

Candle auctions

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1 The scope of the procedure

We define an auction inspired procedure through which an agent A (the so called initial auctioneer) allocates a chore to one agent from a predefined set of sub-agents by letting them arrange the things among themselves including the definition of the compensation for the losing sub-agent.

The compensation is a multiple of the fee f that A fixes at his will and the selected sub-agents are called bidders or players.

The procedure is implemented through a simple algorithm that involves the sub-agents without any further intervention of A .

2 The motivations of the procedure

The proposed procedure is based on certain assumptions on both the auctioneer side and the bidder side. The common assumption is that the number n of bidders is high and potentially infinite.

On **the auctioneer side** it is based on the following assumptions:

- the auctioneer has only a vague idea of the evaluation of a chore and only knows that he needs it carried out and that each of the bidders can carry out the chore;
- the auctioneer has no other sure information about the bidders such as their degree of capability, their costs or their willingness to carry out the chore for him,

therefore he prefers that they find an allocation solution among themselves and that such solution may, on one hand, compensate at the most one of them without, on the other hand, charging excessively any of the others.

On **the bidders side** it aims at defining an adaptive sharing of the final

contribution to the losing bidder (the one who gets the chore at the end of the auction) from the other bidders that exclude an a-priori defined subdivision (such as a uniform subdivision¹ or a proportional subdivision).

In this way we aim at a procedure that is adaptive and potentially more rewarding for the losing bidder without charging too much each of the winning bidders and without hurting in any way the auctioneer

3 The algorithm

The [initial] auctioneer A :

- chooses the n bidders of B ;
- chooses the chore to be auctioned;
- fixes the fee f that each bidder may pay for not accepting the candle (see further on).

The bidders of the set B are chosen by A from a much bigger set and each bidder may be seen as having associated a **type** (see section 3) that defines his behavior in the course of the game as will be explained in section 5.

As to the entity of the fee f we note that it can be fixed freely by A at any value since that value has no effect on A himself. We note that if $f = 0$ the losing bidder has no compensation (see further on) so that we pose the constraint $f > 0$. On the other hand the lower is f the more easily the bidders are going to pay it whereas the higher it is the more unlikely they are going to pay it. Since the value of f affects the behavior of the bidders we can imagine, for simplicity, that A extracts it from a urn containing all the integer values in the interval $[f_{min}, f_{max}]$. The extremes of the interval are exogenous values to be fixed in some way. Further details in section 5.

The algorithm is structured as follows:

- initialization phase;
- auction phase;
- compensation phase.

In the **initialization phase** A in zero time:

¹We say a sum x is shared in a **uniform way** among n actors if each of them is charged with a fraction $\frac{x}{n}$ whereas we say it is shared in a proportional way if for a bid x_i its bidder is charged with a fraction $x \frac{x_i}{X}$ where $X = \sum_{i=1}^n x_i$.

- initializes or switches on a flag that we call candle;
- performs a random initialization of the maximum value L of a counter on an interval (unknown to the bidders) $[0, M]$;
- performs a random selection of one of the bidders, be it b_i ;
- gives the candle to b_i ;
- starts the counter t (that is incremented one tick at a time up to L ticks) from 0.

The **auction phase** is structured as follows. We underline how the counter t is incremented of one tick as stated in each case. At $t = 1$ b_i has the candle:

- (1) b_i selects another bidder b_j at his will and offers him the candle;
- (2) b_j can:
 - (a) accept it so b_j takes the role of b_i , $t = t + 1$, go to (1);
 - (b) refuse it and put the fee f in the common pot, $t = t + 1$, go to (1).

At the generic step t the content of the pot is kf (with $k \in [0, t]$) and this may influence the behavior of both b_i and the others b_j that are contacted by him (see section 5).

At each step the bidder b_i must chose another bidder b_j so to offer him the candle. The choice can be made either to maximize the probability of keeping the candle or to maximize the probability of giving it away: the former case occurs whenever b_i thinks he has a gain from having the candle when the counter expires whereas the latter occurs whenever b_i thinks he has a loss from having the candle when the counter expires.

For further details we refer to section 5.

The generic bidder b_j to which the candle holder b_i offers the candle can either accept it or refuse it according to strategies we describe in section 5. If b_j accepts the candle then he takes the role of b_i (the candle holder) and uses the same search and offer procedure.

If b_j refuses the candle then he must put f in the common pot and b_i goes on with his imposed search and offer procedure.

The private information

Each bidder b_j has the following private information:

- d_j as the damage that derives to b_j from having the candle when the counter expires;
- a vector of values c_j whose elements $c_{j,h}$ denote the damage that derives to b_j from having the candle owned by b_h when the counter expires;

and can use them for his strategies either as the candle holder or as a contacted bidder. We note that:

- $c_{j,j} = d_j$;
- the vectors c_j represent the rows of a square cost matrix C whose dimension coincides with the number of the bidders,
- the values $c_{j,h}$ can be used by b_j to partition the set of the other bidders in disjoint subsets and to order such subsets in increasing order of cost (see section 5).

Besides such information each bidder b_j knows his type t_j whereas has a probability distribution for the type of every other bidder.

For the moment we consider only two possible types: green or conservative (g) and selfish or exploitative (s).

From this we have that whenever b_j faces another bidder b_h he knows that with probability p he is facing a g type bidder and with probability $1 - p$ he is facing an s type bidder.

Termination condition

When the counter t reaches L the bidder who has the candle loses the auction and gets the chore and the amount of fees put in the common pot by the bidders during the auction.

Compensation phase

When the auction is over the candle holder b_i gets the chore and a compensation equal to k^*f where with k^* we denote the number of fees paid by the players different from b_i so that b_i obviously does not count as a gain the sums paid by himself.

We can have the following cases:

- $k^*f > d_i$ so that b_i has a gain,
- $k^*f = d_i$ so that b_i is fully compensated,
- $k^*f < d_i$ so that b_i has a loss.

4 Why we say it is an auction

We define the proposed mechanism as an **auction** since at each step the candle holder b_i can be seen as the “auctioneer” whereas one of the others either “bids” a fixed sum for not getting the candle or gets it and becomes the current “auctioneer”.

In a traditional auction at the end the auctioneer gives away the auctioned good and gets a certain amount of money whereas in this case the final auctioneer gets the candle (i. e. a chore) and a compensation (the “net” common pot²) that some of the others contributed to form in a way that resembles an all pay auction.

5 The properties

The balances

The balances of the various bidders depend heavily on L since the higher is L the higher may be k^* for every bidder. From this we derive that L has a lower bound. They also depend on the variety of the interactions among the players and so on their strategies.

The losing bidder b_i may be worse off if k^* is not high enough.

The winning bidders b_j with $j \neq i$ pay on the average:

$$\frac{L}{n}f \tag{1}$$

and each has a gain of d_j . For each b_j the balance is³:

$$d_j - k_j f \tag{2}$$

where k_j is the number of times bidder b_j refuses the candle and pays the fee, with $0 \leq k_i \leq L$. Every bidder can control the amount of money that he puts into the pot by controlling his number of refusals.

Collusions

Never occur. Who pays the fee to the pot has not the candle and who, at the end, remains with the candle gets the content of the pot as a compensation.

²We call it “net” since we exclude the sums paid the the final auctioneer himself.

³We use the notation k_j to denote the value k^* for bidder b_j since each bidder has his own value k^* as the number of times the other players paid the fee.

The strategies

At this point we must specify, at each step, the possible strategies for both the current candle holder b_i and the players that may be contacted by him and so b_j with $j \neq i$.

(1) For what concerns b_i he has two possible strategies and one constraint:

- (a) **attractive** if he tries to keep the candle for himself by contacting bidders that he is sure will refuse it so paying the fee f so increasing the common pot;
- (b) **repulsive** if he tries to give away the candle by contacting bidders that is sure will prefer to accept it rather than paying the fee f .

The strategy (a) is worth following if some time has already elapsed since the start of the game and/or the fees in the pot are such that b_i is sure to have a gain from getting the candle. This occurs for sure if $k^*f > d_i$. Such strategy may either succeed or fail. If it succeeds it causes a further increase of the common pot of f at the benefit of b_i . Unfortunately it may fail since the contacted player b_j , using his private information, may find the common pot high enough so to accept the candle without paying the fee f .

On the other hand the strategy (b) is worth following if the game is at the start and/or the common pot is low so the best thing b_i to do is to pass the candle to a bidder b_j such that $d_i - k^*f > c_{i,j}$. In this case the best bidder b_j is the one with the minimum $c_{i,j}$. In order to prevent b_i from continuously contacting the same b_j we introduced a constraint (see further on).

Such strategy may either succeed or fail. If it fails the bidder b_j pays the fee f and this makes b_i better off whereas if it succeeds it makes b_i , at least temporarily, better off.

We note that the common pot may be low for the following reasons:

- the game is in the initial phases,
- the game has been characterized by the passing of the candle among the players so that few of them paid the fee f .

We note that the common pot may be high for the following reasons:

- the game has already been running for some time,
- the game has been characterized by a lot of refusals so that many of the players paid the fee f .

The constraint we introduced imposes b_i to contact at least half of the bidders in turn, according to whatever ordering he choses, before contacting again the same player so to prevent him from exploiting a refusing bidder by repeatedly contacting him. The easiest ordering is based on the values $c_{i,j}$. Using such values b_i can partition the set B_{-i} of the other bidders in subsets of equal cost bidders so to get an ordering of such subsets and contact their members according to an increasing costs criterion.

- (2) For what concerns the bidders b_j their repertoire of moves is reacher since they can either **accept** or **refuse** the candle.

A player is said to refuse the candle is he does not accept it so we can restrict ourself to the definition of the conditions for the acceptance of the candle.

We recall that we may have either $t_j = g$ or $t_j = s$.

If $t_j = g$ the bidder b_j accepts the candle if he hopes to find a bidder b_h such that $c_{j,h} < c_{j,i}$ even at the risk of remaining with the candle so to suffer the damage d_j independently from the content of the pot. In this case we presumably have $c_{j,h} < d_j < c_{j,i}$ or $c_{j,h} < c_{j,i} < d_j$.

If $t_j = s$ the bidder b_j accepts the candle if he has a gain from getting it so that $k^*f > d_j$.

In the acceptance case b_j becomes the candle holder and, at least for one turn, acts as b_i .

In the refusal case b_j puts the sum f in the common pot and puts himself on waiting for a possible future choice until the counter expiration time.

At this point we have to establish a relation among the two pairs of strategies we have defined:

- attractive and repulsive when the bidder is the candle holder,
- green and selfish when the bidder is not the candle holder,

From the descriptions we have that a $t_j = g$ bidder tends to adopt a repulsive strategy when he plays as the candle holder whereas a $t_j = s$ tends to use an attractive strategy when in the same role.

The timings

The counter starts from 0 and stops at a random value L . The unitary increments occur at every acceptance or refusal so there is no way for the bidders to loose time so to exploit an advantageous position.

6 The experimental side

The model should be implementable through the use of an agents system by defining two populations of agents with the defined types and strategies and by having them play the game for some random amount of time L with the specified timings.

One possible extension, to be tested experimentally, could be the following: each bidder b_j has a randomly fixed sum of money F_j (integer multiple of f) that he can use to pay the fee f so that when that sum is finished he can only accept the candle without any further possibility of refusal.