The lion's share. Is fairness easy? (WIP)

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Introduction

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Note: this talk will be in PPEnglish (Personal Pisa English, a personal dialect of the English spoken in Pisa, no simultaneous translation service is available).

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  (a) half an orange each (how many ways? how many cuts?);
  (b) pulp (for a juice) and peel (as an ingredient for a cake);
  (c) as a set of segments, half (? odd cardinality ?) a set each;
  (d) conflicting requests?

(2) The parties themselves (maybe with the help of a facilitator/mediator):
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Enraged the lion eats the ass, then asks the fox to make the division.

The fox piles all the kill into one great heap except for one tiny morsel.

Delighted at this division, the lion asks: “Who has taught you, my very excellent fellow, the art of division?” to which the fox replies: “I learnt it from the ass, by witnessing his fate”. (Aesop’s Fables)

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(1) Envy-freeness

(1.a) An allocation is envy-free (r1, r2) if every party thinks to have received a portion that is at least as valuable as the portion received by every other party.

(2) Proportionality

(2.a) An allocation is proportional (r1, r2) if every party thinks to have received a portion that is at least worth $\frac{1}{n}$ of the total value.

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Proportionality can be expressed only in terms of utilities

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(3) Equitability

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(4) Equitability

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Formalizing definitions 2

We denote with P_{ij} the value the player i gives to the portion received by player j with $i, j \in N$ (N is the set of players)

(3) Efficiency (Pareto)

(3.a) if $\nexists$ another allocation with evaluations Q such that ($Q_{ii} \geq P_{ii}$ and $\exists i^* \in N$ such that $Q_{i^* i^*} > P_{i^* i^*}$)

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(r3) H. Peyton Young, "Equity. In Theory and Practice", Princeton University Press, 1994

Main cases

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It requires a referee and a knife .

(1) the referee puts the knife on the left edge and moves it in a parallel way towards the right edge,
(2) both Bob and Carol can call "**cut**" at any moment,
(3) the player who calls "**cut**" gets the portion at the left of the knife whereas the other takes the remaining portion.

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(1) A referee moves as before a knife from the left edge to the right edge.

(2) At any time one of the two players, say Bob, can call "**stop**".

(3) A second knife is put on the left edge of the cake and Bob moves both so that when the rightmost is on the right edge the other is where the former was at the stop.

(4) While knives are moving (but the relative distance can vary) Carol can call "**stop**" at any moment.

(5) Chance establishes who gets the inner portion between the knives so that the other player gets the outer portions.

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Proportionality $n > 2$, divisible case

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(5) Bob, Carol and Ted $n = 3$. Bob splits the cake in two for him equal parts A and B. Carol chooses one and gives the other to Bob. Bob and Carol trisects their own pieces. Ted chooses two pieces, one from each triple, leaving the other two pieces to Bob and Carol respectively. Ted is the lone chooser.

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Banach-Knaster last-diminisher for $n > 2$

It guarantees only proportionality.

(1) Players are ordered either by chance or by an arbitrator as $1 \dots n$.

(2) The i-th player cuts a slice of the cake and pass it on to the other players.

(3) Every following player can reduce the slice if he thinks it is too big.

(4) The player who is the last to diminish the slice gets it and exits the game.

(5) The remaining parts of the cake are collected in a smaller cake and if more that one player is left we go back to (2) else the last slice is given to the last player.

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Proportionality $n > 2$, indivisible case

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We need procedures for the cases of indivisible goods whose value is destroyed if they are divided (no partial allocations). We only mention the following procedures:

(1) the Knaster's procedure of sealed bids (efficient but not envy-free) uses an auction like mechanism to allocate goods and distribute money compensations from players who gets more to players who gets less;

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Envy-freeness and equitability for $n = 2$

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(1) the Adjusted-Winner procedure (strategic behaviour);

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(4) more than 2 players: there may be no way to assign points to players that is efficient, equitable and envy-free (so only proportionality is saved).

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AW is based on the assignment from both players of a non null portion of 100 points (common scale) and is made of two consecutive phases:

(1) win, where the single players gets the goods on which they win by putting more points than the other;

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(1) win, where the single players gets the goods on which they win by putting more points than the other;

(2) adjust, where goods are transferred from one player to the other so that both can get the same amount of points;

(2) only the good on which equitability adjustment is made need to be divisible.

Adjusted Winner (AW) for $n = 2$

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Each player puts some points on each b_i.

(1) Each player gets the goods he values strictly more than the other.

(2) If the goods are over and if both players get the same sum of points we are over otherwise we can try to obtain a parity firstly assigning (one after the other) to the lower score player the goods on which they put the same amount of points.

(3) If in this way we get a parity we are over otherwise we must transfer a good (or better a portion of it) from the higher score player (be it 1) to the lower score one (be it 2).

(4) To find such a good we evaluate for each good of 1 the ratio of 1's points to 2's points for that good. We then choose the good with the lowest ratio and split it as α and $1 - \alpha$ so to attain parity.

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Adjusted Winner, an example

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	I	II
A	35	55
B	6	1
C	8	1
D	8	1
E	5	6
F	18	17
G	15	15
H	5	4
Total	100	100

(1) I wins B, C, D, F and H for a total of 45 points whereas II wins A and E for a total of 61 points.

(2) We have a draw on G and we assign it to I to get respectively 60 and 61 points.

(3) We can obtain perfect parity by transferring $\frac{1}{11}$ of E from II to I (we suppose E is perfectly divisible).

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Proportional Allocation (PA) for $n = 2$

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It is a variation of AW. The procedure is:

(1) envy-free so is
(2) proportional;
(3) equitable;
(4) not efficient since the players can exchange fractions of goods so to be both better off.

(1) PA is based on the assignment from both players of a non null portion of 100 points (common scale);
(2) given b_k, if 1 assigns x_k and 2 assigns y_k points ...
(3) 1 gets a fraction $\frac{x_k}{x_k+y_k}$ and 2 gets a fraction $\frac{y_k}{x_k+y_k}$. Fraction: time/value/portion.
(4) If we have b_s and b_t such that $\frac{x_s}{x_t} < \frac{y_s}{y_t}$ it is possible to exchange a fraction α of b_s with a fraction β of b_t so that both players are better off.

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	A	B	C	D	Total points
I	$\frac{1}{4}$ 2.5	$\frac{1}{3}$ 6.66	1 70	0 0	79.16
II	$\frac{3}{4}$ 22.5	$\frac{2}{3}$ 26.66	0 0	1 30	79.16

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	A	B	C	D	Total points
I	0 0	$\frac{1}{2}$ 10	1 70	0 0	80
II	1 30	$\frac{1}{2}$ 20	0 0	1 30	80

Adjusted Winner, the same example (tricky, 0 divisor)

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(1) I wins C;

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(3) we evaluate the ratios $\frac{30}{10}$ and $\frac{40}{20}$ and take the lowest;

(4) we solve $30 + 30 + \alpha 40 = 70 + (1 - \alpha)20$ to get $\alpha = \frac{1}{2}$;

(5) both I and II get 80 points;

(6) in this case all the goods are divisible, in any case at least one must be divisible.

(7) Divide the indivisible: convert into money, probability, time-sharing.

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Example of point assignment for $n = 3$

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	G_1	G_2	G_3	Total
A	40	50	10	100
B	30	40	30	100
C	30	30	40	100

(1) (A,B,C)=(G_1,G_2,G_3): efficiency and equitability but not envy-freeness (A envies B),

(2) (A,B,C)=(G_2,G_1,G_3): efficiency but neither equitability nor envy-freeness (B envies A),

(3) all the items to one player: efficiency but surely neither envy-freeness nor equitability.

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We only mention the following procedures that can be used to divide constructively (specifying an algorithm) an heterogeneous good:

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(2) the Stromquist moving-knife procedure for $n = 3$;
(3) the Levmore-Cook moving-knife procedure for $n = 3$;
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(4) other procedures for $n = 3$ or $n = 4$.

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Legitimate claim: is a claim that is well grounded and sound under given habits and conventions.

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