Veryfing Real–Time Systems

The UPPAAL Model Checker

Introduction to UPPAAL

UPPAAL is a toolbox for modeling, simulation and verification of real–time systems

- Uppsala University + Aalborg University = UPPAAL

Examples of real–time systems are

- real–time controllers
- communication protocols
- multimedia applications

Introduction to Uppaal

Systems modeled as networks of Timed Automata enriched with

- integer variables
- structured data types
- channel syncronisations
- urgency

Properties to be verified can specified in a subset of CTL (computational tree logic)

Introduction to UPPAAL

About UPPAAL :

- first version released in 1995
- it consists of:
  - a graphical description tool
  - a simulator
  - a model–checker
- Java user interface and C++ verification engine
- freely available at `http://www.uppaal.com/`

Network of Timed Automata

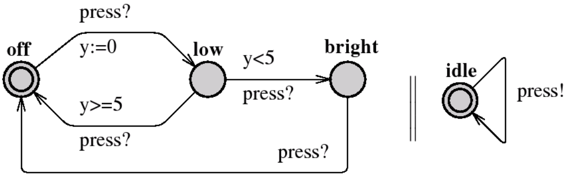
A Timed Automaton is a finite–state machine extended with clock variables.

- A clock variable evaluates to a real number
- All the clocks progress synchronously

A system is modelled as a parallel composition of timed automata

An automaton may perform a transition separately or synchronise with another automaton (channel synchronisation)

Network of Timed Automata



The lamp example

Clock Valuations and Boolean Guards

Let C be a set of clocks. A *clock valuation* is a function $u : C \to \mathbb{R}_{\geq 0}$

Boolean guards are defined as follows:

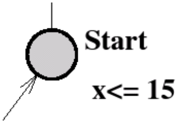
$$\begin{array}{rcl} B(C) & = & \mathit{true} \mid \mathit{false} \mid x \bowtie c \mid x - y \bowtie c \mid \\ & & B(C) \wedge B(C) \mid B(C) \vee B(C) \mid \neg B(C) \end{array}$$

where $x, y \in C$, $c \in \mathbb{N}$, and $\bowtie \in \{<, \leq, =, \geq, >\}$.

Let $g \in B(C)$, we write $u \in g$ if u is a clock valuation satisfying the boolean guard g.

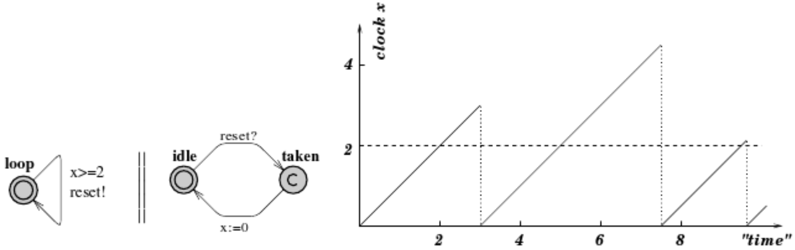
Location Invariants

Boolean guards are used as transition guards, but also as location invariants:



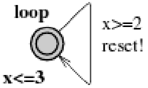
The automaton can be in state **Start** only if the valuation of clock x is smaller than or equal to 15.

Example: Transition Guards and Location Invariants

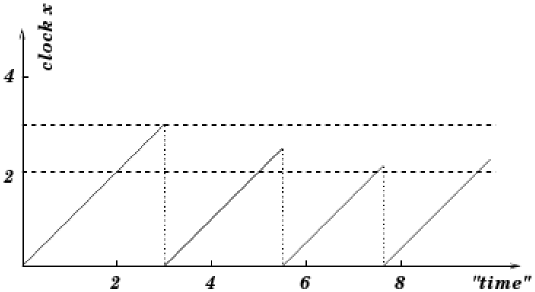


(a) Test. (b) Observer. (c) Behaviour: one possible run.

Example: Transition Guards and Location Invariants



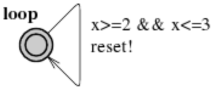
(a) Test.



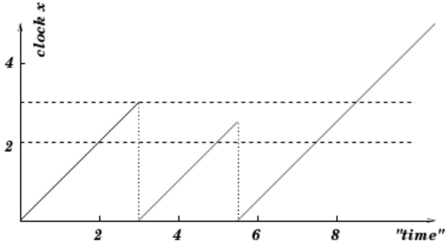
(b) Updated behaviour with an invariant.

The observer automaton is as before

Example: Transition Guards and Location Invariants



(a) Test.



(b) Updated behaviour with a guard and no invariant.

The observer automaton is as before

Definition of Timed Automaton

Definition (Timed Automaton) A Timed Automaton is a tuple (L, ℓ_0, C, A, E, I) , where:

- L is a set of locations
- $\ell_0 \in L$ is the initial location
- C is the set of clocks,
- A is the set of actions (e.g. press!), co–actions (e.g. press?) and internal τ–actions
- $E \in L \times A \times B(C) \times 2^C \times L$ is a set of edges between locations with an action, a guard and a set of clocks to be reset
- $I : L \to B(C)$ assigns invariants to locations

Semantics of a Timed Automaton

Definition (Semantics of a Timed Automaton) Let (L, ℓ_0, C, A, E, I) be a timed automaton. The semantics is defined as a labelled transition system $\langle S, s_0, \rightarrow \rangle$, where $S \subseteq L \times \mathbb{R}^C$ is the set of states, $s_0 = (\ell_0, u_0)$ is the initial state, and $\rightarrow \subseteq S \times \{\mathbb{R}_{\geq 0} \cup A\} \times S$ is the transition relation such that

- $(\ell, u) \xrightarrow{d} (\ell, u + d)$ if $\forall d'. 0 \leq d' \leq d \implies u + d' \in I(\ell)$
- $(\ell, u) \xrightarrow{a} (\ell', u')$ if there exists $e = (\ell, a, g, r, \ell') \in E$ such that $u \in g$, $u' = [r \mapsto 0]u$, and $u' \in I(\ell)$

where for $d \in \mathbb{R}_{\geq 0}$, $u + d$ maps each clock x in C to the value $u(x) + d$, and $[r \mapsto 0]u$ denotes the clock valuation which maps each clock in r to 0 and agrees with u over $C \setminus r$.

Definition of Network of Timed Automata

A *network of Timed Automata* over a common set of clocks and actions consists of n Timed Automata $(L_i, \ell_i^0, C, A, E_i, I_i)$ with $1 \leq i \leq n$.

A *location vector* is a vector $\overline{\ell} = (\ell_1, \ldots, \ell_n)$

Location invariant functions are composed into a common function over location vectors $I(\overline{\ell}) = I_1(\ell_1) \wedge \ldots \wedge I_n(\ell_n)$.

$\overline{\ell}[\ell_i/\ell_i']$ denotes the location vector where the ith element ℓ_i of $\overline{\ell}$ has been replaced by ℓ_i'.

Semantics of a Network of Timed Automata

Semantics of a network of Timed Automata Let $A_i = (L_i, \ell_i^0, C, A, E_i, I_i)$ be a network of Timed Automata. Let $\bar{\ell}_0 = (\ell_1^0, \ldots, \ell_n^0)$ be the initial location vector. The semantics is defined as a transition system $\langle S, s_0, \rightarrow \rangle$, where $S = (L_1, \times \ldots \times L_n) \times \mathbb{R}^C$ is the set of states, $s_0 = (\bar{\ell}_0, u_0)$ is the initial state, and $\rightarrow \subseteq S \times S$ is the transition relation defined by:

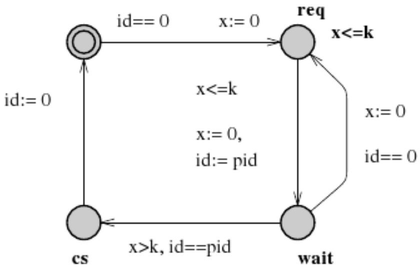
- $(\bar{\ell}, u) \rightarrow (\bar{\ell}, u + d)$ if $\forall d'.0 \leq d' \leq d \implies u + d' \in I(\bar{\ell})$
- $(\bar{\ell}, u) \rightarrow (\bar{[}\ell_i'/\ell_i], u')$ if there exists $(\ell_i, \tau, g, r, \ell_i')$ such that $u \in g, u' = [r \mapsto 0]u$ and $u' \in I(\ell[\ell_i'/\ell_i])$
- $(\ell, u) \rightarrow (\bar{\ell}[\ell_j'/\ell_j, \ell_i'/\ell_i], u')$ if there exist $(\ell_i, c?, g_i, r_i, \ell_i')$ and $(\ell_j, c!, g_j, r_j, \ell_j')$ such that $u \in (g_i \wedge g_j)$, $u' = [r_i \cup r_j \mapsto 0]u$ and $u' \in I(\ell[\ell_j'/\ell_j, \ell_i'/\ell_i])$.

Extensions to Timed Automata

Some of the additional features of the UPPAAL modelling language are the following:

- **Bounded integer variables** are declared as `int[min,max] name`, where `min` and `max` are the lower and upper bound, respectively. Violating a bound leads to an invalid state that is discarded at run–time.
- **Arrays** are arrays. . .
- **Broadcast channels** One sender $c!$ can synchronise with an arbitrary number of receivers $c?$. Any available receiver must syncrhonise. Broadcast sending is never blocking.

Example: Fisher's Mutual Exclusion Protocol



With the following declarations (for 6 processes):

```
int[0,6] id; const k 2; clock x;
```

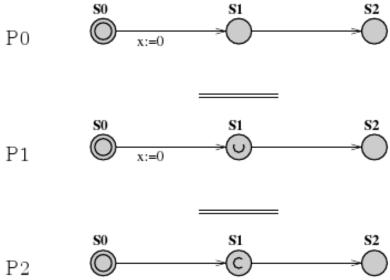
and the following parameter (for 6 processes): `int[1,6] pid;`

Extensions to Timed Automata

Some of the additional features of the UPPAAL modelling language are the following:

- **Urgent locations** Time is not allowed to pass when the system is an urgent location. They are semantically equivalent to adding an extra clock x that is reset on all incoming edges, and having an invariant $x \leq 0$ on the location.
- **Committed locations** A commited location is the same as a urgent location but the next transition must involve an outgoing edge of at least one of the committed locations of the network.

Example: Urgent vs Commit



- When P0 is in S1, time can pass and any edge can be taken.
- When P1 is in S1, time cannot pass, but any edge can be taken.
- When P2 is in S1, time cannot pass and the only edge that can be taken is the one from S1 to S2 in P2.

Examples. . .

Examples included in the UPPAAL package

- The four vikings problem (bridge.xml)
- The train gate (train-gate.xml)

State Formulae

UPPAAL uses a simplified version of CTL as its query language.

The query language consists of path formulae and state formulae.

- State formulae describe individual states
- Path formulae quantify over paths of the model

A state formula is an expression that can be evaluated for a state.

`x>3`   `i==2`   `x<=3 and i==5`

Moreover:

- the state formula $P.\ell$ tests whether the Timed Automaton identified as process P is in a given location ℓ
- the state formula `deadlock` is satisfied for all deadlock states of the network (there are no enabled transitions)

Path Formulae

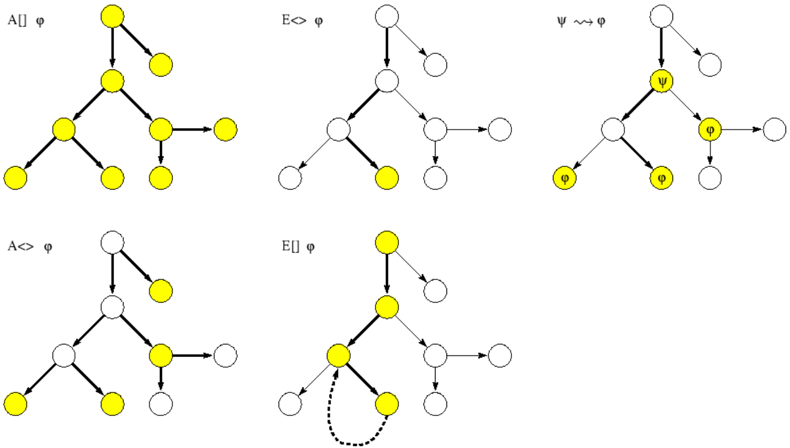
Path formulae have the following syntax:

$$
\begin{aligned}
PF \; ::= \; & A\Box\, \phi \\
\mid \; & A\Diamond\, \phi \\
\mid \; & E\Box\, \phi \\
\mid \; & E\Diamond\, \phi \\
\mid \; & \phi \leadsto \psi \quad \text{that is } A\Box\, (\phi \rightarrow E\Diamond\, \psi)
\end{aligned}
$$

Path formulae can be classified into

- reachability
- safety
- liveness

Path Formulae



Path Formulae

Reachability Properties
They ask whether there exists a path starting at the initial state, such that a state formula ϕ is eventually satisfied

Reachability properties are often used while designing a model to perform sanity checks (e.g. is it possible for a sender to send a message?).

These properties do not by themselves guarantee the correctness of the protocol (i.e. that any message is eventually delivered), but they validate the basic behavior of the model.

A reachability property is expressed by the path formula $E\Diamond\ \phi$.

Path Formulae

Safety Properties
Something bad will never happen! (e.g. in a nuclear power plant the temperature of the core is always (invariantly) under a certain threshold)

A variation: something bad will possibly never happen! (e.g. in a game, a safe state in one in which the player can still win – will possibly not loose)

In UPPAAL these properties are formulated positively (something good is invariantly true) and they are expressed by the path formulae $A\Box\ \phi$ and $E\Diamond\ \phi$

Path Formulae

Liveness Properties
Something good will eventually happen! (e.g. when pressing the *on* button, then eventually the television should turn on)

A variation: if something good happen, then something else will eventually happen! (e.g. in a communication protocol, any message that has been sent should eventually be received)

These properties are formulated as $A\Diamond\ \phi$, and $\phi \leadsto \psi$ (i.e. ϕ *leads to* ψ).

Model Checking Procedure

All path formulae can be expressed as reachability and invariance properties:

- $E\Diamond\ \phi$ is reachability
- $E\Box\ \phi$ is invariance
- $A\Diamond\ \phi = \neg E\Box\ \neg\phi$
- $A\Box\ \phi = \neg E\Diamond\ \neg\phi$

The model–checking procedure implemented in UPPAAL is based on a finite–state symbolic semantics of networks.

- the logic is interpreted with respect to symbolic states of the form $(\bar{\ell}, D)$, where D is a constraint system.
- a symbolic state $(\bar{\ell}, D)$ represents all the states $(\bar{\ell}, u)$ where u satisfies the constraint D

Model Checking Procedure

```
PASSED:= {}
WAITING:= {(l̄_0, D_0)}
repeat
    begin
    get (l̄, D) from WAITING
    if (l̄, D) ⊨ β then return "YES"
    else if D ⊈ D' for all (l̄, D') ∈ PASSED then
        begin
        add (l̄, D) to PASSED
        SUCC:={(l̄_s, D_s) : (l̄, D) ⇝ (l_s, D_s) ∧ D_s ≠ ∅}
        for all (l̄_s', D_s') in SUCC do
            put (l̄_s', D_s') to WAITING
        end
    end
until WAITING={}
return "NO"
```

An algorithm for symbolic reachability analysis. Symbolic invariance should be similar.