Using auctions to allocate chores (WIP)

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Aim: use of auction mechanisms for the allocation of a chore to one of the bidders belonging to a set $\mathscr{B}$.

(1) theoretical considerations (low and informal level);

(2) main features;

(3) an algorithm;

(4) uses and properties;

(5) another algorithm (hints).

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The theoretical background

Auctions to allocate goods: a value for the auctioneer and the bidders.

Features of auction mechanisms that influence both protocol and strategies.

(1) Value: private, common or correlated.

(2) One shot versus multi shot.

(3) Open cry versus sealed bid.

(4) Ascending versus descending.

(5) How the winner is identified and how much he has to pay.

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Classical auction mechanisms

Classical auction mechanisms include:

(1) English auction (multi shot, open cry, ascending);

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What is a chore?

(1) a difficult or disagreeable task;

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Modified auctions

We propose the following auction mechanisms where an auctioneer proposes a chore to a set of bidders.

(1) Dutch auction with negative prices: the auctioneer proposes a chore and an increasing amount ($\leq M$) of money until when one of bidders calls stop and accept the chore.

(2) English auction with negative prices: the auctioneer proposes a chore and a starting amount of money L to the bidders that start bidding lower and lower amounts of money until one of them stops the descent and gets the chore.

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(1) guaranteed success: the goal is reached in a finite amount of time;

(2) Pareto efficiency: no other outcome where one player is better off and none is worse off;

(3) individual rationality: following the rules is in the best interest of players;

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The framing situation

The mechanism we propose is inspired by the following situation.

(1) a commissioning authority wants to implement a controversial plant (an incinerator, a dumping ground, a heavy impact industrial plant, a commercial port or a marina or an airport);

(2) the planned infrastructure is something that nobody wants but whose services may be used by a wide group of other authorities that may include also the commissioning authority;

(3) the commissioning authority can identify a certain number of potential contractors (on the base of technical and economical considerations) over which it has no binding authority but with which it tries to achieve an agreement;

(4) a "negative" approach: according to this approach the potential contractors must take part to an auction and bid so to avoid the auctioned chore.

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In our mechanism the fee **f** plays the following roles:

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The basic structure

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,

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(2) each of the bidders b_i bids a sum x_i for not having the chore,

(3) who bids less gets the chore.

b_1 losing bidder (x_1), winning bidders indexed by the set
$N_{-1} = N \setminus \{1\}$

$$x_1 = min\{x_i \mid i \in N\}$$

$$X = \sum_{j \in N_{-1}} x_j$$

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(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$;

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(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$.

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Main features of the proposed algorithm:

(f1) the auctioneer has no revenue and no loss but only gets the chore allocated (a benefit whose value does not influence in any way the auction since it is not known by the bidders);

(f2) the bidders are in competition among themselves in order to no get the chore;

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Compensations

(1) The auctioneer:

    (a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;

    (b) may have an incentive to be deceitful as to the amount of fees he received;

    (c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders may be forced:

    (d) either to pay $p_j = \frac{x}{k} x_1$;

    (e) or to pay x_1.

    (f) They have an expected loss $0.5 \frac{\pi_j}{k} x_1 + 0.5 \pi_j x_1$ where $\pi_j \in [0,1]$ is the probability with which $j \in H$ (set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

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Strategies of the bidders

For each bidder b_i we have:

(1) m_i evaluation of the chore,

(2) x_i curent bid,

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On the other hand if he makes a bid higher than m_i he is more secure he will not lose the auction but he can run a winner's course like risk: he can be compelled to compensate the loser with a sum of money higher than his evaluation of the chore m_i (so it would have been better for him to get the chore). From this we conclude that each bidder should choose to bid a sum $x_i = m_i$.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.

(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.

(3) Individual rationality through the mechanism of the fee.

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(1) Proper value of the fee: not too low (otherwise all bidders can pay). The higher the better for the auctioneer but not for the bidders (no extra compensation for the losing bidder).

(2) Social welfare, next slide.

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A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).

(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.

(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off otherwise.

(4) If we consider the complete set of bidders we have:

   (a) those who pay the fee suffer a collective loss of mf,

   (b) those who bid suffer a collective loss of $\sum_{i=2}^{n} E[i]$,

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(4) If we consider the complete set of bidders we have:
   (a) those who pay the fee suffer a collective loss of mf,
   (b) those who bid suffer a collective loss of $\sum_{i=2}^{n} E[i]$,
   (c) the losing bidder has an expected utility given by
      $E[1] = mf + \sum_{i=2}^{n} E[i] - m_1$,

   so that the complete set of bidders is worse off by m_1 that, anyway, is the less they can lose since $m_1 < m_i$.

Extensions

(1) More than one losing bidder L:

    (1a) use a random device to choose on of them (back to the lone loser case);

    (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:

    (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.

    (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:

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        (a) the auctioneer evaluates q and r such that $c = qn + r$;

        (b) he performs q times the algorithm, each time with n initial bidders as before,

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(1) The auctioneer **a** starts the game with a starting offer $x = x_0 < M$;

(2) bidders b_i may either accept (by calling "stop") or refuse;

(3) if one b_i accepts the auction is over, go to (5);

(4) if none accepts and $x < M$ then **a** rises the offer as $x = x + \delta$ with $0 < \delta < M - x$, go to (3) otherwise go to (5);

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Reverse auction, the strategies

At this point we have to define the strategies of both **a** and the b_i.

(1) The best strategy of **a** is to use a low value of x_0 and, at each step, to rise it of a small fraction δ with the rate of increment of δ decreasing the more x approaches M.

(2) The bidder b_i's best strategy is to refuse any offer that is lower than m_i and to accept when $x = m_i$ since if he refuses that price he risks to lose the auction in favour of another bidder who accepts that offer.

(3) b_i may use a higher value of $m'_i > m_i$ only if he is sure that the private values of all the other bidders are higher. Since no bidder can be sure of this, each of them has a strong incentive to behave truthfully.

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The end, at last!!

What a grand big trip!!