Barter models (WIP)

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Introduction

We present a family of models that involve:

- a **pair of actors** that aim at
- **bartering** goods from two **privately** owned **heterogeneous pools**.
  - We describe four "basic" models:
    - one-to-one barter model
    - one-to-many barter model
    - many-to-one barter model
    - many-to-many barter model
  - and two "variations on the theme":
    - pure model: nobody shows, hidden items
    - mixed model: shown items, hidden items

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Basic criteria, classical definitions (1)

The starting point is to have fair barters.

- Fairness is measured in function of:
  - (1) envy-freeness,
  - (2) proportionality,
  - (3) equitability,
  - (4) efficiency.
- It is easy to show how in case of two actors envy-freeness and proportionality represent equivalent properties whereas for more than two players envy-freeness $\Rightarrow$ proportionality but not vice versa.

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Basic criteria, classical definitions (2)

(1) Envy-freeness:
- none of the actors involved in an agreement would prefer somebody's else portion, how it derives to him from the agreement, to his own.

(2) Proportionality:
- each of the n players thinks to have received at least $1/n$ of the total value.

(3) Equitability:
- each players thinks to have received the same fraction of the total value of the goods to be allocated.

(4) Efficiency:
- there is no other allocation where one of the players is better off and none of them is worse off.

- Such criteria must be adapted/redefined someway so to be in agreement with their classical definitions.

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For n=2 we want to maintain equivalence between envy-freeness and proportionality.
a_A and l_A denote the values for A himself, respectively, of what A gets and loses from the barter. The same for B.

- Envy-freeness

$$\frac{a_A}{l_A} \geq 1$$

$$\frac{a_B}{l_B} \geq 1$$

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In our models, if a barter actually occurs it is guaranteed to be envy-free.

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$$\frac{a_A}{a_A + l_A} \geq \frac{l_A}{a_A + l_A}$$

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Basic criteria, revisited definitions (3)

I and I' are the ex-ante and ex-post sets of goods of A, J and J' are the ex-ante and ex-post sets of goods of B. If (i, j) denotes the bartered goods in a one-to-one barter, we have:

$$I' = I \setminus \{i\} \cup \{j\}$$

$$J' = J \setminus \{j\} \cup \{i\}$$

On these sets we define for player A the pair $v_A(I')$ and $s_A(J')$ and for player B the pair $v_B(J')$ and $s_B(I')$.

- Equitability for A

$$\frac{v_A(j)}{v_A(I')} \geq \frac{s_A(i)}{s_A(J')}$$

- Equitability for B

$$\frac{v_B(i)}{v_B(J')} \geq \frac{s_B(j)}{s_B(I')}$$

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If both relations hold we say that the barter is equitable.

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We define a barter as equitable for A himself if the percentage value of what he gets is at least equal to the percentage value he gives to what B gets from the barter.

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As to **efficiency** a barter is efficient (Pareto efficient) if there is no other allocations that makes one of the players better off and the other no worse off.

- Efficiency for A of (I_0, J_0) (with l_A and a_A). There is no (I'_0, J'_0) (with l'_A and a'_A) such that:

$$\frac{a_A}{l_A} < \frac{a'_A}{l'_A}$$

- Efficiency for B of (I_0, J_0) (with l_B and a_B). There is no (I'_0, J'_0) (with l'_B and a'_B) such that:

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- if both players get $\frac{a_{A_{max}}}{l_{A_{min}}}$ and $\frac{a_{B_{max}}}{l_{B_{min}}}$ we have an efficient barter whereas if they get $\frac{a_{A_{min}}}{l_{A_{max}}}$ and $\frac{a_{B_{min}}}{l_{B_{max}}}$ the barter is surely inefficient

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Some [more or less] related "classical" solutions

We list here some "classical" solutions it is possible to find in the literature:

(1) strict alternation
(2) balanced alternation
(3) divide and choose
(4) filter and choose
(5) adjusted winner
(6) market games
(7) assignment games
(8) cost games
(9) and [surely] many others ...

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The basic motivation

(1) To describe how an exchange of goods can happen without the intervention of a transferable utility such that represented by money or by any other numerary good.

(2) The actors share only the will to propose pool of goods that they present each other so to perform some barters.

(3) All barters are in kind (simplest case):

  (3.1) the two actors show each other the goods,
  (3.2) each of them chooses one of the goods of the other,
  (3.3) barter or rearrangement and repetition or give up.

(4) Approach more descriptive than normative: no more or less detailed) recipes through which players can attain their best outcomes.

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(2) The actors share only the will to propose pool of goods that they present each other so to perform some barters.

(3) All barters are in kind (simplest case):

  (3.1) the two actors show each other the goods,

  (3.2) each of them chooses one of the goods of the other,

  (3.3) barter or rearrangement and repetition or give up.

(4) Approach more descriptive than normative: no more or less detailed) recipes through which players can attain their best outcomes.

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Some assumptions

(1) The values of the goods cover two overlapping intervals so that a one shot barter is always possible (at least theoretically);
(2) Such goods and the associated values are chosen privately by each actor without any information on the goods and associated values of the other actor;
(3) Such values are fixed and cannot be changed as a function of the request from the other actor;
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To avoid interpersonal comparisons and the use of a common scale we can proceed as follows: we let the two players show each other their goods and ask separately to each of them if he thinks the goods of the other are worth bartering. If both answer affirmatively we are sure that such interval exists otherwise we cannot be sure of its existence. Anyway the bartering process can go on, though with a lower possibility of successful termination.

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Players' private information

We have:

(1) an actor A with a pool $I = \{i_1, \ldots, i_n\}$ of n heterogeneous goods,

(2) an actor B with a pool $J = \{j_1, \ldots, j_m\}$ of m heterogeneous goods,

(3) A assigns a vector v_A of n values to his goods in I and this vector is fixed and cannot be modified,

(4) B assigns a vector v_B of m values to her goods in J and this vector is fixed and cannot be modified

(5) A has a vector s_A of m values of the appraisals of the goods of B from A,

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The basic hypotheses are:

(H1) we are in an additivity case so the value of any set is the sum of the values of its elements,

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We have four types of barter:

(1) **one-to-one** or one good for one good;

(2) **one-to-many** or one good for a basket of goods;

(3) **many-to-one** or a basket of goods for one good;

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Barter models

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The second and the third case are really two symmetric cases.

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A barter involves a pair (i, j) with $i \in I$ and $j \in J$.

One-to-one barter (2)

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Basic facts and relations:

(1) A has a gain $s_A(j)$ but suffers a loss $v_A(i)$,

(2) B has a gain $s_B(i)$ but suffers a loss $v_B(j)$,

(3)
$$u_A(i,j) = s_A(j) - v_A(i)$$

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(5)
$$\textbf{if}(u_A \geq 0) \textbf{ then } accept_A \textbf{ else } refuse_A$$

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One-to-one barter, simultaneous requests

(1) both A and B show each other their goods;
(2) both players negotiate if the barter is [still] possible or not:
   (a) if it is not possible go to step (6);
   (b) if it is possible continue;
(3) both simultaneously perform their choice;
(4) when choices have been made and revealed both may say if each accepts or refuses:
(5) we have the following cases:
   (a) if ($accept_A$ and $accept_B$) then go to (6);
   (b) if ($refuse_A$ and $accept_B$ then)
      (i) either A performs $I \setminus \{i\}$ and if $I \neq \emptyset$ go to step (2) else go to step (6);
      (ii) or A only performs a new choice then go to step (4);
   (c) if ($accept_A$ and $refuse_B$ then)
      (i) either B performs $J \setminus \{j\}$ and if $J \neq \emptyset$ go to step (2) else go to step (6);
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   (d) if ($refuse_A$ and $refuse_B$ then)
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  (c) if ($accept_A$ and $refuse_B$ then)

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(6) end of the barter.

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(4) 1 reveals his choice $i_2 \in I_2$;

(5) 2 can now perform an evaluation of all his possibilities;

(6) if 2 refuses he takes i_2 off his barter set then go to (2);

(7) if 2 accepts he can reveal his choice $i_1 \in I_1$;

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   (a) if ($accept_1$ and $accept_2$) go to step (10);
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   (b) if ($refuse_1$ and $accept_2$) then either 1 performs $I_1 = I_1 \setminus \{i_1\}$ and if $I_1 \neq \emptyset$ go to (2) else go to (10) or 1 only performs and reveals a new choice and then go to (8);

(10) end of the barter.

One-to-one barter, sequential requests

(1) both A and B show each other their goods;

(2) both players negotiate if the barter is [still] possible or not:
  (a) if it is not possible go to step (10);
  (b) if it is possible continue;

(3) there is a chance move to decide who moves first;

(4) 1 reveals his choice $i_2 \in I_2$;

(5) 2 can now perform an evaluation of all his possibilities;

(6) if 2 refuses he takes i_2 off his barter set then go to (2);

(7) if 2 accepts he can reveal his choice $i_1 \in I_1$;

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One-to-many and many-to-one barters

(1) One good versus a basket of goods.

(2) This kind of barter must be agreed on by both actors and can occur only if one of the two actor offers a large pool of "light" goods whereas the other offers a small pool of "heavy" goods.

(3) Otherwise they may decide either to give up (so the bather process neither starts) or to switch to a one-to-one barter or to switch to a many-to-many barter.

(4) A owns "light" goods and requires a single good $j \in J$,

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The two requests may be either simultaneous or sequential.
If we have simultaneous requests both actors can evaluate their respective utilities, soon after the requests have been revealed, as:

(1) $u_A(\hat{I}_0, j) = s_A(j) - v_A(\hat{I}_0)$
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(A) In the case of simultaneous requests the barter goes on as in the *one − to − one* case with simultaneous requests.
(B) In the case of sequential requests the procedure does not use a chance move to assign one of the two actors the right to move first but gives this right to the actor that owns the pool of “heavy” goods. After this first move the barter goes on as in the *one − to − one* case with sequential requests.

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Many-to-many barter

Main facts:

(1) Also this kind of barter must be agreed on by both actors during a pre-barter phase.

(2) A requires $\hat{J}_0 \subset J$

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(4) Differences: use of subsets, management of double refusals, philosophy.

  (4.a) (After the first double refusal) use of a partitioning of the goods in lots each player is disposed to barter,

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The use of the models: disclosing the metaphor

(1) "Positive" framework: both A and B offer goods or positive externalities. In this case both A and B propose what they are almost sure the other will be willing to accept. We note here that what A thinks is a good for B may be a good or have no value or even be a bad for A himself and the same holds also for B.

(2) "Negative" framework: both A and B present bads or chores. In this case we have that A asks B to accept some bads or to carry out some chores in exchange for other bads or chores that B asks A to accept or to carry out. We note here that what A thinks is a bad/chore for B usually is a bad/chore for A himself and the same holds also for B.

(3) "Mixed" framework: goods and bads/chores can be mixed in any proportion. We imagine the following cases:

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In these cases we have an exchange of items where each actors tend to maximize the goods and minimize the bads/chores he/she obtains.

In practice there can be two solutions:

(A) both A and B splits their pools in two subsets, each containing only goods or bads/chores and negotiate separately on them as in the "pure" frameworks;

(B) A and B agree on a many-to-many barter so to be able to obtain more or less balanced subsets of goods and bads/chores.

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- (B) A and B agree on a many-to-many barter so to be able to obtain more or less balanced subsets of goods and bads/chores.

Fairness of the proposed solutions

In the models we have seen so far:

(1) Envy-freeness is guaranteed every time a barter occurs,

(2) Proportionality is guaranteed every time a barter occurs,

(3) Equitability must be verified case by case,

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In conclusion fairness is a by-product of the barter process and is not a-priori guaranteed by its structure.

Hidden goods: alternating requests

All the models we have seen so far are based on the following common structure:

(1) both players show each other the goods they want to barter;

(2) both agree on the type of barter they are going to have;

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Two more models:

(A) "pure model"

(A1) none of the players show anything to the other,
(A2) one shot one-to-one barter model with successive requests;

(B) "mixed" model

(B1) only one of the two players, say A, shows his goods;
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Pure model: nobody shows, hidden items

We have the following situation:

(1) each player in turn proposes a barter (i, j),

(2) each player receiving a proposal may accept, refuse or answer with a counterproposal,

(3) players use functions $eval_A(i, j)$ and $eval_B(i, j)$ to:

(3.1) accept or refuse a proposal according to rules such as:

$$\textbf{if}(eval_A(i, j) \geq 0) \textbf{ then } accept_A \textbf{ else } refuse_A$$

(3.2) establish a strict preference ordering on the proposals:

$$(i, j) \succ_A (i', j') \Leftrightarrow eval_A(i, j) > eval_A(i', j')$$

(4) Use of "history of proposals" to devise new proposals:

(5) for A I_i is the history of his proposals from the root up to that node along that single path,

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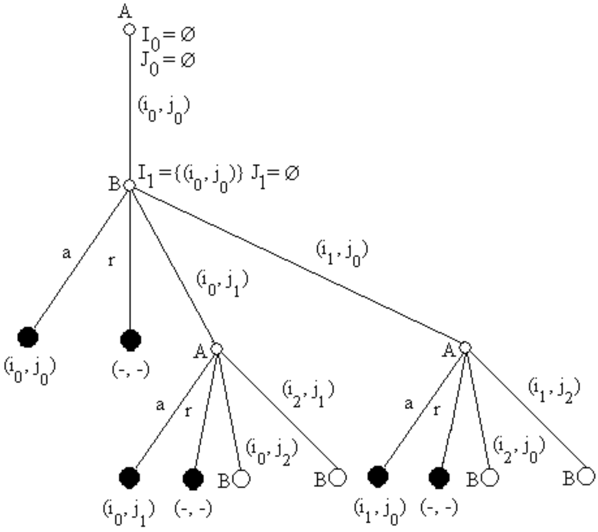
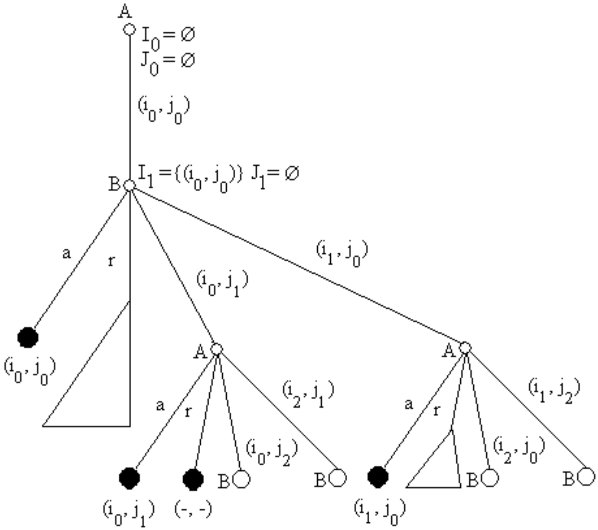
Pure model: nobody shows, hidden items (1)

Part of the barter or "decision tree"



Pure model: nobody shows, hidden items (2)

Modified "decision tree"



Pure model: nobody shows, hidden items (3)

We have therefore identified the following strategies:

(A) **A-conservative** where i_0 is kept whereas the[1] B-side of the barter changes at each step,

(B) **B-conservative** where j_0 is kept whereas the A-side of the barter changes at each step,

(C) **mixed** where at each step both components of a proposal can change.

[1] Given a barter proposal (i, j) we say i the A-side and j the B-side of the barter.

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and such threads can, at least theoretically, last forever.

Each, but not necessarily every, refusal move can be replaced with a completely new barter process where one player implicitly refuses and closes one barter but both players can open a new one by giving a new proposal to the other player. In this way the two players that cannot agree on a line of bartering can change line so to try to reach an agreement starting with a completely different barter proposal. This case cannot, however, be seen as a case of consecutive barters since at the most we can have one successful barter.

[1] Given a barter proposal (i, j) we say i the A-side and j the B-side of the barter.

Mixed model: shown items, hidden items

A shows his goods and B tries to get one or more of them by giving one of her goods to A.
The goods of A are common knowledge between the two players and we have:

(1) A assigns to each of the n goods of his set $I = \{i_1, \ldots, i_n\}$ a value $v_A(i)$;

(2) B assigns to each of the n goods of this set I a value $s_B(i)$;

(3) B knows the value of all her (hidden to A) goods $j \in J$, $v_B(j)$;

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At the very start of the algorithm we have that:

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Mixed model: shown items, hidden items

A shows his goods and B tries to get one or more of them by giving one of her goods to A.
The goods of A are common knowledge between the two players and we have:

(1) A assigns to each of the n goods of his set $I = \{i_1, \ldots, i_n\}$ a value $v_A(i)$;
(2) B assigns to each of the n goods of this set I a value $s_B(i)$;
(3) B knows the value of all her (hidden to A) goods $j \in J$, $v_B(j)$;
(4) A can evaluate (as $s_A(j)$) the single goods of B only after she has made one of her proposals.

At the very start of the algorithm we have that:

(1) A knows his set of goods, I;
(2) A has no idea of the set of goods of B, $J_0 = \emptyset$;
(3) B knows her set of goods, hidden to A, J;
(4) B knows the set of goods of A, I.

Mixed model: shown items, hidden items

The main steps of the algorithm are the followings:

(1) A shows his goods I;

(2) B propose a barter (i_0, j_0) with $i_0 \in I$;

(3) A has the following possibilities:

   (3.1) accept so that the barter occurs,

   (3.2) refuse,

   (3.3) propose a barter (i_1, j_0),

   (3.4) if $J_0 \setminus \{j_0\} \neq \emptyset$ propose (i_0, j_1) with $j_1 \in J_0$.

(4) and so on. . .

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(4) and so on...

A can use the set of B's revealed proposals to create an history of proposals through which he can reply to a proposal of B that is judged unacceptable. In this way B allows A to carry out the barter as in the case where both show each other their goods but for the fact that A is "one step back" since can update the set J_0 only after B has made his proposal.

A refusal may represent for both players an opportunity to start a new barter process with a new proposal that can be built using past proposals of both players.

Extensions, listing

The basic extensions of the proposed models involve essentially:

(1) the possibility of repeated barters between two actors;

(2) the possibility that more than two actors are involved in the barter;

(3) repeated barters involving more than two actors;

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Concluding remarks and future plans

What we have seen: some barter models between two actors that executes a one shot barter through which they exchange the goods of two separate and privately owned pools.
Future plans:

(1) to examine more formally the basic model of one shot barter with all its variants;
(2) to improve the algorithms of the various proposed solutions;
(3) to examine the properties of such solutions and their plausibility;
(4) to develop more thoroughly the pure and mixed models;
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