Iterative barter models

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Abstract
The paper presents two models of iterative barter where two players (A and B) aim at exchanging a pair of items without sharing any common scale of evaluation nor any common numerary good as the money. An item may be either a good or a chore (i. e. a bad) or a service of some kind and may "belong" to either players.
In the first model every player proposes to the other a pair of items in an alternating series of proposals and counter proposals (that dynamically define the acceptable sets of exchangeable items for each player). The barter stops when a player accepts a proposal or both agree that no barter is possible (no counter proposal is made) and so they give up.
In the second model one of the two players (be it A) shows his items to the other whereas the other (B) makes an opening proposal (otherwise the barter ends immediately). After this opening phase the barter goes on as before with the difference that B is fully knowledgeable of the items of A from the offset whereas A knows the items of B dynamically during the course of the process.
In both models the barter process may close with either a success (so that the barter really takes place) or with a failure (so that no barter really takes place).
For the evaluation of the proposed models we introduce, with some modifications owing to hypotheses we made, a set of classical fairness criteria (envy-freeness, equitability and [Pareto] efficiency) and apply them to each model.
We so show how the proposed algorithms may easily satisfy the requirements of envy-freeness and equitability whereas they cannot guarantee in any way that a barter is Pareto efficient.
As to *envy-freeness* we give a necessary and sufficient condition for a successful barter to be envy-free so that if a proposed barter is envy-free it necessarily occurs and if it occurs it is necessarily envy-free.
As to *equitability* we provide two independent conditions, eq_A for player A and eq_B for player B, that are necessary and sufficient conditions for a successful barter to be equitable for each player. We say that successful barter is *equitable* if both conditions are verified otherwise it may be equitable only for one player or simply inequitable.
The condition eq_A (and similarly eq_B) involves a relation between the relative value of what A gets form the barter and the relative value of what he loses from it, both values being measured by A according to his private values system.
Last but not least for the *[Pareto] efficiency* we are able to provide only one

sufficient condition for a successful barter to be efficient and one sufficient condition for a successful barter to be inefficient so that, if the former is violated, there may be at least one exchange of items such that one player is better off with the other not being worse off.
We also give a formal characterization of the general rule of efficiency of a barter and show how, when a barter has occurred, each player may well declare that he would have been better off with another kind of barter so that the current barter is not [Pareto] efficient.
We say therefore that *efficiency* cannot be ex-ante guaranteed and can be verified only ex-post.
From all this we derive that each player may judge an occurred barter as either inequitable or inefficient or both and therefore unfair, since not all the fairness criteria are verified.
The paper closes with a section devoted to possible extensions (such as repeated barters involving one or even more items on each round) and a section devoted to future developments, both on the applicative and on the theoretic side, of the proposed models.