e_i^2) \tag{54}$$

and:

$$\phi(e_i) = \frac{c}{2}(\sum_{j=1}^{N} e_j)^2 \tag{55}$$

4 A *NCGT* theoretic approach

NCGT[6] allows us to describe the strategic interdependence among n players, each one being characterised at least by:

1. a set of pure strategies S_i;
2. an [expected] utility function:

$$u_i \ : \ S \longrightarrow \mathbb{R} \tag{56}$$

with:

$$S = \times_{i=1}^{n} S_i \tag{57}$$

[6]In what follows the basics of *NCGT* are given for granted so that many concepts are used without any, either formal or intuitive, definition.

In what follows we are going to examine some simple examples of non-cooperative games under the following simplifying assumptions:

1. we have only two players;
2. each player has a finite, really very limited set of strategies;
3. players choose their strategy simultaneously;
4. unless stated otherwise, games are single shot games.

The games we examine belong to classical games found in *NCGT* literature and have characteristic names whose origin we do not investigate. All the examples we present are devised from [Mus00] with some modifications. We could have entitled the following sections as "Examples of IEPs".

4.1 A Prisoner's Dilemma game

The first example is a very simple game that represents an application of the Prisoner's Dilemma game. In such a game two players refuse to co-operate so reaching a solution worse than that they could reach by co-operating but have no incentive to act co-operatively. In this case we have a project that brings benefits to two countries/players[7] simultaneously and in a non excludible way: this means that the project is a public good between the two countries and that its benefits are enjoyed by both players (if it is implemented), even by that country that refuses to contribute to its implementation.

A vs. B	c	nc
c	1,1	-1,3
nc	3,-1	0,0

Table 1: Prisoner's Dilemma

In this case (cf. Table 1) we have the following situation:

1. each country can co-operate at the project (strategy c) or not (strategy nc);
2. each country awards a benefit $B = 3$ to the project;
3. the cost of the project is equal to $C = 4$ for both countries.

[7] In what follows we consider the two terms as synonyms.

The joined benefit is equal to $2B = 6$ and is greater than the cost of the project whose implementation is therefore socially efficient. If each country were to implement the project by itself it would incur in a loss equal to $B-C = -1$, if both implement the project they get a gain equal to $B-C/2 = 1$ whereas if the project is abandoned they both have neither a gain nor a loss. The payoffs of both players are those of Table 1.
It is easy to see that we have the following preference structures for both players:

1. $(nc, c) \succ_A (c, c) \succ_A (nc, nc) \succ_A (c, nc)$,
2. $(c, nc) \succ_B (c, c) \succ_B (nc, nc) \succ_B (nc, c)$.

From these structures (but also by using dominated strategies) we see that the only Nash Equilibrium (NE) is the strategy profile (nc, nc) whose payoffs are $(0, 0)$, lower than those that could be attained if the two countries would co-operate $(1, 1)$.
By inspecting Table 1, it is easy to see that if a country would implement the project by itself the other would have only benefits without any cost, it would behave as a **free-rider**. The same is true even if the two country would agree to co-operate before playing the game (so to play both c).In this way both countries would get an higher common benefit. If, after having signed such an agreement to co-operate, A is sure that B will comply with it, A will have a strong incentive to deviate playing nc so to get 3 instead than 1. Since the same holds for B, we again obtain the same NE. In this case a free-rider behaviour (by playing nc) of one of the players would be prevented by an identical move of the other.
The general form of the game of Table 1 is shown in Table 2. For such a

A vs. B	c	nc
c	$B-\frac{C}{2}$,$B-\frac{C}{2}$	B-C,B
nc	B,B-C	0,0

Table 2: Prisoner's Dilemma, general form

game we have:

1. $B < C$;
2. $B > \frac{C}{2}$.

In this situation, since the game is played only once, there is no way to get the two players co-operate and reach the social optimum solution (i.e. the profile (c, c)).

4.1.1 Example

In order to be more concrete we can try to give a practical application of this class of games. Let us suppose we have two countries A and B that share a non renewable resource such as an oil-field or a gas-field or a slowly renewable resource such as an underground aquifer.
Both of them can follow either a **conservative strategy** (c), so that both can benefit of the resource either for a period long enough to switch to other fuels or resources or can benefit of the renewable resource potentially forever, or a **non conservative strategy** (nc) so to deplete the resource. The situation is represented in Table 3. In such a Table we have:

1. if both countries co-operate they share almost evenly the resource so that both get the same benefit $b > 0$;

2. if both countries over-exploit the resource they incur in a loss $l < 0$ since they fall in shortage of the renewable resource;

3. if one over-exploits the resource and the other not, the former gets the whole benefit $B > 0$ whereas the latter incur in a loss $L < 0$.

A vs. B	c	nc
c	b,b	L,B
nc	B,L	l,l

Table 3: Prisoner's Dilemma, depletion vs.conservation game

Since we have $B > b$ and $l > L$ we easily see that, for both players:

$$nc \succ c \tag{58}$$

so that the only Nash equilibrium of the game is the strategy profile (nc, nc).

4.1.2 Possible solutions?

Maintaining the general structure of the game as a one shot game, the possible solutions involve:

1. the intervention of an international punishing authority with:
   (a) direct punishment,
   (b) indirect punishment as sanctions;

2. the intervention of an international funding authority.

In the case of direct punishment such an authority, on condition that it is accepted by both players, would punish the lack of co-operation (or strategy nc) with a penalty equal to the benefit each country would gain from the project. In this case Table 2 would be changed in Table 4.

A vs. B	c	nc
c	$B - \frac{C}{2}$,$B - \frac{C}{2}$	B-C,0
nc	0,B-C	-B,-B

Table 4: Prisoner's Dilemma, with international punishing authority

If we look at Table 4 we easily see that the only NE corresponds now to the following strategy profile:

$$(c, c) \tag{59}$$

so that the threat of a heavy penalty would convince both players to adopt a co-operative strategy.
The main flaw with this solution is that it requires both the presence of an international punishing authority with a real power to impose penalties (this fact itself renders this an impracticable solution) and that both countries declare truthfully the benefit they expect from the project. Since the penalty each of them has to pay in case of non-cooperation is equal to such benefit both have the incentive to declare a lower benefit so to pay less without co-operating anyway.

A vs. B	c	nc
c	$B - \frac{C}{2} + C'$,$B - \frac{C}{2} + C'$	B-C+C',B
nc	B,B-C+C'	0,0

Table 5: Prisoner's Dilemma, with international funding authority

In the second case (cf. Table 5) there we would be the need of an international funding authority that should give to each player, as sunk capital, a sum C' only if it co-operates such that playing c gives higher payoffs than playing nc or such that:

1. $B - \frac{C}{2} + C' > B$,
2. $B - C + C' > 0$,

From the first condition we get:

$$C' > \frac{C}{2} \tag{60}$$

that satisfies also the second condition, since $B > \frac{C}{2}$.
Such a solution is clearly an impracticable solution because the funding authority would bear more than the cost of the project (it should pay $2C' > C$ to the two co-operating countries) whereas the two countries would enjoy the benefits. Moreover, if the money transfer occurs in advance, there is no truly effective way to prevent both countries to get the money and after refuse to co-operate whereas, if it should occur after the project has been implemented, there should be no reason for the funding authority to pay. Since this is known also by the two players they would refuse to implement the project before getting the funds.

4.2 A reassurance game

We now proceed with describing a game similar to the preceding but where a co-operation is possible, mainly for technological reasons.

A vs. B	c	nc
c	4,4	-8,0
nc	0,-8	0,0

Table 6: A reassurance game

Again we have two countries and each of them:

1. must provide for some public good whose cost is $C = 8$;
2. gets a benefit $B = 12$ if and only if both countries provide for the same public good;
3. if only one country provides for or if none of them provides for the public good then they get no benefit at all but the supplying country suffers the cost C.

All this accounts for the payoffs of Table 6. We have, indeed:

1. if both countries co-operate (c) and provide for the public good, each of them gets a benefit $B - C = 4$;

2. if only one country co-operates it gets a loss $-C = -8$ whereas the other gets a null payoff since the basic condition has been violated;

3. if none of them co-operates (nc) then both get a null payoff.

By inspecting Table 6 it is easy to see that we have two NE and precisely the following strategy profiles:

1. (c, c),

2. (nc, nc).

Of those NE the first one strictly Pareto dominates the second so that it is the only one who is implemented. We can note, moreover, that:

1. if a country sends signals indicating a will of no-cooperation the other must adapt to the circumstance and not co-operate, otherwise it would suffer a loss;

2. if, before playing the game, the two countries agree to co-operate none of them, playing the game, has an incentive to deviate since, in any case, it would get a lower payoff. This prevents any free-riding from both countries. In this case we get a self-reinforcing agreement without the need of any international authority.

From all this follows that an ex-ante agreement between the two countries is self-reinforcing: none of them has an incentive either to deviate or to be a free-rider at the expense of the other.
All we have said is true if players choose their strategies $s_i \in S_i = \{c, nc\}$ $(i = 1, 2)$ simultaneously and independently from each other but remains true even if the two players are engaged in a dynamic game and one of them moves first and the other moves knowing the other player's move, as can be shown by applying a backward induction to the game in extensive form.

4.2.1 Examples

Before going on we try to give, also for this family of games, some more or less realistic examples involving a pair of neighbouring countries A and B.

1. Suppose A is richer in hi-tech raw materials but lacks in technology whereas B has very advanced technologies but is very poor in hi-tech raw materials. If they both co-operate (in the sense that A gives his materials to B and gets them back as finished goods at very favourable prices) they can get high payoffs, if they do not co-operate they can

attain very low payoff whereas if only one co-operates gets a loss and the other gets a small gain.

If A does not co-operate and B does, A gets a small gain from exploiting his raw materials with his inefficient technology whereas B gets a high loss since he has to provide for the raw materials from other and more distant countries.

If B does not co-operate and A does, B gets neither a gain nor a loss from this interaction whereas A gets a high loss since he has to look for more distant countries that can efficiently transform his raw materials.

2. Suppose A and B share a river and want to build a dam for producing electric power. If each of them builds his own half of the dam they both get a high payoff, if only one builds a half of the dam and the other not, the builder gets a high loss (since the half of the dam is useless) whereas if none of them starts the construction none of them incurs in a loss nor gets a gain.

3. If A and B possess complementary exclusive technologies they can attain high payoffs only if they both co-operate whereas if only one co-operates that county incurs in the loss of the investments without being able to use the technology (that requires the contribution of both countries to work properly) while the other has neither a gain nor a loss as it happens if non of the two countries co-operate.

4.2.2 Asymmetries

For the game of Table 6 we have supposed symmetric costs C and benefits B for the two countries. Now we proceed with considering the following cases of asymmetric benefits B_A, B_B and costs C_A, C_B:

1. $C_A > C_B$ and $B_A > B_B$ or "who spends more benefits more";

2. $C_A > C_B$ and $B_A < B_B$ or "who spends more benefits less";

3. $C_A < C_B$ and $B_A > B_B$ or "who spends less benefits more";

4. $C_A < C_B$ and $B_A < B_B$ or "who spends less benefits less".

Again, only if both two countries co-operate they get the benefit otherwise they get a cost or nothing at all, as we have already seen. In all the four cases we suppose:

1. $B_A > C_A$,

2. $B_B > C_B$,

otherwise we would get, for one or both countries:

$$nc \succeq c \tag{61}$$

so that the game would be a Prisoner's Dilemma game with all the consequences of the case.
The situation is that of Table 7.

A vs. B	c	nc
c	$B_A - C_A$,$B_B - C_B$	$-C_A, 0$
nc	$0, -C_B$	0,0

Table 7: A reassurance game in general form

In the first and last case nothing changes and the two countries are both better off if they co-operate, though one can be better off than the other (in the first case we have $B_A - C_A > B_B - C_B$ whereas in the last we have $B_A - C_A < B_B - C_B$).
In the second case we get:

$$B_A - C_A < B_B - C_B \tag{62}$$

so that country B is better off than country A.
In the third case we get:

$$B_A - C_A > B_B - C_B \tag{63}$$

so that country A is better off than country B.
Condition $C_A > C_B$ and $B_A < B_B$ may arise if country A has a lower environmental damage to repair but has to build more infrastructures to implement the project or suffers a higher cost of work force.
Condition $C_A < C_B$ and $B_A > B_B$ may arise if country A has a higher environmental damage to repair that can be repaired with the use of lower cost technologies.

4.3 A battle of the sexes game

Another "classical" or paradigmatical class of games is the class of the so called **battle of the sexes** games. Within these games we have two players that wish to co-operate but the preference of each goes to his/her own

project.
So to be more concrete, let us suppose to have two countries, A and B, each of them with a project to affect in some way the environment so that both countries can get a benefit form it. Such projects are mutually exclusive, for either economical or technological reasons, so that only one can be implemented (or even none if no agreement occurs).
We have[8]:

1. country A has a project P_A with a benefit $B_A^A > B_A^B$;
2. country B has a project P_B with a benefit $B_B^B > B_B^A$.

Each project has the same cost $C_A = C_B = C$ to be evenly shared between the two countries. This is a strong hypothesis that can be removed in more realistic applications.
In order that each project is feasible only if both countries agree on one of them we must impose:

1. $C > B_A^A > B_A^B > C/2$,
2. $C > B_B^B > B_B^A > C/2$.

Such conditions prevents each country from implementing the project without the intervention of the other. Since we have:

1. $2C > B_A^A + B_A^B > C$,
2. $2C > B_B^B + B_B^A > C$.

the total benefit is greater that the cost of a single project (so that it is socially efficient to implement it) but is lower than the cost of the two projects. The problem, at this point, is how the two countries can choose which project they are going to implement.
If we represent the situation as a game in strategic form where the two players move simultaneously we get Table 8.

It should be obvious that if both countries implement a distinct project (so that A implements P_A and B implements P_B or vice-versa) they both incur in a loss, generally heavier if the implemented project is the one preferred by the other country, but anyhow a loss. The only way for both countries to get a gain is to engage both in the same project though A would prefer P_A and, obviously, B would prefer P_B.
By inspecting Table 8 it is easy to identify two NE[9]:

[8] A superscript identifies the country whereas a subscript identifies the project.
[9] In all these examples we consider only NE in pure strategies. The main reason is that this hypothesis greatly simplifies the analysis.

A vs. B	P_A	P_B
P_A	$B_A^A - \frac{C}{2}, B_A^B - \frac{C}{2}$	$B_A^A - C, B_B^B - C$
P_B	$B_B^A - C, B_A^B - C$	$B_B^A - \frac{C}{2}, B_B^B - \frac{C}{2}$

Table 8: A battle of the sexes game

1. the former associated to the profile of strategies (P_A, P_A);
2. the latter associated to the profile of strategies (P_B, P_B).

Since the social utility in the first NE is equal to:

$$B_A^A + B_A^B - C \tag{64}$$

and in the second NE is equal to:

$$B_B^A + B_B^B - C \tag{65}$$

we could choose the NE with the higher social utility. In all cases where we have:

$$B_A^A + B_A^B = B_B^A + B_B^B \tag{66}$$

the choice of the more suitable NE depends on the development of some convention between the two countries. For instance, a way to choose could be to develop the favoured project of the poorer country. Another way would be to let the richer country act as a Stackelberg leader and choose first the project to develop. In this case the other should follow and choose the same project, since otherwise both would incur in a loss, heavier to tolerate for the poorer country.

All these considerations are beyond a purely non cooperative game theoretic approach and involve either a sort of agreement between the two countries or the existence of a force relation between them where one of the two is wealthier that the other and can influence its choices.

4.3.1 Example

An example of application of this family of games could be the following. Let us suppose we have two countries that are planning to build some power plant and that, owing to the global level of present and estimated energy demand, only one plant will be sufficient over a long range of time.

Now suppose that country A is rich in natural gas whereas country B is rich in coal. In this setting, country A will be favourable to the implementation of a natural gas power plant whereas country B will be favourable to the implementation of a coal power plant.

4.4 Chicken games

As we have seen, if two countries get engaged in a Prisoner's Dilemma game they both behave non-cooperatively so to attain a sub-optimal equilibrium. In this case, if the game is a single shot game, no co-operation is possible and any eventual agreement in that direction would be violated by both countries (and so is neither stable nor self-reinforcing).
Sometime it can happen that an environmental project can be realised even by only one country without any loss but with a little gain (and with the other acting as a free-rider). To examine such a situation we introduce another class of games, the so called **chicken games**. The name of these games derives from the fact that each player tries to convince the other to give up and behave like a chicken. Possible outcomes of such games are that both players give up (so both co-operate) or insist (and so non co-operate) so to get the worst payoffs.
Again, we have two countries (A and B) whose strategies are (we suppose again to be in a static framework):

$$S_A = S_B = \{c, nc\} \tag{67}$$

so that each country can either co-operate (c) or non co-operate (nc).

A vs. B	c	nc
c	$B - \frac{C}{2}, B - \frac{C}{2}$	$B - C, B$
nc	B, $B - C$	0,0

Table 9: First case of chicken game

The cost of the project (that can be shared between the two countries) is C and the benefit for each country is:

$$B = B_A = B_B > C \tag{68}$$

so that each country can develop the project by itself. The project is public good for the two countries in the sense that, even if only one country implements it, also the other enjoys its benefits and there is no way to prevent this from occurring. Table 9 shows the payoffs for the two players in all the outcomes of the game. If both players act non co-operatively they both get a null payoff. If only one engages in the project, the other act as a free-rider. If both act co-operatively they share the cost and get the benefit.
The possibility of free-riding incentives each country to carry the other to implement the project. By inspecting Table 9 it is easy to detect two NE and namely:

1. (nc, c);

2. (c, nc).

In both NE one of the two countries act as a free-rider since it gets the full benefit without paying any cost. Again we face the problem of which of the two NE will be realised.
A possible solution relies on a pre-play communication between the two countries since each of them tries to convince the other that there will be no co-operation and to urge the other of the necessity to implement the project by itself. The main problem with this solution resides in the possibility that neither countries engage in the project that remains undone.
The same framework can be used in a context similar to the one of Prisoner's Dilemma game and so whenever, if each country implements a common good project by itself, it gets a loss but gets an even greater loss if the project is left unimplemented.
In this case let's suppose we have two countries A and B that must reduce or get rid of a common environmental damage.
We have:

1. countries A and B with strategies $S_A = S_B = \{c, nc\}$;

2. cost C (to be shared between the countries if both co-operate) of the project and benefit $C > B_A = B_B = B > C/2$;

3. a loss or damage $D < 0$ if the project is not implemented.

We design a static game of complete information and represent it in the strategic form of Table 10. For the game of Table 10 we have to specify a

A vs. B	c	nc
c	$B - \frac{C}{2}, B - \frac{C}{2}$	$B - C$,B
nc	B, $B - C$	D,D

Table 10: Another case of chicken game

relation between the value of $D < 0$ and the value of $B - C < 0$. In many cases, indeed, an even approximate evaluation of D may be very complex if not impossible at all. We have, essentially, two meaningful cases:

1. $D > B - C$;

2. $D < B - C$.

A vs. B	c	nc
c	1,1	-1,3
nc	3, -1	-0.5,-0.5

Table 11: A chicken game, numerical example

In Table 11 we show a numerical example with $B = 3$, $C = 4$ and $D = -0.5$. In the first case, we are in an identical situation of a Prisoner's Dilemma game: for both players strategy nc strongly dominates strategy c so that the only NE of the game is (nc, nc). In this case a reduced common damage tends to be ignored and not to be fixed.
In the second case, by inspecting Table 10, it is easy to see that we have the following NE:

1. (nc, c),
2. (c, nc).

In this case each country can act as a free-rider so to force the other either at implementing the project or at suffering an higher loss. Which of the two NE comes true depends on the relations between the two countries and their environmental sensibility but can depend also on technological and economical capabilities. A pre-play communication phase between the two players, even in absence of binding agreements, can influence the outcome of the game if a country succeed in persuading the other country of its will of non co-operation.
It can happen that two countries would suffer a different loss in case the project gets aborted. The situation is depicted in Table 12.

A vs. B	c	nc
c	$B - \frac{C}{2}, B - \frac{C}{2}$	$B - C$,B
nc	B, $B - C$	D_1, D_2

Table 12: Yet another case of chicken game

In this case we have:

1. $D_1 < 0$ and $D_2 < 0$;
2. $D_1 > D_2$, so country B's loss is greater than A's loss;
3. $D_1 > B - C$;

4. $D_2 < B - C$.

In this case strategy nc for country A strictly dominates strategy c. Reducing the game, it is easy to see that the only NE in this case is:

$$(nc, c) \tag{69}$$

so that the country with the higher environmental damage will implement the project by itself getting a lower loss and allowing the other to act as a free-rider (and get the full benefit B from the project). Similar considerations hold in the other situation where country A's loss is greater than B's loss.

4.4.1 Example

Again to give an idea of the practical utility of this family of games we give an example.
In this case we imagine two countries A and B that share a polluted lake that in the past both countries used for fishing.
At the present time, neither A nor B can use lake for fishing purposes. The damage from keeping the status quo of the lake is the same for country A and B but both countries would benefit greatly from the fact that is the other country who undertakes any action for the cleaning of the lake.
If country B implements a project for cleaning, at least partially, the water of the lake it gets a benefit B but it has also to bear the cost C of the project whereas country A only gets the benefit B as a pure free-rider.
Similar considerations hold if player A decides to clean the lake even without the co-operation of B.
If the project would be carried out by the two countries they would share the cost: this behaviour is in contrast with the self interest of both countries.
Of course this is a somewhat unrealistic example because it omits both to mention other problems associated with lake pollution and to identify the responsibility of the pollution: country A or B or both or also a third country C through a river?

4.5 Possible ways to reach a co-operation

Up to this point we have presented games involving two countries that interact only once in a single shot game. In the cases we have examined co-operation is hard to achieve and free-riding has strong incentives since players cannot be credibly threatened of retaliations.
We propose here three possible solutions within the *NCGT* theoretic approach and namely:

1. the use of contracts;
2. the use of communication;
3. the use of repeated interactions among players within the same game.

The first two solutions ([Mye91]) introduce either a contract or a communication among players so that they are convinced to play strategies that yield an outcome better than the one they could get by acting selfishly.
The last solution shows how, whenever players do not know in advance how may times they will be playing together, they can be attracted by a co-operative profile of strategies.

4.6 Games with contracts and communication

As we have seen, in a Prisoner's Dilemma game we get a Nash equilibrium where the outcome is worse than the co-operative outcome. In such games one possibility is to transform the game so to include better outcomes. One way to attain such a transformation is through communication among the players that can coordinate their moves and even sign binding agreements whose effect is reflected in the static or strategic form structure of a game.
We suppose to have games with simultaneous and independent moves in which communication and coordination are part of the strategies of the players.
This approach, however, can be used only in simpler cases whereas, in more complex cases, it is more useful to leave communication and coordination possibilities out of the game model and make use of implicit communication opportunities.
As a first example ([Mye91]) we examine a case in which we have no communication between the players but they can sign jointly binding contracts to coordinate their strategies. In this case we speak of **games with contracts** ([Mye91]).
We examine a Prisoner's Dilemma game in strategic form (cf. Table 13).

A vs. B	x_2	y_2
x_1	2,2	0,6
y_1	6,0	1,1

Table 13: A Prisoner's Dilemma game

Table 14 represents the generalised form of such a game. In this case we have:

$$y_1 \succ_A x_1 \tag{70}$$

and:

$$y_2 \succ_A x_2 \tag{71}$$

only if $z < 6$.

A vs. B	x_2	y_2
x_1	z,z	0,6
y_1	6,0	1,1

Table 14: A Prisoner's Dilemma game, generalised version

The game of Table 13 has a unique Nash equilibrium (y_1, y_2) which gives payoffs $(1, 1)$ worse than the payoff they could get from the co-operative solution (x_1, x_2). The players, each acting alone, cannot attain such a solution. If the two players, with the aid of an outside intervener, were to sign a contract that, whenever signed by both of them, would bind them tho choose (x_1, x_2) and, if signed by only one, would let that player choose either y_1 or y_2 we could get the game of Table 15.

A vs. B	x_2	y_2	s_2
x_1	2,2	0,6	0,6
y_1	6,0	1,1	1,1
s_1	6,0	1,1	2,2

Table 15: Binding contracts in a strategic form game

In such a game s_1 and s_2 are the strategies each player chooses if the contract is signed. It easy to see that:

1. $s_1 \succeq_A y_1$;
2. $s_1 \succ_A x_1$;
3. $s_2 \succeq_B y_2$;
4. $s_2 \succ_B x_2$;

so that the only Nash equilibrium of the game of Table 15 is (s_1, s_2), where both sign the contract and get payoffs $(2, 2)$. Moreover, by examining the game of Table 15, we can see that none of the players has an incentive to deviate autonomously from such an equilibrium that, therefore, proves stable. For the game of Table 13 the two players can get an even better payoff if they use a contract to commit themselves to use a **correlated strategy** (also called a jointly randomised strategy). Beyond the contract associated to the strategy profile (s_1, s_2) the two players could arrange for another contract that, if signed by both players, would implement the mixed strategy:

$$\frac{1}{2}[x_1, y_2] + \frac{1}{2}[x_2, y_1] \tag{72}$$

with a payoff equal to 3 for each player. Under the hypotheses that:

1. each player makes his signing decision independently from the other;

2. each player can sign only one contract;

we get the transformed game of Table 16. The second contract is associated

A vs. B	x_2	y_2	s_2	$\hat{s}_2$
x_1	2,2	0,6	0,6	0,6
y_1	6,0	1,1	1,1	1,1
s_1	6,0	1,1	2,2	1,1
$\hat{s}_1$	6,0	1,1	1,1	3,3

Table 16: Use of two contracts

to the strategy profile $(\hat{s}_1, \hat{s}_2)$ with payoffs $(3, 3)$. In such a modified (more precisely extended) game we have two Nash equilibria in pure strategies:

1. $(\hat{s}_1, \hat{s}_2)$,

2. (s_1, s_2)

and a Nash equilibrium in mixed strategies where both players have a probability distribution:

$$(\frac{2}{3}, \frac{1}{3}) \tag{73}$$

over, respectively, $s_1, \hat{s}_1$ and $s_2, \hat{s}_2$. If we consider only **pure strategies** of the game of Table 16 we have that in every case, by both signing one of the two contracts, the two players get at least the same payoffs they would get in the unreachable solution (x_1, x_2) of Table 13.

The situations we have depicted so far are somewhat unrealistic since, in more realistic settings, the involved players have a wide spectrum of contracts they can sign and, moreover, the act of signing a contract is the last move of a potentially long bargaining process.

A possible solution, to which we only allude here, is twofold:

1. to keep the strategic structure of the game unchanged, without explicitly introducing in it the strategies associated to the signing of a contract;

2. to develop a new solution concept that is able to take such contract signing options into account.

According to ([Mye91]) we define a **game with contracts** whenever the players have:

1. the options they get from the formal structure of the game;

2. the options to bargain with each other and sign contracts.

Each contract, from its side, binds the signers (that form the set $S \subset N$ if N is the set of all players) to some correlated strategy that may depend on the structure of S. According to this approach we keep the actual structure of the contract-signing options implicit and introduce an appropriate solution concept ([Mye91]).
Following ([Mye91]), we define the concept of **correlated strategy** for the game:

$$\Gamma = (N, (C_i)_{i \in N}, (u_i)_{i \in N}) \tag{74}$$

in strategic form as any probability distribution on the set of possible combinations of pure strategies that the players can choose in Γ.
In this case we consider a subset $S \subset N$ of the players (a "coalition") and define a correlated strategy for S as any probability distribution over:

$$\Delta(C_S) \tag{75}$$

with the following notation:

1. $C_S = \times_{i \in S} C_i$,

2. $C = C_N$,

3. $C_{-i} = C_{N-i}$.

A set S of players can implement a **correlated strategy** τ_S with the support of a trustworthy **mediator** that ([Mye91]) designates randomly a **profile of pure strategies** $c_s \in C_S$ so that the probability of any $c_s = (C_i)_{i \in S} \in C_S$ is $\tau_S(c_S)$. Under these assumptions, the mediator would suggest each player $i \in S$ to implement strategy $c_i \in c_S$.
If we now switch to the whole set of players N in game Γ we can define, for

each $i \in N$ and any given correlated strategy $\mu \in \Delta(C)$, the expected payoff of player i:

$$U_i(\mu) = \sum_{c \in C} \mu(c) u_i(c) \tag{76}$$

whereas, for the whole set of players, we define the following vector of payoffs allocation:

$$U(\mu) = (U_i(\mu))_{i \in N} \tag{77}$$

the players get from implementing μ.
We can now define a **contract** as any vector $\tau = (\tau)_{S \subseteq N}$ such that:

$$\tau \in \times_{S \subseteq N} \Delta(C_S) \tag{78}$$

In this setting ([Mye91]), for any contact τ we have that τ_S represents the correlated strategy that the players of the "coalition" S would implement if they sign that contract. Another important point is that for any allocation in the set (a closed and convex subset of $\mathbb{R}^n$):

$$\{U(\mu) \mid \mu \in \Delta(C)\} \tag{79}$$

there is a contract such that, if all the players (the so called "grand coalition") sign it, they get the corresponding payoff allocation.
Given a contract τ we note, however, that it can happen that no all the players would sign it. In the case of the game of Table 13 we note that, if the contract would commit the players to choose (x_1, y_2), player A would not sign it because he would be better off by not signing the contract and choosing y_1, independently from the choice made by player B. We note, indeed, that the payoff 1 for player A is the worse best outcome for that player or his **min max** value. It is easy to see that the same holds also for player B.
Formally, we have ([Mye91]) that the **min max value** of player i (v_i) is the best expected payoff that the player can get against the worse (for him) correlated strategy that the other players of S (or in $S \setminus \{i\}$) can use against him.
We can guess that a player will not sign any contract from which he gets an expected payoff lower than his min max value. From this point of view we define a correlated strategy $\mu \in \Delta(C)$ as **individually rational** iff we have:

$$U_i(\mu) \geq v_i \tag{80}$$

for every $i \in N$. We have, in this way, a set of $n = \mid N \mid$ constraints, called **individual rationality constraints**, on the correlated strategy μ.
As a last step, we suppose that the players decide, one independently from

the others, which contract to sign. As a general fact we can prove (for details cf. [Mye91]) that for every individually rational correlated strategy μ there is a contract τ with $\tau_N = \mu$ such that the fact that all players sign such a contract is an equilibrium of the strategic game with implicit contract-signing options.
If we consider player i we have that τ_{N-i} is the min max strategy of the other players against i so that, if i is the only players who does not sign the contract, his payoff is at least equal to his v_i: under this condition also i will sign the contract that all other players are expected to sign.
On the other hand, if we are in an equilibrium of a contract-signing game, none of the signers can get a payoff that is strictly lower than his v_i otherwise he could be better off not to sign the contract and use the strategy that guarantees him to get v_i (this follows directly from the definition of equilibrium).
We have, now, the final result that defines the set:

$$\{U(\mu) \ | \ \mu \in \Delta(C) \, and \, U_i(\mu) \geq v_i \ \forall i \in N\} \tag{81}$$

as the set of payoff allocations that can be obtained at the equilibria of these games in which player i has the option of either not sign any contract or choose a strategy in C_i.
At this point a few comments are in order.

1. We have seen that one central point is represented by the role of the mediator that randomly designates a profile of pure strategies $c_S \in C_S$. In practical settings of IEA the role of the mediator could be played by an International Organisation that all the players recognise as "super partes", since no enforcing power is needed.
   The main problem with the mediator is, however, that he must know all the pure strategies a player has at his own disposal. We can call this property **observability** and may represent a major problem since each player can either not know or conceal or modify some of those strategies so to obtain some strategic advantage.

2. Another key point is the problem of free-riders, both inner and outer free-riders. We remind that the former players are those players who sign a contract but do not respect its ties whereas the latter are those who does not sign a contract but benefit, at no cost, of its spillovers. For this problem the type of games we introduced has no real solution.

3. We have seen that an equilibrium is individually stable since no player $i \in S$ alone has an incentive to deviate from it. Nothing has been said (and can be said) about the possibility that a subset $S' \subset S$ of players (a "sub coalition") decide to deviate from the correlated strategy.

In many cases ([Mye91]) players cannot sign binding contracts and the reasons may be the following:

1. the strategies of the players are unobservable to the mediator or to the legal enforcer of the contracts;
2. there is no effective way to punish players who infringe a contract either since the available punishments are inadequate or since it is very hard to detect any violation;
3. the strategies of some of the players involve inalienable rights (such as **sovereignty**, **alimentary security** and the like).

In such situations players may have, however, the possibility to communicate and coordinate with each other. In such cases ([Mye91]) we speak of **games with communication** if the player have at their disposal:

1. the explicit strategies specified by the structure of the game,
2. a set of implicit communication options.

Again, following ([Mye91]), we start with a simple example and ten try to generalise.
Table 17 presents a game in strategic form with three Nash equilibria:

1. (x_1, x_2) in pure strategies with payoffs $(5, 1)$;
2. (y_1, y_2) in pure strategies with payoffs $(1, 5)$;
3. a randomised equilibrium with $\sigma_1 = \sigma_2 = (1/2, 1/2)$ and expected payoffs $(2.5, 2.5)$.

A vs. B	x_2	y_2
x_1	5,1	0,0
y_1	4,4	1,5

Table 17: An example of game in strategic form

In this case the social optimum (y_1, x_2), with payoffs $(4, 4)$, cannot be achieved without a binding contract because it is not an equilibrium of the game (so that each player has an incentive to deviate from it).

In this case, if the two players communicate, they can use a correlated strategy to attain a better payoff, better than the expected payoffs $(2.5, 2.5)$.
If they implement the following correlated strategy:

$$0.5[x_1, x_2] + 0.5[y_1, y_2] \tag{82}$$

(based, for instance, on the common observation of a fair coin toss) they get an expected payoff of $(3, 3)$. We note that the event is not binding but, notwithstanding this, gives rise to a self-enforcing plan since none of the players can attain a better payoff by unilaterally deviating from it. If player B follows the plan (and play a "mixed strategy" $\sigma_B = (1/2, 1/2)$) and player A deviates from it, player A gets:

1. an expected payoff equal to $2.5 < 3$ if chooses x_1;
2. an expected payoff equal to $2.5 < 3$ if chooses y_1;

In this case the communication is the common observation of an event that defines a correlated strategy. Another way for the two players to get a better payoff is with the intervention of a **mediator**.
A **mediator** ([Mye91]) is not a player (in the sense that he has neither strategies nor receives payoffs) but has the task to let players communicate and share information.
In this case:

1. if the mediator suggests both players a randomised strategy such as:

$$\frac{1}{3}[x_1, x_2] + \frac{1}{3}[y_1, y_2] + \frac{1}{3}[y_1, x_2] \tag{83}$$

2. if each player receive only his own recommendation from the mediator;
3. if both players obey the mediator M since both expect that also the other obeys;

then, even if the mediator's recommendation has no binding force, the situation in which both players follow the recommendation of the mediator represent a Nash equilibrium of the transformed game with communication.
We can have the following cases:

1. if M recommends A to play y_1 then A can think that B has received a recommendation to play $0.5x_2 + 0.5y_2$ so that the expected payoff for A would be the same both for x_1 and for y_1 and this incentives A to obey to M;

2. if M recommends A to play x_1 then A would think that player B has been recommended to play x_2 so that his best response is x_1 and, again, A obeys the mediator.

In all the cases A obeys M since he expects also B obeys M. Dual arguments apply to player B. In this way both players reach a self-enforcing behaviour where both obey to M so that they get the following expected payoffs:

$$\frac{1}{3}(5,1) + \frac{1}{3}(1,5) + \frac{1}{3}(4,4) = (10/3, 10/3) \tag{84}$$

An essential point ([Mye91]) is that each player gets only a partial information since if, for instance, A know s that B has been told to play x_2 then A will refuse to play y_1 even if recommended by M and will play x_1 (so to attain a pure strategy Nash equilibrium).
In this case we have that a correlated strategy can be implemented only in presence of a mediator or of a noisy communication (so that one player does receive only his own recommendation and is in the dark about other players' recommendations).
If, on the other hand, players can communicate among themselves so that all the available strategies are directly observable then ([Mye91]) the only self-enforcing plans that, in absence of contracts, the players can implement are randomisations of the Nash equilibria of the original game as the correlated strategy we have already seen.
As a general rule ([Mye91]) for any finite game in strategic form:

$$\Gamma = (N, (C_i)_{i \in N}, (u_i)_{i \in N}) \tag{85}$$

if there is a **mediator** that tries to coordinate the actions of the players, the mediator at least would need to let any player $i \in N$ know a recommended strategy $c_i \in C_i$. If the game we are modelling is without contracts we have:

1. every player i knows only his recommended strategy and not the ones recommended to the others;

2. every player i is free to choose any strategy in C_i after hearing the mediator's recommendation. In this case the set of each player's available strategies is enlarged since it includes also all the possible mappings:

$$\delta_i : C_i \longrightarrow C_i \tag{86}$$

of a recommended strategy on the set of the available strategies C_i. In this case the player can obey the mediator but can also deviate from this recommendation and choose a different strategy.

At this point we suppose all the players know the probability distribution with which the mediator determines his recommendations:

$$\mu \in \Delta(C) \tag{87}$$

Under this hypothesis we have that:

$$\mu(c) \tag{88}$$

is the probability of any pure strategy profile:

$$c = (c_i)_{i \in N} \tag{89}$$

that the mediator would recommend the the players.
We define, at this point, an **equilibrium condition** ([Mye91]) as follows. It is an equilibrium for all players to follow mediator's recommendation iff, for all $i \in N$ and for all possible mappings δ_i we have:

$$U_i(\mu) = \sum_{c \in C} \mu(c) u_i(\delta_i(c_i), c_{-i}) \tag{90}$$

where:

$$U_i(\mu) = \sum_{c \in C} \mu(c) u_i(c) \tag{91}$$

is the expected utility of the generic player i.
If relation (90) is satisfied, for all $i \in N$ and for all possible mappings δ_i, we speak of a **correlated equilibrium**. We note, with ([Mye91]), that a correlated equilibrium is any correlated strategy for the players of Γ that is self-enforcing and can be implemented with the help of a mediator that, confidentially, suggests a strategy to each player.
Condition (90) can be shown to be equivalent to a system of inequalities:

$$\sum_{c_{-i} \in C_{-i}} \mu(c)(u_i(c) - u_i(e_i, c_{-i})) \geq 0 \tag{92}$$

for all $i \in N$, $c_i \in C_i$, $e_i \in C_i$ and with $c = (c_i, c_{-i})$. Relations like (92) can be rewritten as:

$$\sum_{c_{-i} \in C_{-i}} \mu(c) u_i(c) \geq \sum_{c_{-i} \in C_{-i}} \mu(c) u_i(e_i, c_{-i}) \tag{93}$$

meaning that the expected utility if all the players obey to the mediator is no worse that the expected utility when all the players obey but one (indeed e_i is a disobedient action). The right side of (93) occurs if player i chooses

a disobedient action e_i when getting recommendation c_i from the mediator. Relations (92) and the equivalent (93) are called **strategic incentive constraints** since ([Mye91]) they represent a set of constraints over the correlated strategy of the mediator so that all the players find it convenient to obey him.
We note that, for any finite game in strategic form:

1. the set of correlated strategies is a compact and convex set;
2. such a set can be described with a finite set of linear inequalities;
3. a vector $\mu \in \mathbb{R}^N$ is a correlated equilibrium if satisfies the strategic incentive constraints and the following **probability constraints**:
   (a) $\sum_{e \in C} \mu(e) = 1$,
   (b) $\mu(c) \geq 0$ for all $c \in C$.

Before some concluding remarks we briefly discuss here an example taken from ([Mye91]). We examine the game of Table 18 in the light of what we have said till now. If we want to find a correlated equilibrium that maximises

A vs. B	x_2	y_2
x_1	5,1	0,0
y_1	4,4	1,5

Table 18: An example of game in strategic form

the expected sum of the players' payoffs we must solve the following linear programming problem:

$$\begin{cases} max\ 6\mu(x_1,x_2) + 0\mu(x_1,y_2) + 8\mu(y_1,x_2) + 6\mu(y_1,y_2) \\ \\ s.t. \\ \\ (5-4)\mu(x_1,x_2) + (0-1)\mu(x_1,y_2) \geq 0 \\ (4-5)\mu(y_1,x_2) + (1-0)\mu(y_1,y_2) \geq 0 \\ (1-0)\mu(x_1,x_2) + (4-5)\mu(y_1,x_2) \geq 0 \\ (0-1)\mu(x_1,y_2) + (5-4)\mu(y_1,y_2) \geq 0 \\ \mu(x_1,x_2) + \mu(x_1,y_2) + \mu(y_1,x_2) + \mu(y_1,y_2) = 1 \\ \mu(x_1,x_2) \geq 0 \\ \mu(x_1,y_2) \geq 0 \\ \mu(y_1,x_2) \geq 0 \\ \mu(y_1,y_2) \geq 0 \end{cases} \tag{94}$$

Problem (94) can be easily solved with standard techniques so to obtain the following optimum solution:

$$\mu(x_1, x_2) = \mu(y_1, y_2) = \mu(y_1, x_2) = \frac{1}{3} \quad \mu(x_1, y_2) = 0 \tag{95}$$

so that the sought for correlated equilibrium is:

$$\frac{1}{3}[x_1, x_2] + \frac{1}{3}[y_1, y_2] + \frac{1}{3}[y_1, x_2] \tag{96}$$

In this case we have that strategic incentive constraints require that for each player the expected payoffs are upperly bounded by:

$$\frac{1}{3}[5, 1] + \frac{1}{3}[1, 5] + \frac{1}{3}[4, 4] = \frac{10}{3} \tag{97}$$

for player A and:

$$\frac{1}{3}[5, 1] + \frac{1}{3}[1, 5] + \frac{1}{3}[4, 4] = \frac{10}{3} \tag{98}$$

for player B so that the expected sum of both players' payoff is upperly bounded by:

$$\frac{10}{3} + \frac{10}{3} = \frac{20}{3} \tag{99}$$

At this point, as usual, before closing the topic and the section some comments are in order. More details on this type of equilibrium can be found on ([Mye91]).

1. The key point is the reason why the analysis has focused on communication systems with a mediator in which it is rational for all players to obey the mediator. The reason, ([Mye91]), is that such communication systems can simulate **any equilibrium** of **any game** that can be generated from **any given strategic form game** by adding **any communication system**. The details of this equivalence can be found in ([Mye91]) and represent an application of the **revelation principle** for strategic form games.

2. Again, as in the case of games with contracts, given a set of players, a mediator must be identified such as that all the players may wish to obey his recommendations. In cases of multipart negotiations this requirement may be (more or less) easily satisfied but there are cases such as:

   (a) the set of players contains all the possible players;

   (b) the set of players includes all the trustworthy mediators of the members of the set;

   where this requirement represents a hard problem by itself.

3. The requirement of confidentiality may be hard to satisfy in real situations so that, in many cases, the private recommendations to each player can influence the strategic decisions of the others. We recall here that each player can indeed decide independently either to obey or disobey the mediator's recommendation. If confidentiality is not perfect, informative asymmetries among the players may arise so that some of them may refuse to obey since, individually, they have better actions o choose. For the game of Table 18 in case of this asymmetric situation:

   (a) player B knows only his recommendation, say x_2;

   (b) player A knows both his recommendation, say y_1, and B's recommendation;

   we have that A will disobey and choose x_1 so that the outcome of the game is no more a correlated equilibrium but a traditional Nash equilibrium in pure strategies.

4. If the set of players N is seen as a "grand coalition" and stability concerns only the convenience of a single player to disobey whereas all the others obey, nothing is said about the possibility that a subset $S \subset N$ of players (a "subcoalition") decides to disobey to their recommendations.

5. The set of correlated strategies can contain more then one element (up to an infinite set of elements) so there is the problem of the choice of the "best" equilibrium, as traditionally happens in all the cases where a game has more than one Nash equilibrium.

6. The last point that we mention here is related to infinite strategy spaces available to the players since all we have said is based on the assumption of finite strategy spaces. We note that the hypothesis of a finite set of players is of no harm since, within the scope of International Environmental Agreements or Problems the set N is upperly bounded for any definition of player we can adopt in practise (i. e. nations, groups of nations or even single human beings).

4.7 Repeated games

In the present section we are going to examine games that are repeated, between the same players, more than once. The underlying hypothesis is that we have a single shot game, the so called **stage game**, that is repeated, identical to itself, a certain number of times.
Basically we have two cases:

1. the number of repetitions N is high but finite;

2. the number of repetitions is potentially infinite or, that is the same, players do not know when the game will end so that, after any play of the game, the game can continue with probability p and can end with probability $1 - p$.

The analysis of this class of games starts with a very simple Prisoner's Dilemma game. Again we have two countries A and B that strategically interact for the implementation of a common project P with:

1. a total cost C;

2. a benefit B for each country.

As we have already seen, we have the following relations:

$$C > B > \frac{C}{2} \tag{100}$$

The single shot game, to be used as a stage game, is described by Table 19.

A vs. B	c	nc
c	$B - \frac{C}{2}$,$B - \frac{C}{2}$	B-C,B
nc	B,B-C	0,0

Table 19: Prisoner's Dilemma game as a stage game

If we define the following quantities:

1. $R = B - \frac{C}{2}$,

2. $S = B - C$,

3. $P = 0$,

we have the following ordering:

$$B > R > P > S \tag{101}$$

In general we have $P > 0$ but small enough to preserve the aforesaid ordering. The game of Table 19 has a single NE $((nc, nc))$ that differs from the social optimum that could be attained through co-operation $((c, c))$. For simplicity we can rewrite Table 19 as Table 20.

A vs. B	c	nc
c	R,R	S,B
nc	B,S	P,P

Table 20: Prisoner's Dilemma game as a stage game, simplified form

Now let's consider the possibility that the two countries interact twice through the same game, playing it twice. The game of Table 20 is a stage game of such twice repeated new game. The timing of the game is the following:

1. the two countries play the game once;
2. the outcome is observed;
3. the game is played again;
4. the payoffs are distributed to the players.

Under the hypothesis of a discount factor equal to 1, by using backward induction, both players know that the outcome of the second repetition of the game is $((nc, nc))$ with payoffs P, P so that the payoffs of the first stage game are those of Table 21.

A vs. B	c	nc
c	R+P,R+P	S+P,B+P
nc	B+P,S+P	P+P,P+P

Table 21: Prisoner's Dilemma game as first stage of a two stage game

It is easy to see that, since we simply added a positive quantity to each payoff of the stage game, the outcome of the new game remains unchanged and stuck to $((nc, nc))$. If the two countries play the game twice they get the same outcome they get if they play it once: a non co-operative result. It is easy to see that the same result holds also if the game is repeated T times, the only difference being that in this case we have to sum to each payoff of the stage game the quantity:

$$\sum_{i=1}^{T} P \tag{102}$$

Things can change if we imagine the two players do not know how many times they are going to interact through such a game so that they can imagine T as potentially being equal to $+\infty$.
In this case we want to verify if there is any possibility that, in this particular type of repeated game, we can obtain a co-operation between the two countries.
We suppose the two countries A and B adopt a "tit for tat" strategy and we aim at verifying under which conditions such a strategy is feasible (i. e. it gives better payoff that the traditional non co-operative strategy).
The "tit for tat" strategy is structured as follows:

1. each country adopts a co-operative strategy c at $t = 0$;

2. at a generic $t > 0$, if the country B (or A) always played c before, A (or B) should play c otherwise it should play nc as well as for any subsequent play of the game.

If we denote with r the **interest rate** (also called discount rate) and with:

$$\frac{1}{1+r} \tag{103}$$

the **discount factor** we can compare the payoffs for each play of the game with the payoffs at $t = 0$.
In this way, if a player chooses to play the co-operative strategy c at each repetition of the game it gets the following total payoff:

$$\sum_{i=0}^{\infty}\left(\frac{R}{1+r}\right) = R\frac{1+r}{r} \tag{104}$$

(where we have used the sum of an infinite geometrical series). If, on the other hand, a player chooses a non co-operative strategy nc at t and is forced to go on playing nc owing to the retaliation of the other country it gets a total payoff equal to:

$$B + \sum_{i=1}^{\infty}\left(\frac{P}{1+r}\right) = B + \frac{P}{r} \tag{105}$$

(again we have used the sum of an infinite geometrical series and a small trick to arrange the initial value of the summation).
By comparing the two resulting payoffs we see for what values of r it is better to act co-operatively and for what values it is better to act selfishly. If r is such that:

$$R\frac{1+r}{r} > B + \frac{P}{r} \tag{106}$$

or:

$$r < \frac{R-P}{B-R} \tag{107}$$

co-operation is a desirable strategy otherwise not. In this way future benefits play a bigger role that today's and this encourages co-operation: if interest rate is high current benefits are exalted with respect to future benefits so today's non co-operative strategies are preferred to co-operative ones since it reduces the weight of future retaliation punishments.
The present result has a general validity that holds whenever the stage game has a unique NE. Two possible extensions include:

1. the case of more than two players, as it usually happens in cases where a group of countries negotiate an **International Environmental Agreement**;

2. the case where the stage game has more than one NE, that we are not going to examine in the present notes.

Whenever we have more than two countries that interact within an infinitely repeated game again co-operation is hardly established and, in many cases, we can assist to a transformation of the game: it can happen that we start with a Prisoner's Dilemma game and end with a chicken game.
According to this point of view, we start describing a repeated Prisoner's Dilemma game among N players. In this case we denote as:

$$\pi_c(\nu) \tag{108}$$

the "stage" profit of a co-operating country when further ν countries co-operate and as:

$$\pi_{nc}(\nu) \tag{109}$$

the "stage" profit of a non co-operating country when ν countries co-operate. Since we are in a repeated Prisoner's Dilemma game, for any value of $\nu \in [0, N-1]$, we must have:

$$\pi_{nc}(\nu) > \pi_c(\nu) \tag{110}$$

(so that non co-operation for a country gives a greater benefit than co-operation and represent a dominant strategy) and also:

$$\pi_c(N-1) > \pi_{nc}(0) \quad (111)$$

so that a situation of full co-operation gives a greater benefit than a situation in which every country acts selfishly (otherwise there should be no incentive at all to co-operate).

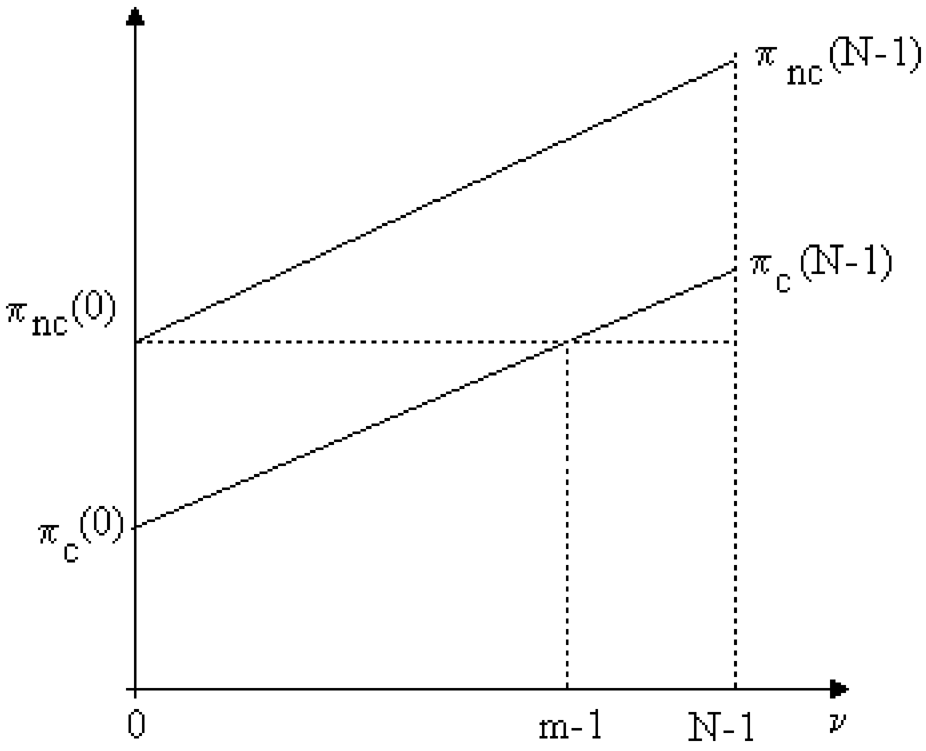

Figure 1: *Profits and coalitions [Mus00]*

In addition to the preceding inequalities we have the following hypotheses:

1. function π_c increases with ν;
2. function π_{nc} increases with ν;
3. all the countries have the same functions π_c and π_{nc} (symmetry).

From equation (110) we have that the curve of function π_{nc} lies above the graph of function π_c. From equation (111) we obtain a constraint on the

relative position of the two curves.
The situation is depicted in figure 1.
If we have that:

$$\pi_{nc}(0) < \pi_c(N-1) \tag{112}$$

so that the benefit that a country gets from global co-operation is greater than the benefit it gets acting selfishly, from the nature of the interactions, it can happen that (cf. figure 1):

$$\pi_{nc}(0) = \pi_c(m-1) \tag{113}$$

In this case, acting together with other $m-1$ countries gives the same benefit than when nobody co-operates. Under this hypothesis we can see that a coalition of $m+1$ countries can successfully reach a common agreement since:

$$\pi_{nc}(0) < \pi_c(m) \tag{114}$$

Such a coalition cannot be said stable since, even if only one country withdraws, the agreement breaks since countries have no benefit from co-operation.
Now let's evaluate the total benefit a country gets from entering such a coalition and compare it with what gets in case of non co-operation.
Defining, again, the discount factor as:

$$\delta = \frac{1}{1+r} \tag{115}$$

(where r is the **discount rate**) we can evaluate the present value of future benefits from entering a coalition (of agreement's signers) as:

$$\sum_{i=0}^{\infty} \delta^i \pi_c(m) = \sum_{i=0}^{\infty} (\frac{1}{1+r})^i \pi_c(m) = \pi_c(m)\frac{1+r}{r} \tag{116}$$

If a country does not co-operate at $i=0$ whereas other m countries do we can have a "tit for tat" strategy so that for $i>0$ there is no co-operation. In this case the non co-operating (and punished) country obtainf the following present value benefit:

$$\pi_{nc}(m) + \sum_{i=1}^{\infty} \delta^i \pi_{nc}(0) = \pi_{nc}(m) + \pi_{nc}(0)(\sum_{i=0}^{\infty} \delta^i - 1) \tag{117}$$

and, therefore:

$$\pi_{nc}(m) + \pi_{nc}(0)\frac{1}{r} \tag{118}$$

If we impose that the benefit defined by relation (116) is greater that the benefit defined by relation (118) we get:

$$r < \frac{\pi_c(m) - \pi_{nc}(0)}{\pi_{nc}(m) - \pi_c(m)} \tag{119}$$

If disequation (119) holds, a group of $m+1$ can settle an environmental agreement about a common problem.
Now the bad news. The main problems with the aforesaid solution are the followings:

1. that it involves only $m+1$ countries out of N;
2. that such a number is lower the steeper is π_c and the higher is $\pi_c(0)$.

From the first point we have that $N-(m+1)$ stay out of the agreement and, notwithstanding this, they get the benefits of the agreement at no cost. This is an incentive to act as a free-rider and not to join a coalition. Moreover, we have that each country has a strong interest that other countries co-operate and sign the agreement. The game is now a **chicken game** where every country urges the others to achieve an agreement, having no intention to join.
This is a major problem since there can be a lot of coalitions involving $m+1$ countries out of N and every country has an incentive both to stay out from each of them and, at the same time, that one of such coalitions forms so the get its benefits as a free-rider.
As to the second point we note that, under the constraints:

1. $\pi_c(0) < \pi_{nc}(0)$,
2. $\pi_c(n) < \pi_{nc}(n)$ for $n \in [0, N]$,

we have:

1. the more each country wishes to co-operate in a singleton situation (or the higher is $\pi_c(0)$) the smaller will be a coalition;
2. the higher is π_c', or the marginal benefit of each country from co-operation, the smaller will be a coalition;
3. even if $\pi_c(0)$ and π_c' are small enough so that only a **grand coalition** (i.e. a coalition involving the full set of N countries) could be possible. only the problem of free-riding would clear up by itself whereas the problem of stability would still be on since, by definition, we would have:

$$\pi_{nc}(0) = \pi_c(N-1) \tag{120}$$

so that no country would have a real incentive at co-operation.

5 A CGT theoretic approach

5.1 Introduction

The $NCGT$ theoretic approach has been examined in the previous section where we have tried to show how co-operation among countries can be a hard matter.
In this section we are going to use a CGT theoretic approach to examine the formation of International Environmental Agreements (IEAs) involving a set of countries i. e. a coalition.

5.2 A coalition against environmental pollution

In this section we are going to examine a co-operative game based on the RSG paradigm. Such a game is based on two stages:

1. at the first stage we have a **coalition game** during which countries decide to adhere or not to a coalition;
2. at the second stage we have a **pollution game** where the countries of the coalition act as a single player against all the other that act selfishly as singletons.

At this level we want to describe and solve the problem of forming groups (coalitions) of countries who sign international agreements so to attain a co-operative solution, more efficient from the point of view of the involved countries.
We suppose to have a set S of n polluting countries (a **coalition**), each country contributing with a quantity e_i to the global pollution. The total pollution amounts to:

$$X = \sum_i e_i \tag{121}$$

Such a pollution causes damages both in country i itself and in all the other countries. We suppose that the self induced damage has the following expression:

$$m_i X = m_i \sum_j e_j \tag{122}$$

where $m_i \geq 0$ represents the marginal damage for country i:

$$m_i = \frac{d}{de_i} m_i \sum_j e_j \tag{123}$$

. We have:

1. a global marginal damage $M = \sum_i m_i$;

2. a total ordering on the marginal damages, since we can always reorder the countries so to get:

$$m_1 \geq m_2 \geq \cdots \geq m_n \tag{124}$$

The main problem is that each polluting country gets a benefit $B(e_i)$ from its contribution e_i. Under classical hypotheses we have:

1. $B'(e_i) > 0$ so that the marginal benefit is positive (and so the benefit is increasing in e_i);

2. $B''(e_i) < 0$ so that the marginal benefit is decreasing in e_i.

In this way we have that B is a strictly concave function.
As to the country i we have, therefore:

$$W_i(e_i, e_{-i}) = B(e_i) - m_i X \tag{125}$$

as the net benefit of country i as a function of the polluting strategies of the other countries (represented by the term e_{-i}) with:

$$X = e_i + e_{-i} \tag{126}$$

whereas we have the following expression of the welfare in case of a cost:

$$W_i(e_i, e_{-i}) = B(e_i) - m_i X - C(e_i) \tag{127}$$

In absence of any co-operation country i maximises its own net benefit by evaluating:

$$\frac{dW_i(e_i, e_{-i})}{de_i} = 0 \tag{128}$$

so to get, from equation (125) the marginal damage:

$$B_i'(e_i^0) = m_i \tag{129}$$

where e_i^0 represents the non co-operative level of emissions of country i, for all $i \in S$.
This solution, that represents a NE of the non co-operative solution of the problem, has two major drawbacks:

1. it neglects the damages that other countries cause to country i;

2. it neglects the damage that country i causes to the other countries;

so that solution (129) is not socially efficient for the whole set S of countries. To get such a solution the countries should work to reach the co-operative solution that maximises:

$$\sum_i W_i(e_i, e_{-i}) = \sum_i (B(e_i) - m_i X) \tag{130}$$

Considering the expression for X, it is easy to see that the first order condition applied to relation (130) gives the following result:

$$B'(e_i^*) = \sum_j m_j = M \tag{131}$$

so that the value of admissible emissions e_i^* for country i at the social optimum is such that the marginal benefit $B'(e_i^*)$ for country i is equal to the global marginal environmental damage M.

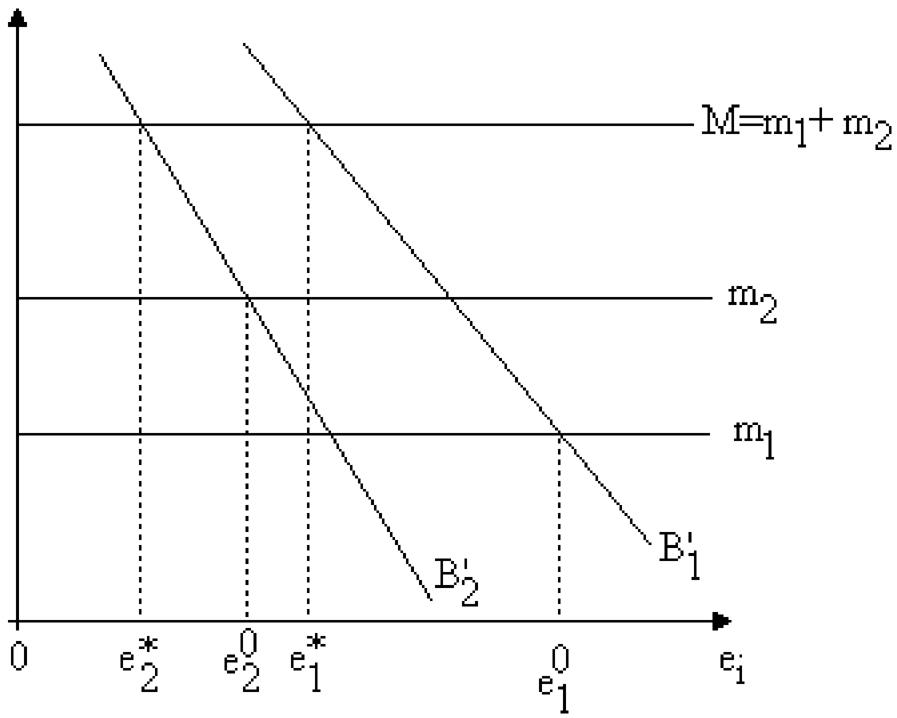

Figure 2: *Case of two asymmetric countries [Mus00]*

Figure 2 presents the case of two countries 1 and 2. If they act non co-operatively they maximise their own benefits at the emission levels e_1^0 and e_2^0. If they both act co-operatively they consider the global value $M = m_1 + m_2$

so that their emission levels drop to lower values e_1^* and e_2^*. In this case, as to the global pollution (or global emission), we have:

$$X^* = e_1^* + e_2^* < X^0 = e_1^0 + e_2^0 \tag{132}$$

The global pollution in the co-operative case X^* is lower than that in the non co-operative case X^0.
The situation we have depicted is a little bit unrealistic since it neglects the abatement costs each country has to tackle. For the moment we compare the following two situations (cf. figure 3) of two countries (namely 1 and 2) with a status quo of, respectively, $\bar{e}_1$ and $\bar{e}_2$ levels of emission.

1. If the two countries decide to act independently one from the other they will lower their emission levels, respectively, at e_1^0 e_2^0. In this case the drops (cf. figure 3 (a)) are equal to:

   (a) $\Delta e_1 = \bar{e}_1 - e_1^0$,

   (b) $\Delta e_2 = \bar{e}_2 - e_2^0$.

2. If, on the other hand, they decide to act co-operatively they sign an agreement by which they reduce their respective emission levels to e_1^* e_2^* so to obtain the following bigger drops (cf. figure 3 (b)):

   (a) $\Delta e_1 = \bar{e}_1 - e_1^*$,

   (b) $\Delta e_2 = \bar{e}_2 - e_2^*$.

As we have already seen, in the second case the value of the global pollution X is lower than in the first case (cf. equation (132)).
For the moment we have dealt with marginal benefit curves and emission reduction levels. It is time to introduce something about unitary abatement cost functions $C_i(e_i)$. Traditional assumptions on such curves dictate that:

1.
$$C_i'(e_i) > 0 \tag{133}$$

2.
$$C_i''(e_i) > 0 \tag{134}$$

From equation (134) we have that C_i is not linear so that the higher the pollution level the higher the unitary cost of pollution abatement.
A closer examination of figures 3 (a) and (b) shows some of the problems of such solutions.

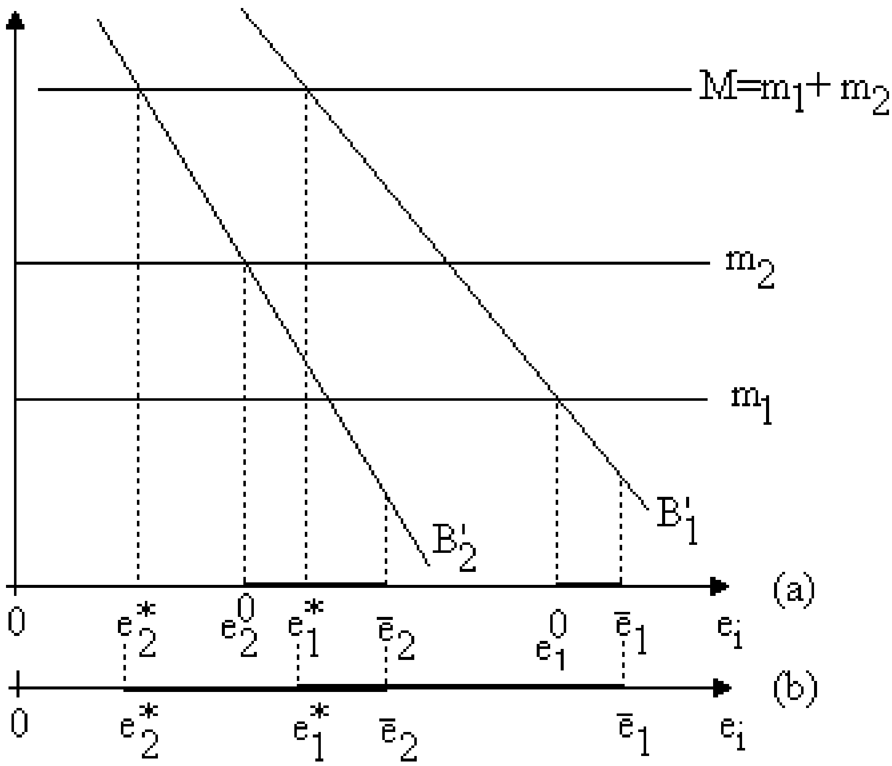

Figure 3: *Case of two co-operating or non co-operating countries [Mus00]*

1. We can have countries whose initial pollution level is lower than that of the others. This is the case of country 2 compared with country 1. In this case it is easier that country 2 (maybe a developed and technologically advanced country) engages in a reduction plan whereas the other (a developing country) may find it hard to face the costs of a pollution reduction plan, owing to the high costs and the lack of proper technology.

2. On the other hand, in the co-operative case, the needed abatement levels are higher so that also the associated costs are much higher. In this case if we want that countries join the effort there must be some sort of support, at least for the less developed countries, so to encourage their participation.

3. In both cases, the country with higher pollution levels faces, at the same time, bot an higher reduction level and, therefore, higher costs. This represents an asymmetry among countries that we are going to

discuss in greater details shortly.

4. Usually co-operative solutions (involving more that $m = 2$ countries on a total of $n > m$) suffer the following problems:

   (a) stability, since countries tend to withdraw from too costly agreements so that in many cases it is necessary to make concessions in order that countries not only join an agreement but also stay within it;

   (b) internal free-riding whenever a country joins an agreement but tends not to satisfy its obligations;

   (c) external free-riding whenever a country does not join an agreement but benefits, at no cost, of the efforts made by the signers.

5.3 Drawbacks of agreements

One of the main problems associated with the setting up of an agreement among a set of countries is that, in many cases, a country may suffer a loss passing form a non co-operative equilibrium to a co-operative one. Let's examine a simple case, switching, for a moment, to the *NCGT* and representing a simple game in strategic form.
We suppose we have two countries A and B with the classical strategies of a static game in strategic form:

1. $S_A = \{c, nc\}$,

2. $S_B = \{c, nc\}$.

A vs. B	c	nc
c	b_1, c_2	d_1, a_2
nc	a_1, d_2	c_1, b_2

Table 22: Co-operation is hard

If we have the payoffs of Table 22 with the following orderings:

$$a_1 > b_1 > c_1 > d_1 \tag{135}$$

$$a_2 > b_2 > c_2 > d_2 \tag{136}$$

we easily see that:

$$nc \succ_1 c \tag{137}$$

$$nc \succ_2 c \tag{138}$$

so that (nc, nc) is the only Nash Equilibrium of the game. If the two countries would decide to switch to the co-operating solution (c, c) one of the major obstacle is represented by the fact that, for country B, the switching from the former to the latter implies a loss equal to $b_2 - c_2$ whereas country A gets, from the same transaction, a net benefit equal to $b_1 - c_1$. Moreover it is not always assured that the global benefit is bigger than the selfish (or non co-operative) benefit or, formally, that:

$$b_1 + c_2 > c_1 + b_2 \tag{139}$$

In order for (139) to be satisfied we must have (under the hypotheses $b_1 - c_1 > 0$ and $b_2 - c_2 > 0$ that are satisfied from (135) and (135)):

$$b_1 - c_1 > b_2 - c_2 \tag{140}$$

but relations (135) and (136) are not enough for (139) to be always verified. This situation derives mainly from an asymmetry among the various countries owing to:

1. different marginal benefit B_i' each country i gets from its emission levels e_i;
2. different marginal damage m_i;
3. different technological capabilities and so different cost function $C_i(e_i)$.

A vs. B	c	nc
c	$b_1 - \epsilon, c_2 + \epsilon$	d_1, a_2
nc	a_1, d_2	c_1, b_2

Table 23: Incentives to co-operation

A possible solution, in cases where relation (139) is verified, may be found under the form of a transfer of resources ($\epsilon > 0$) from country A to country B (cf. Table 23) so that the following conditions are satisfied:

1. for country A we have $b_1 - \epsilon > c_1$;
2. for country B we have $c_2 + \epsilon > b_2$.

In this case we have, again:

$$b_1 + c_2 > c_1 + b_2 \tag{141}$$

We note, however, that also the game of Table 23, under the orderings:

$$a_1 > b_1 - \epsilon > c_1 > d_1 \tag{142}$$

$$a_2 > c_2 + \epsilon > b_2 > d_2 \tag{143}$$

has the only Nash equilibrium at (nc, nc). This means that:

1. even if two countries agree to co-operate (so to adopt the strategy profile (c, c)) both have strong incentives to unilaterally deviate from it;
2. if transfer ϵ occurs before the two countries effectively co-operate, country B can obtain it and then adopt a non co-operative strategy since, in many cases, there is no effective way for A to get the transferred good back;
3. if transfer ϵ occurs after the two countries effectively co-operate, country A can refuse to carry out the promised transfer.

Another bunch of problems arise as to the nature of transfer ϵ. Usually there are three kinds of transfers:

1. money transfers;
2. in-kind transfers;
3. technological transfers.

All these kinds of transfers suffer drawbacks. Among these the major drawback is that, after a transfer has occurred, if the beneficiary country does not fulfil the associated obligations there is no effective way, for the donor country, to get back the transferred good.

5.4 Something more on coalitions

In order for an IEA to be ratified and go into effect, in many cases, it is necessary that a minimal set of countries agree on it and form a stable coalition.
We call these countries **initial signers** (IS) and define a **stable coalition** a coalition that none of the belonging country has any incentive to leave. More

formally we speak, in this case, of **internal stability** whereas a coalition is termed as **externally stable** if no country outside of the coalition has incentives to join it.
Such a coalition, therefore, should not be **externally stable**, in the sense that it should give incentives to non initial signers countries (NIS) to join the coalition.
This can happen essentially in three ways:

1. because a country $j \in NIS$ thinks that it can get bigger benefits joining the coalition than staying out of it;

2. because a country $j \in NIS$ obtains by countries in IS an $in - out$ transfer that is an incentive to join the coalition;

3. because a country $j \in NIS$ gives a transfer to another country $j \in NIS$ so that it joins the coalition ($out - out$ transfer).

As to the second point we only note that each transfer reduces the net benefit of the countries in IS so that usually this is not the right way to reach neither a much wider coalition nor a grand coalition (or the coalition that includes all the countries).
We already noted some of the problems associated with the use of transfers to enlarge a coalition beyond IS. Here we add two more problems:

1. in many cases it is hard to quantify the entity of the transfer as a function of the (political, social, economical and technological) characteristics of the receiving country in order to incentive it to join the coalition;

2. in many cases the existence of transfers, as incentives to join the coalition, can spur strategic behaviours from candidate countries. Such behaviours may be summarised as follows: a country $j \in NIS$ can decide to stay out of the coalition as long as it does not receive transfers of a given amount to join, maybe threatening to carry out a damaging environmental policy (owing to causes beyond control) so to rise the price of its adhesion. In this case we have a bargaining process among the countries of IS and the candidate country $j \in NIS$ that must equitably close either with j joining the coalition or staying out of it. The risk of such strategic behaviours can increase in presence of asymmetries and lack of information among the countries.

5.5 A new solution: issues linkage

A new way to favour the formation of a coalition of countries that sign an IEA is to bind the coalition forming game to some other game among the same countries so that within the global game the co-operative strategy profile results preferable to any other (or at least at no cost or with some limited benefit).
In this way the transfers are not seen as a sunk cost but as a exchange goods between the two games. In the present section we are going to examine a simple example from [Mus00].
We suppose to have two countries A and B engaged in two negotiation processes:

1. an environmental negotiation for the reduction of greenhouse gases;

2. an economical negotiation for th adhesion to a free trade agreement.

We want to model such negotiations with two single shot static games and assuming that, in each game, both countries have the following strategies:

1. $S_A = \{c, nc\}$;

2. $S_B = \{c, nc\}$.

The first game, Γ_1, is an environmental negotiation game whose strategic form representation is given in Table 24

A vs. B	c	nc
c	b_1, c_2	d_1, a_2
nc	a_1, d_2	c_1, b_2

Table 24: Environmental negotiation game

If we have the following orderings:

$$a_1 > b_1 > c_1 > d_1 \tag{144}$$

$$a_2 > b_2 > c_2 > d_2 \tag{145}$$

we can easily see that:

$$nc \succ_A c \tag{146}$$

$$nc \succ_B c \tag{147}$$

so that (nc, nc) is the only Nash Equilibrium of the game Γ_1.
The other game, Γ_2, is an economical negotiation game whose strategic form

A vs. B	c	nc
c	γ_1, β_2	δ_1, α_2
nc	α_1, δ_2	β_1, γ_2

Table 25: Economical negotiation game

representation is given in Table 25.
If we have the following orderings:

$$\alpha_1 > \beta_1 > \gamma_1 > \delta_1 \tag{148}$$

$$\alpha_2 > \beta_2 > \gamma_2 > \delta_2 \tag{149}$$

we can easily see that:

$$nc \succ_A c \tag{150}$$

$$nc \succ_B c \tag{151}$$

so that (nc, nc) is, again, the only Nash Equilibrium of the game Γ_2.
If the two games are played separately, in both cases the two countries attain a non co-operative solution whereas the switching to the co-operative solution would impose a loss (either $b_2 - c_2$ or $\beta_1 - \gamma_1$) to one of the two countries. If, moreover, we add the conditions:

1. $b_1 + c_2 > c_1 + b_2$,
2. $\gamma_1 + \beta_2 > \beta_1 + \gamma_2$,

it is easy to see why the two countries should prefer a co-operation that is not attained owing to the loss each one of them incurs in switching from non co-operation to co-operation.
If the two games are played together we switch to a more complicated but more rich structure where the two players have the following strategies:

$$S_A = S_B = \{nc, nc; nc, c; c, nc; c, c\} \tag{152}$$

or:

$$S_A = S_B = \{c, nc\} \times \{c, nc\} \tag{153}$$

The first element of each strategy identifies the choice of the player in Γ_1 whereas the second element identifies the choice of the player in Γ_2 so that, for instance, nc, nc is the case where a player never co-operates. The payoffs in the new game are easily obtained by appropriately summing the payoffs of the two separate games so to obtain the payoffs of Table 26. If we suppose

A vs. B	nc, nc	nc, c	c, nc	c, c
nc, nc	$c_1+\beta_1, b_2+\gamma_2$	$c_1+\alpha_1, b_2+\delta_2$	$a_1+\beta_1, d_2+\gamma_2$	$a_1+\alpha_1, d_2+\delta_2$
nc, c	$c_1+\delta_1, b_2+\alpha_2$	$c_1+\gamma_1, b_2+\beta_2$	$a_1+\delta_1\ d_2+\alpha_2$	$a_1+\gamma_1, d_2+\beta_2$
c, nc	$d_1+\beta_1, a_2+\gamma_2$	$d_1+\alpha_1, a_2+\delta_2$	$b_1+\beta_1, c_2+\gamma_2$	$b_1+\alpha_1, c_2+\delta_2$
c, c	$d_1+\delta_1, a_2+\alpha_2$	$d_1+\gamma_1, a_2+\beta_2$	$b_1+\delta_1, c_2+\alpha_2$	$b_1+\gamma_1, c_2+\beta_2$

Table 26: Composed game

that in cases where neither countries co-operate we have null payoffs so that:

$$c_1 = b_2 = \beta_1 = \gamma_2 = 0 \tag{154}$$

we get Table 27. From (154) and the orderings on the payoffs we get:

A vs. B	nc, nc	nc, c	c, nc	c, c
nc, nc	$0, 0$	α_1, δ_2	a_1, d_2	$a_1+\alpha_1, d_2+\delta_2$
nc, c	δ_1, α_2	γ_1, β_2	$a_1+\delta_1\ d_2+\alpha_2$	$a_1+\gamma_1, d_2+\beta_2$
c, nc	d_1, a_2	$d_1+\alpha_1, a_2+\delta_2$	b_1, c_2	$b_1+\alpha_1, c_2+\delta_2$
c, c	$d_1+\delta_1, a_2+\alpha_2$	$d_1+\gamma_1, a_2+\beta_2$	$b_1+\delta_1, c_2+\alpha_2$	$b_1+\gamma_1, c_2+\beta_2$

Table 27: Composed game, reduced Table

1. $d_1 < 0$,
2. $d_2 < c_2 < 0$,
3. $\delta_1 < \gamma_1 < 0$,
4. $\delta_2 < 0$.

By examining Table 27 we can easily see that the two countries have, in the aggregate game:

1. a Nash Equilibrium for the following strategy profile $(nc, nc; nc, nc)$;
2. a co-operative solution associated to the following strategy profile $(c, c; c, c)$.

Seemingly nothing has changed but if the following relations are satisfied:

1. $$b_1 + \gamma_1 > 0 \tag{155}$$

2.
$$c_2 + \beta_2 > 0 \tag{156}$$

we can make the following considerations:

1. in game Γ_1 the switching from the solution (nc, nc) to the solution (c, c) is associated to a loss from country B so to incentive that country a transfer is needed;

2. in game Γ_2 the switching from the solution (nc, nc) to the solution (c, c) is associated to a loss from country A so to incentive that country a transfer is needed;

3. in the compound game both countries are better off in the solution (c, c) than they are in the solution (nc, nc) so that the switching can occur without any compensatory transfer from the richer to the poorer country.

Relations (155) and (156) are more easily satisfied in all cases where countries A and B are characterised by symmetric conditions with regard to both negotiation so that, for instance, we have:

1. $b_1 = \beta_2$,

2. $c_2 = \gamma_1$.

In this case both countries get the same positive benefit from the co-operative solution.
We have seen how linking a negotiation (an issue) with another one can make unnecessary any transfer between involved countries in order to encourage the co-operation among them. This, of course, leaves opened, among the others, the following interlinked problems.

1. A **problem of stability**: like in any Prisoner's Dilemma game, how to prevent that a country violates or denounces an agreement since it has incentives to do so.

2. A **problem of size and scale**: given a set of countries and a set of linked issues define what is the best way to carry on the negotiations: bilateral vs. multilateral negotiations with the aid of one or more mediators and/or one arbitrator.

3. A **problem of complexity**: linking two negotiations together increases the complexity of the overall process and this may be even more

evident if one or both negotiations are characterised by pre-existing constraints. For instance we can have two parallel negotiations, ν_1 and ν_2, but the latter lies within the scope of some wider and pre-existing negotiation π, that poses some non negotiable constraints on the former.

4. A **problem of transferability**: an agreement that requires a certain kind of economical structure can be more easily concluded and respected if the necessary technologies are available and can be transfered among the involved countries. This has been the case of Montreal Protocol where alternative products and technologies have been swapped among countries so to obtain (at least officially) a reduction of the production a some gases that were damaging the ozone layer.

6 A few notes about free-riding, transfers and related topics

In this section we present a few remarks ([Fin00]) about some of the "political" aspects of what we have been examining up to this concluding section.
First of all we face some fundamental problems and questions then we step to examine the main flaw of IEA, the problem of free-riding, and close the section with some comments on sanctioning and monitoring.

6.1 "Hardness" of co-operation

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