Using auctions to allocate chores (Work In Progress)

Lorenzo Cioni

Department of "Computer Science"
University of Pisa
lcioni@di.unipi.it

AIRO 2008, September 7-11 2008, Ischia, Italy

Introduction

Aim: to show the use of auction mechanisms for the allocation of a chore to one of the bidders belonging to a set $\mathscr{B}$.

(1) theoretical considerations (low and informal level);

(2) main features;

(3) an algorithm (a rule);

(4) uses and properties;

(5) another algorithm (hints).

Introduction

Aim: to show the use of auction mechanisms for the allocation of a chore to one of the bidders belonging to a set $\mathscr{B}$.

(1) theoretical considerations (low and informal level);

(2) main features;

(3) an algorithm (a rule);

(4) uses and properties;

(5) another algorithm (hints).

Introduction

Aim: to show the use of auction mechanisms for the allocation of a chore to one of the bidders belonging to a set $\mathscr{B}$.

(1) theoretical considerations (low and informal level);

(2) main features;

(3) an algorithm (a rule);

(4) uses and properties;

(5) another algorithm (hints).

Introduction

Aim: to show the use of auction mechanisms for the allocation of a chore to one of the bidders belonging to a set $\mathscr{B}$.

(1) theoretical considerations (low and informal level);

(2) main features;

(3) an algorithm (a rule);

(4) uses and properties;

(5) another algorithm (hints).

Introduction

Aim: to show the use of auction mechanisms for the allocation of a chore to one of the bidders belonging to a set $\mathscr{B}$.

(1) theoretical considerations (low and informal level);

(2) main features;

(3) an algorithm (a rule);

(4) uses and properties;

(5) another algorithm (hints).

Introduction

Aim: to show the use of auction mechanisms for the allocation of a chore to one of the bidders belonging to a set $\mathscr{B}$.

(1) theoretical considerations (low and informal level);

(2) main features;

(3) an algorithm (a rule);

(4) uses and properties;

(5) another algorithm (hints).

Introduction

Aim: to show the use of auction mechanisms for the allocation of a chore to one of the bidders belonging to a set $\mathscr{B}$.

(1) theoretical considerations (low and informal level);

(2) main features;

(3) an algorithm (a rule);

(4) uses and properties;

(5) another algorithm (hints).

The theoretical background

Auctions to allocate goods: a good has a value for the auctioneer and the bidders.

Features of auction mechanisms that influence both protocol and strategies.

(1) Value of the auctioned good: private, common or correlated.

(2) One shot versus multi shot.

(3) Open cry versus sealed bid.

(4) Ascending versus descending.

(5) How the winner is identified and how much he has to pay.

The theoretical background

Auctions to allocate goods: a good has a value for the auctioneer and the bidders.

Features of auction mechanisms that influence both protocol and strategies.

(1) Value of the auctioned good: private, common or correlated.

(2) One shot versus multi shot.

(3) Open cry versus sealed bid.

(4) Ascending versus descending.

(5) How the winner is identified and how much he has to pay.

The theoretical background

Auctions to allocate goods: a good has a value for the auctioneer and the bidders.

Features of auction mechanisms that influence both protocol and strategies.

(1) Value of the auctioned good: private, common or correlated.

(2) One shot versus multi shot.

(3) Open cry versus sealed bid.

(4) Ascending versus descending.

(5) How the winner is identified and how much he has to pay.

The theoretical background

Auctions to allocate goods: a good has a value for the auctioneer and the bidders.
Features of auction mechanisms that influence both protocol and strategies.

(1) Value of the auctioned good: private, common or correlated.
(2) One shot versus multi shot.
(3) Open cry versus sealed bid.
(4) Ascending versus descending.
(5) How the winner is identified and how much he has to pay.

The theoretical background

Auctions to allocate goods: a good has a value for the auctioneer and the bidders.

Features of auction mechanisms that influence both protocol and strategies.

(1) Value of the auctioned good: private, common or correlated.
(2) One shot versus multi shot.
(3) Open cry versus sealed bid.
(4) Ascending versus descending.
(5) How the winner is identified and how much he has to pay.

The theoretical background

Auctions to allocate goods: a good has a value for the auctioneer and the bidders.
Features of auction mechanisms that influence both protocol and strategies.

(1) Value of the auctioned good: private, common or correlated.
(2) One shot versus multi shot.
(3) Open cry versus sealed bid.
(4) Ascending versus descending.
(5) How the winner is identified and how much he has to pay.

The theoretical background

Auctions to allocate goods: a good has a value for the auctioneer and the bidders.

Features of auction mechanisms that influence both protocol and strategies.

(1) Value of the auctioned good: private, common or correlated.

(2) One shot versus multi shot.

(3) Open cry versus sealed bid.

(4) Ascending versus descending.

(5) How the winner is identified and how much he has to pay.

Classical auction mechanisms (direct auctions, positive prices) include:

(1) English auction (multi shot, open cry, ascending);
(2) Dutch auction (multi shot, open cry, descending);
(3) First price auction (one shot, sealed, highest wins and pays);
(4) Second price or Vickrey auction (one shot, sealed, highest bid wins but pays second highest bid);
(5) All pay auctions and many many more....

Classical auction mechanisms

Classical auction mechanisms (direct auctions, positive prices) include:

(1) English auction (multi shot, open cry, ascending);
(2) Dutch auction (multi shot, open cry, descending);
(3) First price auction (one shot, sealed, highest wins and pays);
(4) Second price or Vickrey auction (one shot, sealed, highest bid wins but pays second highest bid);
(5) All pay auctions and many many more....

Classical auction mechanisms

Classical auction mechanisms (direct auctions, positive prices) include:

(1) English auction (multi shot, open cry, ascending);
(2) Dutch auction (multi shot, open cry, descending);
(3) First price auction (one shot, sealed, highest wins and pays);
(4) Second price or Vickrey auction (one shot, sealed, highest bid wins but pays second highest bid);
(5) All pay auctions and many many more....

Classical auction mechanisms

Classical auction mechanisms (direct auctions, positive prices) include:

(1) English auction (multi shot, open cry, ascending);
(2) Dutch auction (multi shot, open cry, descending);
(3) First price auction (one shot, sealed, highest wins and pays);
(4) Second price or Vickrey auction (one shot, sealed, highest bid wins but pays second highest bid);
(5) All pay auctions and many many more....

Classical auction mechanisms

Classical auction mechanisms (direct auctions, positive prices) include:

(1) English auction (multi shot, open cry, ascending);
(2) Dutch auction (multi shot, open cry, descending);
(3) First price auction (one shot, sealed, highest wins and pays);
(4) Second price or Vickrey auction (one shot, sealed, highest bid wins but pays second highest bid);
(5) All pay auctions and many many more....

Classical auction mechanisms

Classical auction mechanisms (direct auctions, positive prices) include:

(1) English auction (multi shot, open cry, ascending);
(2) Dutch auction (multi shot, open cry, descending);
(3) First price auction (one shot, sealed, highest wins and pays);
(4) Second price or Vickrey auction (one shot, sealed, highest bid wins but pays second highest bid);
(5) All pay auctions and many many more....

Classical auction mechanisms

Classical auction mechanisms (direct auctions, positive prices) include:

(1) English auction (multi shot, open cry, ascending);
(2) Dutch auction (multi shot, open cry, descending);
(3) First price auction (one shot, sealed, highest wins and pays);
(4) Second price or Vickrey auction (one shot, sealed, highest bid wins but pays second highest bid);
(5) All pay auctions and many many more....

... no limit to ingenuity ....

Classical auction mechanisms

Classical auction mechanisms (direct auctions, positive prices) include:

(1) English auction (multi shot, open cry, ascending);
(2) Dutch auction (multi shot, open cry, descending);
(3) First price auction (one shot, sealed, highest wins and pays);
(4) Second price or Vickrey auction (one shot, sealed, highest bid wins but pays second highest bid);
(5) All pay auctions and many many more....

... no limit to ingenuity ....

The concept of Chore

What is a chore?

(1) a difficult or disagreeable task;
(2) seller of the chore (auctioneer);
(3) [buyer?] bidder (do not want the chore);
(4) negative value for the auctioneer and every potential bidder, a chore is something that nobody wants.

The concept of Chore

What is a chore?

(1) a difficult or disagreeable task;
(2) seller of the chore (auctioneer);
(3) [buyer?] bidder (do not want the chore);
(4) negative value for the auctioneer and every potential bidder, a chore is something that nobody wants.

The concept of Chore

What is a chore?

(1) a difficult or disagreeable task;

(2) seller of the chore (auctioneer);

(3) [buyer?] bidder (do not want the chore);

(4) negative value for the auctioneer and every potential bidder, a chore is something that nobody wants.

The concept of Chore

What is a chore?

(1) a difficult or disagreeable task;
(2) seller of the chore (auctioneer);
(3) [buyer?] bidder (do not want the chore);
(4) negative value for the auctioneer and every potential bidder, a chore is something that nobody wants.

The concept of Chore

What is a chore?

(1) a difficult or disagreeable task;
(2) seller of the chore (auctioneer);
(3) [buyer?] bidder (do not want the chore);
(4) negative value for the auctioneer and every potential bidder, a chore is something that nobody wants.

The concept of Chore

What is a chore?

(1) a difficult or disagreeable task;

(2) seller of the chore (auctioneer);

(3) [buyer?] bidder (do not want the chore);

(4) negative value for the auctioneer and every potential bidder, a chore is something that nobody wants.

Modified auctions

We propose the following modified auction mechanisms where an auctioneer proposes a chore to a set of bidders.

(1) Dutch auction with negative prices: the auctioneer proposes a chore and an increasing amount ($x_0 < x_1 < \cdots < x_n \cdots \leq M$) of money until when one of the bidders calls stop and accepts the chore.

(2) English auction with negative prices: the auctioneer proposes a chore and a starting amount of money L to the bidders that start bidding lower and lower amounts of money until one of them (the last who makes an offer) stops the descent and gets the chore.

(3) A sort of first price auction with negative prices: the auctioneer proposes a chore, each of the bidders makes a bid and the one who bids less gets the chore.

Modified auctions

We propose the following modified auction mechanisms where an auctioneer proposes a chore to a set of bidders.

(1) Dutch auction with negative prices: the auctioneer proposes a chore and an increasing amount $(x_0 < x_1 < \cdots < x_n \cdots \leq M)$ of money until when one of the bidders calls stop and accepts the chore.

(2) English auction with negative prices: the auctioneer proposes a chore and a starting amount of money L to the bidders that start bidding lower and lower amounts of money until one of them (the last who makes an offer) stops the descent and gets the chore.

(3) A sort of first price auction with negative prices: the auctioneer proposes a chore, each of the bidders makes a bid and the one who bids less gets the chore.

Modified auctions

We propose the following modified auction mechanisms where an auctioneer proposes a chore to a set of bidders.

(1) Dutch auction with negative prices: the auctioneer proposes a chore and an increasing amount $(x_0 < x_1 < \cdots < x_n \cdots \leq M)$ of money until when one of the bidders calls stop and accepts the chore.

(2) English auction with negative prices: the auctioneer proposes a chore and a starting amount of money L to the bidders that start bidding lower and lower amounts of money until one of them (the last who makes an offer) stops the descent and gets the chore.

(3) A sort of first price auction with negative prices: the auctioneer proposes a chore, each of the bidders makes a bid and the one who bids less gets the chore.

Modified auctions

We propose the following modified auction mechanisms where an auctioneer proposes a chore to a set of bidders.

(1) Dutch auction with negative prices: the auctioneer proposes a chore and an increasing amount $(x_0 < x_1 < \cdots < x_n \cdots \leq M)$ of money until when one of the bidders calls stop and accepts the chore.

(2) English auction with negative prices: the auctioneer proposes a chore and a starting amount of money L to the bidders that start bidding lower and lower amounts of money until one of them (the last who makes an offer) stops the descent and gets the chore.

(3) A sort of first price auction with negative prices: the auctioneer proposes a chore, each of the bidders makes a bid and the one who bids less gets the chore.

Modified auctions

We propose the following modified auction mechanisms where an auctioneer proposes a chore to a set of bidders.

(1) Dutch auction with negative prices: the auctioneer proposes a chore and an increasing amount ($x_0 < x_1 < \cdots < x_n \cdots \leq M$) of money until when one of the bidders calls stop and accepts the chore.

(2) English auction with negative prices: the auctioneer proposes a chore and a starting amount of money L to the bidders that start bidding lower and lower amounts of money until one of them (the last who makes an offer) stops the descent and gets the chore.

(3) A sort of first price auction with negative prices: the auctioneer proposes a chore, each of the bidders makes a bid and the one who bids less gets the chore.

Really only (1) (very briefly, end of the talk) and (3) (more thoroughly, core of the talk).

Modified auctions

We propose the following modified auction mechanisms where an auctioneer proposes a chore to a set of bidders.

(1) Dutch auction with negative prices: the auctioneer proposes a chore and an increasing amount $(x_0 < x_1 < \cdots < x_n \cdots \leq M)$ of money until when one of the bidders calls stop and accepts the chore.

(2) English auction with negative prices: the auctioneer proposes a chore and a starting amount of money L to the bidders that start bidding lower and lower amounts of money until one of them (the last who makes an offer) stops the descent and gets the chore.

(3) A sort of first price auction with negative prices: the auctioneer proposes a chore, each of the bidders makes a bid and the one who bids less gets the chore.

Really only (1) **(very briefly, end of the talk) and** (3) **(more thoroughly, core of the talk).**

Mechanism performance criteria

As to the **performance criteria** we use:

(1) guaranteed success: the goal is reached in a finite amount of time;

(2) Pareto efficiency: no other outcome where one player is better off and none is worse off;

(3) individual rationality: following the rules on an auction type is in the best interest of the players as well as not to attend an auction;

(4) stability: incentives for the players to behave in a certain way, Nash Equilibria;

(5) simplicity: such a way is easily understandable by players [with bounded rationality].

Mechanism performance criteria

As to the **performance criteria** we use:

(1) guaranteed success: the goal is reached in a finite amount of time;

(2) Pareto efficiency: no other outcome where one player is better off and none is worse off;

(3) individual rationality: following the rules on an auction type is in the best interest of the players as well as not to attend an auction;

(4) stability: incentives for the players to behave in a certain way, Nash Equilibria;

(5) simplicity: such a way is easily understandable by players [with bounded rationality].

Mechanism performance criteria

As to the **performance criteria** we use:

(1) guaranteed success: the goal is reached in a finite amount of time;

(2) Pareto efficiency: no other outcome where one player is better off and none is worse off;

(3) individual rationality: following the rules on an auction type is in the best interest of the players as well as not to attend an auction;

(4) stability: incentives for the players to behave in a certain way, Nash Equilibria;

(5) simplicity: such a way is easily understandable by players [with bounded rationality].

As to the **performance criteria** we use:

(1) guaranteed success: the goal is reached in a finite amount of time;

(2) Pareto efficiency: no other outcome where one player is better off and none is worse off;

(3) individual rationality: following the rules on an auction type is in the best interest of the players as well as not to attend an auction;

(4) stability: incentives for the players to behave in a certain way, Nash Equilibria;

(5) simplicity: such a way is easily understandable by players [with bounded rationality].

Mechanism performance criteria

As to the **performance criteria** we use:

(1) guaranteed success: the goal is reached in a finite amount of time;

(2) Pareto efficiency: no other outcome where one player is better off and none is worse off;

(3) individual rationality: following the rules on an auction type is in the best interest of the players as well as not to attend an auction;

(4) stability: incentives for the players to behave in a certain way, Nash Equilibria;

(5) simplicity: such a way is easily understandable by players [with bounded rationality].

Mechanism performance criteria

As to the **performance criteria** we use:

(1) guaranteed success: the goal is reached in a finite amount of time;

(2) Pareto efficiency: no other outcome where one player is better off and none is worse off;

(3) individual rationality: following the rules on an auction type is in the best interest of the players as well as not to attend an auction;

(4) stability: incentives for the players to behave in a certain way, Nash Equilibria;

(5) simplicity: such a way is easily understandable by players [with bounded rationality].

As to the **performance criteria** we use:

(1) guaranteed success: the goal is reached in a finite amount of time;

(2) Pareto efficiency: no other outcome where one player is better off and none is worse off;

(3) individual rationality: following the rules on an auction type is in the best interest of the players as well as not to attend an auction;

(4) stability: incentives for the players to behave in a certain way, Nash Equilibria;

(5) simplicity: such a way is easily understandable by players [with bounded rationality].

Mechanism design criteria

Traditional **design criteria** include possibility of collusions among bidders, entry deterrence and predation.

Mechanism design criteria

Traditional **design criteria** include possibility of collusions among bidders, entry deterrence and predation.
We do not use these, for theoretical reasons (they are uninfluential).

Mechanism design criteria

Traditional **design criteria** include possibility of collusions among bidders, entry deterrence and predation.
We do not use these, for theoretical reasons (they are uninfluential).
Rather we use:

(1) [auctioneer] strategies for fixing the fee.

(2) [bidders] profitability of bidding untruthfully.

(3) considerations on social welfare.

Mechanism design criteria

Traditional **design criteria** include possibility of collusions among bidders, entry deterrence and predation.
We do not use these, for theoretical reasons (they are uninfluential).
Rather we use:

(1) [auctioneer] strategies for fixing the fee.
(2) [bidders] profitability of bidding untruthfully.
(3) considerations on social welfare.

Mechanism design criteria

Traditional **design criteria** include possibility of collusions among bidders, entry deterrence and predation.
We do not use these, for theoretical reasons (they are uninfluential).
Rather we use:

(1) [auctioneer] strategies for fixing the fee.
(2) [bidders] profitability of bidding untruthfully.
(3) considerations on social welfare.

Mechanism design criteria

Traditional **design criteria** include possibility of collusions among bidders, entry deterrence and predation.
We do not use these, for theoretical reasons (they are uninfluential).
Rather we use:

(1) [auctioneer] strategies for fixing the fee.

(2) [bidders] profitability of bidding untruthfully.

(3) considerations on social welfare.

Mechanism design criteria

Traditional **design criteria** include possibility of collusions among bidders, entry deterrence and predation.
We do not use these, for theoretical reasons (they are uninfluential).
Rather we use:

(1) [auctioneer] strategies for fixing the fee.

(2) [bidders] profitability of bidding untruthfully.

(3) considerations on social welfare.

The framing situation

The mechanism we propose is inspired by the following situation (first price auction with negative prices).

(1) a commissioning authority wants to implement a controversial plant (an incinerator, a dumping ground, a heavy impact industrial plant, a commercial port or a marina or an airport);

(2) the planned infrastructure is something that nobody wants but whose services may be used by a wide group of other authorities that may include also the commissioning authority;

(3) the commissioning authority can identify a certain number of potential contractors (on the basis of technical and economical considerations) over which it has no binding authority but with which it tries to achieve an agreement;

(4) we propose a "negative" approach: according to this approach the potential contractors must take part to an auction and bid so to avoid the auctioned chore or pay the fee so to be excluded from the auction.

The framing situation

The mechanism we propose is inspired by the following situation (first price auction with negative prices).

(1) a commissioning authority wants to implement a controversial plant (an incinerator, a dumping ground, a heavy impact industrial plant, a commercial port or a marina or an airport);

(2) the planned infrastructure is something that nobody wants but whose services may be used by a wide group of other authorities that may include also the commissioning authority;

(3) the commissioning authority can identify a certain number of potential contractors (on the basis of technical and economical considerations) over which it has no binding authority but with which it tries to achieve an agreement;

(4) we propose a "negative" approach: according to this approach the potential contractors must take part to an auction and bid so to avoid the auctioned chore or pay the fee so to be excluded from the auction.

The framing situation

The mechanism we propose is inspired by the following situation (first price auction with negative prices).

(1) a commissioning authority wants to implement a controversial plant (an incinerator, a dumping ground, a heavy impact industrial plant, a commercial port or a marina or an airport);

(2) the planned infrastructure is something that nobody wants but whose services may be used by a wide group of other authorities that may include also the commissioning authority;

(3) the commissioning authority can identify a certain number of potential contractors (on the basis of technical and economical considerations) over which it has no binding authority but with which it tries to achieve an agreement;

(4) we propose a "negative" approach: according to this approach the potential contractors must take part to an auction and bid so to avoid the auctioned chore or pay the fee so to be excluded from the auction.

The framing situation

The mechanism we propose is inspired by the following situation (first price auction with negative prices).

(1) a commissioning authority wants to implement a controversial plant (an incinerator, a dumping ground, a heavy impact industrial plant, a commercial port or a marina or an airport);
(2) the planned infrastructure is something that nobody wants but whose services may be used by a wide group of other authorities that may include also the commissioning authority;
(3) the commissioning authority can identify a certain number of potential contractors (on the basis of technical and economical considerations) over which it has no binding authority but with which it tries to achieve an agreement;
(4) we propose a "negative" approach: according to this approach the potential contractors must take part to an auction and bid so to avoid the auctioned chore or pay the fee so to be excluded from the auction.

The framing situation

The mechanism we propose is inspired by the following situation (first price auction with negative prices).

(1) a commissioning authority wants to implement a controversial plant (an incinerator, a dumping ground, a heavy impact industrial plant, a commercial port or a marina or an airport);
(2) the planned infrastructure is something that nobody wants but whose services may be used by a wide group of other authorities that may include also the commissioning authority;
(3) the commissioning authority can identify a certain number of potential contractors (on the basis of technical and economical considerations) over which it has no binding authority but with which it tries to achieve an agreement;
(4) we propose a "negative" approach: according to this approach the potential contractors must take part to an auction and bid so to avoid the auctioned chore or pay the fee so to be excluded from the auction.

The framing situation

The mechanism we propose is inspired by the following situation (first price auction with negative prices).

(1) a commissioning authority wants to implement a controversial plant (an incinerator, a dumping ground, a heavy impact industrial plant, a commercial port or a marina or an airport);

(2) the planned infrastructure is something that nobody wants but whose services may be used by a wide group of other authorities that may include also the commissioning authority;

(3) the commissioning authority can identify a certain number of potential contractors (on the basis of technical and economical considerations) over which it has no binding authority but with which it tries to achieve an agreement;

(4) we propose a "negative" approach: according to this approach the potential contractors must take part to an auction and bid so to avoid the auctioned chore or pay the fee so to be excluded from the auction.

Basic features (1), the auctioneer ≪**skip**≫

An auctioneer wants to allocate a chore to one of the bidders of a set $\mathscr{B}$.

Basic features (1), the auctioneer ≪**skip**≫

An auctioneer wants to allocate a chore to one of the bidders of a set $\mathscr{B}$.

The auctioneer can:

(1) identify the heaviest/highest priority chore for him to carry out;

(2) identify a set of bidders whom he expects are willing to compete for not getting the chore (but can perform it) and

(3) fix an exclusion fee.

Basic features (1), the auctioneer ≪**skip**≫

An auctioneer wants to allocate a chore to one of the bidders of a set $\mathscr{B}$.

The auctioneer can:

(1) identify the heaviest/highest priority chore for him to carry out;

(2) identify a set of bidders whom he expects are willing to compete for not getting the chore (but can perform it) and

(3) fix an exclusion fee.

Basic features (1), the auctioneer ≪**skip**≫

An auctioneer wants to allocate a chore to one of the bidders of a set $\mathscr{B}$.

The auctioneer can:

(1) identify the heaviest/highest priority chore for him to carry out;

(2) identify a set of bidders whom he expects are willing to compete for not getting the chore (but can perform it) and

(3) fix an exclusion fee.

An auctioneer wants to allocate a chore to one of the bidders of a set $\mathscr{B}$.

The auctioneer can:

(1) identify the heaviest/highest priority chore for him to carry out;

(2) identify a set of bidders whom he expects are willing to compete for not getting the chore (but can perform it) and

(3) fix an exclusion fee.

Basic features (1), the auctioneer ≪skip≫

An auctioneer wants to allocate a chore to one of the bidders of a set $\mathscr{B}$.

The auctioneer can:

(1) identify the heaviest/highest priority chore for him to carry out;

(2) identify a set of bidders whom he expects are willing to compete for not getting the chore (but can perform it) and

(3) fix an exclusion fee.

The auctioneer therefore identifies the bidders, or the n members of $\mathscr{B}$, indexed by a set $N = \{1, \ldots, n\}$ (ex-ante fixed by the auctioneer).

Basic features (2), the bidders ≪**skip**≫

The auctioneer therefore identifies the bidders, or the n members of $\mathscr{B}$, indexed by a set $N = \{1, \ldots, n\}$ (ex-ante fixed by the auctioneer).

The chosen bidders may show two possible behaviors:

(1) pay the exclusion fee so to avoid bidding,

(2) accept to bid and use the best bidding strategy.

Basic features (2), the bidders ≪**skip**≫

The auctioneer therefore identifies the bidders, or the n members of $\mathscr{B}$, indexed by a set $N = \{1, \ldots, n\}$ (ex-ante fixed by the auctioneer).

The chosen bidders may show two possible behaviors:

(1) pay the exclusion fee so to avoid bidding,

(2) accept to bid and use the best bidding strategy.

Basic features (2), the bidders ≪**skip**≫

The auctioneer therefore identifies the bidders, or the n members of $\mathscr{B}$, indexed by a set $N = \{1, \ldots, n\}$ (ex-ante fixed by the auctioneer).

The chosen bidders may show two possible behaviors:

(1) pay the exclusion fee so to avoid bidding,

(2) accept to bid and use the best bidding strategy.

Basic features (2), the bidders ≪**skip**≫

The auctioneer therefore identifies the bidders, or the n members of $\mathscr{B}$, indexed by a set $N = \{1, \ldots, n\}$ (ex-ante fixed by the auctioneer).

The chosen bidders may show two possible behaviors:

(1) pay the exclusion fee so to avoid bidding,

(2) accept to bid and use the best bidding strategy.

Basic features (3), the role and meaning of the fee ≪skip≫

In our mechanism the fee **f** plays the following roles:

(1) allows the members of $\mathscr{B}$ (that have been selected against their will) to escape from the auction (implements "individual rationality");

(2) works as a possible further compensation for the losing bidder (if $m > 0$).

Basic features (3), the role and meaning of the fee ≪skip≫

In our mechanism the fee **f** plays the following roles:

(1) allows the members of $\mathscr{B}$ (that have been selected against their will) to escape from the auction (implements "individual rationality");

(2) works as a possible further compensation for the losing bidder (if $m > 0$).

Basic features (3), the role and meaning of the fee ≪skip≫

In our mechanism the fee **f** plays the following roles:

(1) allows the members of $\mathscr{B}$ (that have been selected against their will) to escape from the auction (implements "individual rationality");

(2) works as a possible further compensation for the losing bidder (if $m > 0$).

Basic features (3), the role and meaning of the fee ≪skip≫

In our mechanism the fee **f** plays the following roles:

(1) allows the members of $\mathscr{B}$ (that have been selected against their will) to escape from the auction (implements "individual rationality");

(2) works as a possible further compensation for the losing bidder (if $m > 0$).

Of n bidders m prefer to pay the fee but $k = n - m$ bidders of $\hat{\mathscr{B}}$ prefer to bid.

Basic features (3), the role and meaning of the fee ≪skip≫

In our mechanism the fee **f** plays the following roles:

(1) allows the members of $\mathscr{B}$ (that have been selected against their will) to escape from the auction (implements "individual rationality");

(2) works as a possible further compensation for the losing bidder (if $m > 0$).

Of n bidders m prefer to pay the fee but $k = n - m$ bidders of $\hat{\mathscr{B}}$ prefer to bid.
If the auction is void the auctioneer must refund the sums he received since he cannot keep them for himself and there is no losing bidder to compensate.

Basic features (3), the role and meaning of the fee ≪skip≫

In our mechanism the fee **f** plays the following roles:

(1) allows the members of $\mathscr{B}$ (that have been selected against their will) to escape from the auction (implements "individual rationality");

(2) works as a possible further compensation for the losing bidder (if $m > 0$).

Of n bidders m prefer to pay the fee but $k = n - m$ bidders of $\hat{\mathscr{B}}$ prefer to bid.

If the auction is void the auctioneer must refund the sums he received since he cannot keep them for himself and there is no losing bidder to compensate.

The basic structure ≪skip≫

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,
(2) each of the bidders b_i bids a sum x_i for not having the chore,
(3) who bids less gets the chore.

The basic structure ≪skip≫

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,

(2) each of the bidders b_i bids a sum x_i for not having the chore,

(3) who bids less gets the chore.

The basic structure ≪skip≫

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,

(2) each of the bidders b_i bids a sum x_i for not having the chore,

(3) who bids less gets the chore.

The basic structure ≪skip≫

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,

(2) each of the bidders b_i bids a sum x_i for not having the chore,

(3) who bids less gets the chore.

The basic structure ≪**skip**≫

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,

(2) each of the bidders b_i bids a sum x_i for not having the chore,

(3) who bids less gets the chore.

b_1 losing bidder (x_1), winning bidders indexed by the set $N_{-1} = N \setminus \{1\}$

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,

(2) each of the bidders b_i bids a sum x_i for not having the chore,

(3) who bids less gets the chore.

b_1 losing bidder (x_1), winning bidders indexed by the set $N_{-1} = N \setminus \{1\}$

$$x_1 = min\{x_i \mid i \in N\} \text{ lowest bid}$$

The basic structure ≪skip≫

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,

(2) each of the bidders b_i bids a sum x_i for not having the chore,

(3) who bids less gets the chore.

b_1 losing bidder (x_1), winning bidders indexed by the set $N_{-1} = N \setminus \{1\}$

$$x_1 = min\{x_i \mid i \in N\} \ \textit{lowest bid}$$

$$X = \sum_{j \in N_{-1}} x_j$$

The basic structure ≪skip≫

The basic structure is the following:

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$,

(2) each of the bidders b_i bids a sum x_i for not having the chore,

(3) who bids less gets the chore.

b_1 losing bidder (x_1), winning bidders indexed by the set $N_{-1} = N \setminus \{1\}$

$$x_1 = min\{x_i \mid i \in N\} \ \ \textit{lowest bid}$$

$$X = \sum_{j \in N_{-1}} x_j$$

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;
(2) each b_i makes his bid x_i,
(3) the auctioneer collects the bids and reveals them once they have all been collected;
(4) the bidder who bid less gets the chore;
(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;

(2) each b_i makes his bid x_i,

(3) the auctioneer collects the bids and reveals them once they have all been collected;

(4) the bidder who bid less gets the chore;

(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;

(2) each b_i makes his bid x_i,

(3) the auctioneer collects the bids and reveals them once they have all been collected;

(4) the bidder who bid less gets the chore;

(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;

(2) each b_i makes his bid x_i,

(3) the auctioneer collects the bids and reveals them once they have all been collected;

(4) the bidder who bid less gets the chore;

(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;
(2) each b_i makes his bid x_i,
(3) the auctioneer collects the bids and reveals them once they have all been collected;
(4) the bidder who bid less gets the chore;
(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;

(2) each b_i makes his bid x_i,

(3) the auctioneer collects the bids and reveals them once they have all been collected;

(4) the bidder who bid less gets the chore;

(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;
(2) each b_i makes his bid x_i,
(3) the auctioneer collects the bids and reveals them once they have all been collected;
(4) the bidder who bid less gets the chore;
(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

Main features of the proposed algorithm:

(f1) the auctioneer has no revenue and no loss but only gets the chore allocated (a benefit whose value does not influence in any way the auction since it is not known by the bidders);
(f2) the bidders are in competition among themselves in order to no get the chore;
(f3) one of the bidders loses the auction and gets the chore but is compensated by the all the other participants for his loss.

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;
(2) each b_i makes his bid x_i,
(3) the auctioneer collects the bids and reveals them once they have all been collected;
(4) the bidder who bid less gets the chore;
(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

Main features of the proposed algorithm:

(f1) the auctioneer has no revenue and no loss but only gets the chore allocated (a benefit whose value does not influence in any way the auction since it is not known by the bidders);
(f2) the bidders are in competition among themselves in order to no get the chore;
(f3) one of the bidders loses the auction and gets the chore but is compensated by the all the other participants for his loss.

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;
(2) each b_i makes his bid x_i,
(3) the auctioneer collects the bids and reveals them once they have all been collected;
(4) the bidder who bid less gets the chore;
(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

Main features of the proposed algorithm:

(f1) the auctioneer has no revenue and no loss but only gets the chore allocated (a benefit whose value does not influence in any way the auction since it is not known by the bidders);
(f2) the bidders are in competition among themselves in order to no get the chore;
(f3) one of the bidders loses the auction and gets the chore but is compensated by the all the other participants for his loss.

The algorithm

(1) the auctioneer presents the chore to the bidders $b_i \in \hat{\mathscr{B}}$ that decided to attend the auction;
(2) each b_i makes his bid x_i,
(3) the auctioneer collects the bids and reveals them once they have all been collected;
(4) the bidder who bid less gets the chore;
(5) the other bidders compensate him for this and the auctioneer gives him the total fee he received from the bidders of the set $\mathscr{B} \setminus \hat{\mathscr{B}}$ (those selected who did not pay the fee).

Main features of the proposed algorithm:

(f1) the auctioneer has no revenue and no loss but only gets the chore allocated (a benefit whose value does not influence in any way the auction since it is not known by the bidders);
(f2) the bidders are in competition among themselves in order to no get the chore;
(f3) one of the bidders loses the auction and gets the chore but is compensated by the all the other participants for his loss.

Compensations

(1) The auctioneer:

(a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;

(b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);

(c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:

(d) either to pay $p_j = \frac{x_j}{X} x_1$,

(e) or to pay x_1 if they belong to the set H (see below).

(f) Winning bidders have an expected loss $0.5 \frac{x_j}{X} x_1 + 0.5 \pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Compensations

(1) The auctioneer:

(a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;

(b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);

(c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:

(d) either to pay $p_j = \frac{x_j}{X} x_1$,

(e) or to pay x_1 if they belong to the set H (see below).

(f) Winning bidders have an expected loss $0.5 \frac{x_j}{X} x_1 + 0.5 \pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Compensations

(1) The auctioneer:

(a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;

(b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);

(c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:

(d) either to pay $p_j = \frac{x_j}{X} x_1$,

(e) or to pay x_1 if they belong to the set H (see below).

(f) Winning bidders have an expected loss $0.5 \frac{x_j}{X} x_1 + 0.5 \pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Compensations

(1) The auctioneer:

- (a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;
- (b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);
- (c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:

- (d) either to pay $p_j = \frac{x_j}{X} x_1$,
- (e) or to pay x_1 if they belong to the set H (see below).
- (f) Winning bidders have an expected loss $0.5 \frac{x_j}{X} x_1 + 0.5 \pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Compensations

(1) The auctioneer:
   (a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;
   (b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);
   (c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:
   (d) either to pay $p_j = \frac{x_j}{X} x_1$,
   (e) or to pay x_1 if they belong to the set H (see below).
   (f) Winning bidders have an expected loss $0.5 \frac{x_j}{X} x_1 + 0.5 \pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Compensations

(1) The auctioneer:

  (a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;
  (b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);
  (c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:

  (d) either to pay $p_j = \frac{x_j}{X} x_1$,
  (e) or to pay x_1 if they belong to the set H (see below).
  (f) Winning bidders have an expected loss $0.5 \frac{x_j}{X} x_1 + 0.5 \pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Compensations

(1) The auctioneer:
  (a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;
  (b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);
  (c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:
  (d) either to pay $p_j = \frac{x_j}{X} x_1$,
  (e) or to pay x_1 if they belong to the set H (see below).
  (f) Winning bidders have an expected loss $0.5 \frac{x_j}{X} x_1 + 0.5 \pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Compensations

(1) The auctioneer:

  (a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;
  (b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);
  (c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:

  (d) either to pay $p_j = \frac{x_j}{X} x_1$,
  (e) or to pay x_1 if they belong to the set H (see below).
  (f) Winning bidders have an expected loss $0.5\frac{x_j}{X}x_1 + 0.5\pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Compensations

(1) The auctioneer:

  (a) may manage the sum $m \times f$ to compensate the losing bidder on behalf of those who preferred to pay;
  (b) may have an incentive to be deceitful as to the amount of fees he received (as in Second Price Auctions);
  (c) may make use of a random device to choose one from two compensation schemes.

(2) The winning bidders (did attend but did not get the chore) may be forced:

  (d) either to pay $p_j = \frac{x_j}{X} x_1$,
  (e) or to pay x_1 if they belong to the set H (see below).
  (f) Winning bidders have an expected loss $0.5\frac{x_j}{X}x_1 + 0.5\pi_j x_1$ where $\pi_j \in \{0, 1\}$ is the characteristic function that says if $j \in H$ or not (H is the set of winning bidders who bid the highest bid x_n, so that $x_n > x_j \forall j \notin H$).

Strategies of the bidders

For each bidder b_i we have:

(1) m_i evaluation of the chore,

(2) x_i current bid,

(3) $x_i - m_i$ bidder's utility.

Strategies of the bidders

For each bidder b_i we have:

(1) m_i evaluation of the chore,

(2) x_i current bid,

(3) $x_i - m_i$ bidder's utility.

Strategies of the bidders

For each bidder b_i we have:

(1) m_i evaluation of the chore,

(2) x_i current bid,

(3) $x_i - m_i$ bidder's utility.

Strategies of the bidders

For each bidder b_i we have:

(1) m_i evaluation of the chore,

(2) x_i current bid,

(3) $x_i - m_i$ bidder's utility.

Strategies of the bidders

For each bidder b_i we have:

(1) m_i evaluation of the chore,

(2) x_i current bid,

(3) $x_i - m_i$ bidder's utility.

For every bidder b_i bidding $x_i = m_i$ is the best strategy.

Strategies of the bidders

For each bidder b_i we have:

(1) m_i evaluation of the chore,

(2) x_i current bid,

(3) $x_i - m_i$ bidder's utility.

For every bidder b_i bidding $x_i = m_i$ is the best strategy.

The intuition is the following. Making a bid x_i lower that m_i is not convenient to b_i since if he loses the auction and gets the chore he may get a low compensation, lower than his evaluation of the chore.

For each bidder b_i we have:

(1) m_i evaluation of the chore,

(2) x_i current bid,

(3) $x_i - m_i$ bidder's utility.

For every bidder b_i bidding $x_i = m_i$ is the best strategy.

The intuition is the following. Making a bid x_i lower that m_i is not convenient to b_i since if he loses the auction and gets the chore he may get a low compensation, lower than his evaluation of the chore.

On the other hand if he makes a bid higher than m_i he is more secure he will not lose the auction but he can run a winner's course like risk: he can be compelled to compensate the loser with a sum of money higher than his evaluation of the chore m_i (so it would have been better for him to get the chore). From this we conclude that each bidder should choose to bid a sum $x_i = m_i$.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.

(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.

(3) Individual rationality is implemented through the mechanism of the fee.

(4) Stability: the best strategy for each bidder is to bid his own evaluation.

(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.

(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.

(3) Individual rationality is implemented through the mechanism of the fee.

(4) Stability: the best strategy for each bidder is to bid his own evaluation.

(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.

(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.

(3) Individual rationality is implemented through the mechanism of the fee.

(4) Stability: the best strategy for each bidder is to bid his own evaluation.

(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.

(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.

(3) Individual rationality is implemented through the mechanism of the fee.

(4) Stability: the best strategy for each bidder is to bid his own evaluation.

(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.

(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.

(3) Individual rationality is implemented through the mechanism of the fee.

(4) Stability: the best strategy for each bidder is to bid his own evaluation.

(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.
(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.
(3) Individual rationality is implemented through the mechanism of the fee.
(4) Stability: the best strategy for each bidder is to bid his own evaluation.
(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.

(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.

(3) Individual rationality is implemented through the mechanism of the fee.

(4) Stability: the best strategy for each bidder is to bid his own evaluation.

(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Design criteria.

(1) Proper value of the fee: not too low (otherwise all bidders can pay). The higher the better for the auctioneer but not for the bidders (no extra compensation for the losing bidder).

(2) Social welfare, next slide.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.
(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.
(3) Individual rationality is implemented through the mechanism of the fee.
(4) Stability: the best strategy for each bidder is to bid his own evaluation.
(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Design criteria.

(1) Proper value of the fee: not too low (otherwise all bidders can pay). The higher the better for the auctioneer but not for the bidders (no extra compensation for the losing bidder).
(2) Social welfare, next slide.

Performance and design criteria satisfaction

Performance criteria.

(1) Termination guaranteed but not success (void auction). If fee properly fixed then guaranteed success.

(2) Pareto efficiency: all the bidders are satisfied and there is no solution where one is better off and none is worse off.

(3) Individual rationality is implemented through the mechanism of the fee.

(4) Stability: the best strategy for each bidder is to bid his own evaluation.

(5) Simplicity: this strategy can be easily implemented also by bidders with bounded rationality.

Design criteria.

(1) Proper value of the fee: not too low (otherwise all bidders can pay). The higher the better for the auctioneer but not for the bidders (no extra compensation for the losing bidder).

(2) Social welfare, next slide.

A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).

(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.

(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.

(4) If we consider the complete set of bidders we have:

(a) those who pay the fee suffer a collective loss of $m \times f$,

(b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,

(c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$.

A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).

(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.

(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.

(4) If we consider the complete set of bidders we have:

   (a) those who pay the fee suffer a collective loss of $m \times f$,
   (b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,
   (c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$.

A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).

(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.

(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.

(4) If we consider the complete set of bidders we have:
   (a) those who pay the fee suffer a collective loss of $m \times f$,
   (b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,
   (c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$.

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).

(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.

(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.

(4) If we consider the complete set of bidders we have:

   (a) those who pay the fee suffer a collective loss of $m \times f$,

   (b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,

   (c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$.

A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).

(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.

(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.

(4) If we consider the complete set of bidders we have:
   (a) those who pay the fee suffer a collective loss of $m \times f$,
   (b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,
   (c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$,

A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).
(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.
(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.
(4) If we consider the complete set of bidders we have:
    (a) those who pay the fee suffer a collective loss of $m \times f$,
    (b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,
    (c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$,

A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).
(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.
(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.
(4) If we consider the complete set of bidders we have:
    (a) those who pay the fee suffer a collective loss of $m \times f$,
    (b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,
    (c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$,

A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).
(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.
(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.
(4) If we consider the complete set of bidders we have:
   (a) those who pay the fee suffer a collective loss of $m \times f$,
   (b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,
   (c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$,

A few notes on social welfare

Situation before the auction and that after the auction. Before the auctioneer and every bidder have a welfare w_i. Then we examine the situation after the auction.

(1) Auction not void: the welfare of the auctioneer can only increase (he succeeds in allocating a chore at no cost, gets a benefit and suffers no loss of any kind).

(2) Auction void: he is worse off, incentives to choose properly the bidders and in fixing properly the exclusion fee.

(3) Losing bidder: is best off if at least one bidder pays the exclusion fee is no worse off (if $|H|=1$) otherwise.

(4) If we consider the complete set of bidders we have:
   - (a) those who pay the fee suffer a collective loss of $m \times f$,
   - (b) those who bid suffer a collective loss of $\sum_{i=2}^{k} E[i]$,
   - (c) the losing bidder has an expected utility given by $E[1] = m \times f + \sum_{i=2}^{k} E[i] - m_1$,

   so that the complete set of bidders is worse off by m_1 that, anyway, is the less they can lose since $m_1 < m_i$.

Extensions ≪skip≫

(1) More than one losing bidder L:

- (1a) use a random device to choose on of them (back to the lone loser case);
- (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:

- (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
- (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:
  - (a) the auctioneer evaluates q and r such that $c = qn + r$;
  - (b) he performs q times the algorithm, each time with n initial bidders as before;
  - (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

(1) More than one losing bidder L:
   (1a) use a random device to choose on of them (back to the lone loser case);
   (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:
   (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
   (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:

Extensions ≪**skip**≫

(1) More than one losing bidder L:

- (1a) use a random device to choose on of them (back to the lone loser case);
- (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:

- (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
- (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:
  - (a) the auctioneer evaluates q and r such that $c = qn + r$;
  - (b) he performs q times the algorithm, each time with n initial bidders as before;
  - (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

Extensions ≪skip≫

(1) More than one losing bidder L:
   (1a) use a random device to choose on of them (back to the lone loser case);
   (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:
   (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
   (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:

Extensions ≪skip≫

(1) More than one losing bidder L:
  (1a) use a random device to choose on of them (back to the lone loser case);
  (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:
  (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
  (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:
    (a) the auctioneer evaluates q and r such that $c = qn + r$;
    (b) he performs q times the algorithm, each time with n initial bidders as before;
    (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

Extensions ≪skip≫

(1) More than one losing bidder L:
  - (1a) use a random device to choose on of them (back to the lone loser case);
  - (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:
  - (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
  - (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:
    - (a) the auctioneer evaluates q and r such that $c = qn + r$;
    - (b) he performs q times the algorithm, each time with n initial bidders as before;
    - (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

Extensions ≪skip≫

(1) More than one losing bidder L:
  (1a) use a random device to choose on of them (back to the lone loser case);
  (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:
  (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
  (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:
    (a) the auctioneer evaluates q and r such that $c = qn + r$;
    (b) he performs q times the algorithm, each time with n initial bidders as before;
    (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

(1) More than one losing bidder L:
  (1a) use a random device to choose on of them (back to the lone loser case);
  (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:
  (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
  (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:
    (a) the auctioneer evaluates q and r such that $c = qn + r$;
    (b) he performs q times the algorithm, each time with n initial bidders as before;
    (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

(1) More than one losing bidder L:
  (1a) use a random device to choose on of them (back to the lone loser case);
  (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:
  (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
  (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:
    (a) the auctioneer evaluates q and r such that $c = qn + r$;
    (b) he performs q times the algorithm, each time with n initial bidders as before;
    (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

Extensions ≪skip≫

(1) More than one losing bidder L:
  (1a) use a random device to choose on of them (back to the lone loser case);
  (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:
  (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.
  (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:
    (a) the auctioneer evaluates q and r such that $c = qn + r$;
    (b) he performs q times the algorithm, each time with n initial bidders as before;
    (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

Extensions ≪skip≫

(1) More than one losing bidder L:

  (1a) use a random device to choose on of them (back to the lone loser case);

  (1b) set up an auction among the bidders of L so to choose a single loser.

(2) To allocate a set of chores $\mathscr{C}$ to a set of bidders $\mathscr{B}$:

  (2a) $|\mathscr{C}| = c \leq n$ (with $n = |N|$) it is possible to use c rounds to allocate at the most one chore to each bidder so that a bidder who gets a chore at step k exits the allocation process but not the compensation phase.

  (2b) $|\mathscr{C}| = c > n$ there are necessarily bidders who get more than one chore. Proposed algorithm:

    (a) the auctioneer evaluates q and r such that $c = qn + r$;

    (b) he performs q times the algorithm, each time with n initial bidders as before;

    (c) the remaining r chores are allocated with one more execution reserved to the r bidders who got the r lower total sums of chore values.

Reverse auction: paying more and more to allocate a chore

The auctioneer offers the chore and a sum of money and raises the offer (up to an upper bound M) until when one of the bidders accepts it and gets both the chore and the money.

Reverse auction: paying more and more to allocate a chore

The auctioneer offers the chore and a sum of money and raises the offer (up to an upper bound M) until when one of the bidders accepts it and gets both the chore and the money.
M private information of the auctioneer, not known by the bidders.
x current offer, utility $M - x$ for the auctioneer.

Reverse auction: paying more and more to allocate a chore

The auctioneer offers the chore and a sum of money and raises the offer (up to an upper bound M) until when one of the bidders accepts it and gets both the chore and the money.
M private information of the auctioneer, not known by the bidders.
x current offer, utility $M - x$ for the auctioneer.
Each bidder has the minimum sum he is willing to accept m_i (private data) so that $x - m_i$ may be seen as a measure of the utility of bidder b_i.

Reverse auction: paying more and more to allocate a chore

The auctioneer offers the chore and a sum of money and raises the offer (up to an upper bound M) until when one of the bidders accepts it and gets both the chore and the money.
M private information of the auctioneer, not known by the bidders.
x current offer, utility $M - x$ for the auctioneer.
Each bidder has the minimum sum he is willing to accept m_i (private data) so that $x - m_i$ may be seen as a measure of the utility of bidder b_i.
$F = \{i \mid m_i \leq M\}$, feasible set, the problem may have a solution only if $F \neq \emptyset$.

Reverse auction: paying more and more to allocate a chore

The auctioneer offers the chore and a sum of money and raises the offer (up to an upper bound M) until when one of the bidders accepts it and gets both the chore and the money.

M private information of the auctioneer, not known by the bidders.

x current offer, utility $M - x$ for the auctioneer.

Each bidder has the minimum sum he is willing to accept m_i (private data) so that $x - m_i$ may be seen as a measure of the utility of bidder b_i.

$F = \{i \mid m_i \leq M\}$, feasible set, the problem may have a solution only if $F \neq \emptyset$.

Reverse auction, the algorithm

(1) The auctioneer **a** starts the game with a starting offer $x = x_0 < M$;

(2) bidders b_i may either accept (by calling "stop") or refuse;

(3) if one b_i accepts the auction is over, go to (5);

(4) if none accepts and $x < M$ then **a** rises the offer as $x = x + \delta$ with $0 < \delta < M - x$ (residual utility), go to (3) otherwise go to (5);

(5) end.

Reverse auction, the algorithm

(1) The auctioneer **a** starts the game with a starting offer $x = x_0 < M$;

(2) bidders b_i may either accept (by calling "stop") or refuse;

(3) if one b_i accepts the auction is over, go to (5);

(4) if none accepts and $x < M$ then **a** rises the offer as $x = x + \delta$ with $0 < \delta < M - x$ (residual utility), go to (3) otherwise go to (5);

(5) end.

Reverse auction, the algorithm

(1) The auctioneer **a** starts the game with a starting offer $x = x_0 < M$;

(2) bidders b_i may either accept (by calling "stop") or refuse;

(3) if one b_i accepts the auction is over, go to (5);

(4) if none accepts and $x < M$ then **a** rises the offer as $x = x + \delta$ with $0 < \delta < M - x$ (residual utility), go to (3) otherwise go to (5);

(5) end.

Reverse auction, the algorithm

(1) The auctioneer **a** starts the game with a starting offer $x = x_0 < M$;

(2) bidders b_i may either accept (by calling "stop") or refuse;

(3) if one b_i accepts the auction is over, go to (5);

(4) if none accepts and $x < M$ then **a** rises the offer as $x = x + \delta$ with $0 < \delta < M - x$ (residual utility), go to (3) otherwise go to (5);

(5) end.

Reverse auction, the algorithm

(1) The auctioneer **a** starts the game with a starting offer $x = x_0 < M$;

(2) bidders b_i may either accept (by calling "stop") or refuse;

(3) if one b_i accepts the auction is over, go to (5);

(4) if none accepts and $x < M$ then **a** rises the offer as $x = x + \delta$ with $0 < \delta < M - x$ (residual utility), go to (3) otherwise go to (5);

(5) end.

Reverse auction, the algorithm

(1) The auctioneer **a** starts the game with a starting offer $x = x_0 < M$;

(2) bidders b_i may either accept (by calling "stop") or refuse;

(3) if one b_i accepts the auction is over, go to (5);

(4) if none accepts and $x < M$ then **a** rises the offer as $x = x + \delta$ with $0 < \delta < M - x$ (residual utility), go to (3) otherwise go to (5);

(5) end.

Reverse auction, the strategies

At this point we have to define the strategies of both **a** and the b_i.

(1) The best strategy of **a** is to use a low value of x_0 and, at each step, to rise it of a small fraction δ with the rate of increment of δ decreasing the more x approaches M.

(2) The bidder b_i's best strategy is to refuse any offer that is lower than m_i and to accept when $x = m_i$ since if he refuses that price he risks to lose the auction in favor of another bidder who accepts that offer.

(3) b_i may use a higher value of $m_i' > m_i$ only if he is sure that the private values of all the other bidders are higher. Since no bidder can be sure of this, each of them has a strong incentive to behave truthfully.

Reverse auction, the strategies

At this point we have to define the strategies of both **a** and the b_i.

(1) The best strategy of **a** is to use a low value of x_0 and, at each step, to rise it of a small fraction δ with the rate of increment of δ decreasing the more x approaches M.

(2) The bidder b_i's best strategy is to refuse any offer that is lower than m_i and to accept when $x = m_i$ since if he refuses that price he risks to lose the auction in favor of another bidder who accepts that offer.

(3) b_i may use a higher value of $m_i' > m_i$ only if he is sure that the private values of all the other bidders are higher. Since no bidder can be sure of this, each of them has a strong incentive to behave truthfully.

Reverse auction, the strategies

At this point we have to define the strategies of both **a** and the b_i.

(1) The best strategy of **a** is to use a low value of x_0 and, at each step, to rise it of a small fraction δ with the rate of increment of δ decreasing the more x approaches M.

(2) The bidder b_i's best strategy is to refuse any offer that is lower than m_i and to accept when $x = m_i$ since if he refuses that price he risks to lose the auction in favor of another bidder who accepts that offer.

(3) b_i may use a higher value of $m'_i > m_i$ only if he is sure that the private values of all the other bidders are higher. Since no bidder can be sure of this, each of them has a strong incentive to behave truthfully.

Reverse auction, the strategies

At this point we have to define the strategies of both **a** and the b_i.

(1) The best strategy of **a** is to use a low value of x_0 and, at each step, to rise it of a small fraction δ with the rate of increment of δ decreasing the more x approaches M.

(2) The bidder b_i's best strategy is to refuse any offer that is lower than m_i and to accept when $x = m_i$ since if he refuses that price he risks to lose the auction in favor of another bidder who accepts that offer.

(3) b_i may use a higher value of $m_i' > m_i$ only if he is sure that the private values of all the other bidders are higher. Since no bidder can be sure of this, each of them has a strong incentive to behave truthfully.

At this point we have to define the strategies of both **a** and the b_i.

(1) The best strategy of **a** is to use a low value of x_0 and, at each step, to rise it of a small fraction δ with the rate of increment of δ decreasing the more x approaches M.

(2) The bidder b_i's best strategy is to refuse any offer that is lower than m_i and to accept when $x = m_i$ since if he refuses that price he risks to lose the auction in favor of another bidder who accepts that offer.

(3) b_i may use a higher value of $m'_i > m_i$ only if he is sure that the private values of all the other bidders are higher. Since no bidder can be sure of this, each of them has a strong incentive to behave truthfully.

References

Steven J. Brams and Alan D. Taylor. *Fair division. From cake-cutting to dispute resolution*. Cambridge University Press, 1996.

Vito Fragnelli. *Teoria dei Giochi. Parte Prima*. Materiali didattici, internet version, in Italian, 2005.

Paul Klemperer. What Really Matters in Auction Design. *Journal of Economic Perspectives*, 16(1):169–189, 2002. Internet version.

Paul Klemperer. A Survey of uction Theory. *Journal of Economic Surveys*, 13(3):227–286, 1999. Internet version.

Roger B. Myerson. *Game Theory. Analysis of conflict*. Harvard University Press, 1991.

Fioravate Patrone. *Decisori (razionali) interagenti. Una introduzione alla teoria dei giochi*. Edizioni plus, 2006.

Anatol Rapoprt. *Decision Theory and Decision Behaviour. Normative and Decisive Approaches*. Kluver Academic Publishers, 1989.

Michael Wooldridge. *An Introduction to MultiAgent Systems*. John Wiley and Sons, 2002.

The end, at last!!

The end, at last!!

What a grand big trip!!