# Auctions and barters

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*Abstract*
In this paper we face the problem of the fair sharing of goods and bads (also collectively termed items) among a set of players that cannot (or do not want to) use a common cardinal scale for their evaluation owing to the very qualitative and non economical nature of the items themselves.
To solve this problem we present two families of protocols and use a set of classical fairness criteria (for barter protocols) and performance criteria (for auction protocols) for their evaluation.
As to the fairness criteria we use envy-freeness, proportionality, equitability and [Pareto] efficiency with some modifications and adjustments in order to make them suitable for the new contexts.
The performance criteria that we use include: guaranteed success, [Pareto] efficiency, individual rationality, stability and simplicity.
As to the families of protocols we have:

- a family $F_1$ of protocols that are based on auctions mechanisms and that can involve any number of players as an auctioneer and a set of bidders;
- a family $F_2$ of protocols that are based on barter mechanisms and that involve a pair of players at a time but can involve an arbitrary number of such pairs.

Family $F_1$ contains three types of auction mechanisms: ($a_1$) a sort of Dutch auction with negative prices, ($a_2$) a sort of English auction with negative prices and ($a_3$) a sort of first price auction with negative prices.
In mechanism ($a_1$) the auctioneer tries to allocate a bad to one bidder by rising his offer up to a maximum value M whereas in mechanism ($a_2$) the auctioneer starts with an offer L and the bidders make lower and lower offerings until one of them wins the auction and gets the bad and the money. In mechanism ($a_3$) the bidders bid for not getting a bad that is assigned to the losing bidder (the one who bid less than the others) together with a compensation from all the other bidders.
Of each mechanism we provide a description and the best strategy. Once the mechanisms have been described we also prove how the first two mechanisms are really equivalent and define some relations between them and the last one. We also apply the performance criteria to such mechanisms for their evaluation and prove under which conditions they are satisfied.
On the other hand family $F_2$ contains two subfamilies of models. In this paper we present such models in their basic two players A and B version and then we discuss how they can be extended to the interactions between *n* pairs ($A_i$ $B_i$) of actors.
The former subfamily contains a set of *explicit barter models* whereas the latter

contains an *implicit barter model* and a *mixed barter model*.
In the *explicit barter models* the players A and B show each other the set of items that each of them is willing to barter within a procedure that is characterized by either simultaneous or consecutive requests from one player to the other in which the barter may involve either a single item or a subset of items.
An explicit barter is an iterative procedure that may end either with a success (and so with an exchange) or with a failure but, at each step, may also involve a reduction of the items each player is willing to barter.
In the implicit barter case none of the players show his items to the other so that each player, in his turn, proposes to the other a pair of items (i, j) that he is willing to barter so that the other may either accept or reply with a counter proposal. The barter ends when an agreement is reached or both agree to give up since they decide that no barter is possible. During the barter both players reveal to the other the items they are willing to barter and this can ease the reaching of an agreement.
Last but not least in the mixed barter model we have that one player (be it A) shows his items to B that, on the other hand, behaves as in the implicit case. Also in this case the barter goes on as a series of proposals and counteproposals with an incremental definition of bartering set of player B.
All these models are presented in detail, discussed and evaluated using the fairness criteria. In this way we show how the proposed models satisfy envy-freeness for sure whereas equitability and efficiency may fail to be verified unless certain conditions are satisfied.
The paper closes with a section devoted to the discussion of the possible extensions to the models we presented, their practical applications and a section devoted to future research plans.