# On the Dimensioning of Ad Hoc Sensor Networks

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***Abstract*— In this paper, we address the problem of dimensioning wireless sensors networks in order to guarantee an expected connectivity. This means finding the minimum number of sensors to guarantee an expected mean connectivity on a given area.**
**We propose an equation which approximates the number of sensors to be dropped in that area in order to achieve a given expected connectivity $\overline{K}$. Our equation is generic, and works with any statisical distibution function $f(x, y)$ of the position of nodes on the sensed area.**
**The equation is validated both theoretically and by simulation. In particular, we prove an upper bound for the error introduced by the approximation which is independent of the type of distribution $f(x, y)$. We also simulate the dropping of sensors in a unitary square area with uniform and Gaussian distribution functions and compare the results with the dimensioning suggested by our equation. For a given $\overline{K}$, we find that the dimensioning equation produced networks with a mean connectivity of $\overline{K}$ in 97% of cases.**

## I. INTRODUCTION

In the Homogeneous Topology Control research area the problem of finding an Equation for the dimensioning of a sensor, or ad hoc, network with a fixed communication range is very important [10]. Deploying a wrong number of sensors in an area can affect the functionality of the network itself and cause a loss of time and money for the deploying entity. Dropping few sensors can bring to disconnected or partitioned networks. As a consequence the data acquired by some sensors could not be retrieved to sinks or sent to other sensors: situation that brings to a useless network. On the other side, dropping too many sensors can produce an high concentration of nodes in an area, which in turn can cause frequent packet collisions (channel contention).
The idea behind our solution is based on the study of the density of the nodes in an area. We define the density of the nodes in an area $a$ as the number of sensors that are present in that area. We propose an equation which approximates the number of sensors to be dropped in that area in order to achieve a given mean connectivity $\overline{K}$. Our equation is generic, and works with any statisical distibution function $f(x, y)$ of the position of nodes on the sensed area.
With this new approach we propose three things: the first one is a model to study, based on a physical characteristic of the network that is *density*; the second is about the study of that characteristic in non-uniformly distributed networks (with the study of the special case of *Gaussian* deployed networks); and the last one is to offer a tool, our formula, to help the designers of sensors network in the dimensioning of the number of devices to be deployed to acquire a wished connectivity.
On line strategies [13], to acquire the same results, in our opinion, are only partially applicable: once sensors are deployed, if their density is too high they can start a wake-sleep strategy to reduce density. If density is too low we must deploy other sensors.
We think that these tasks are not impossible to perform: we simply think that starting with the right number of sensors is a better choice.
The paper is structured as follows. In Sec. II, we expose some results in Homogeneous Topology Control that are related to the ones presented in this paper. In Sec. III we present the sensor network model used in this paper. In Sec. IV we expose the basic ideas behind our work using an uniform distribution of sensors. In Sec. V, we discuss the limitations of the previous result for different distributions and we generalize the model for arbitrary distributions. The result of the last model is fully compatible with uniform distribution result. That Section also presents a theoretical upper bound to the error. In Sec. VI, we present some simulation results that validate our work and in Sec. VII, we show a realistic application scenario in which our method can be used to establish the size of a sensor network. Sec. VIII resumes the work in the paper and Sec. IX concludes giving a small outline of our future research

based on the results of this paper.

## II. RELATED WORKS

In the Homogeneous Topology Control research area the problem of dimensioning sensors, or ad hoc, networks is usually formulated as follows: *given a number of nodes deployed in an area, find the best transmitting range for all the transmitting radios to guarantee some given properties, such as connectivity*. Most researches approach this problem only studying the desired properties of the network once it is generalized as a *Random Geometric Graph* (see [3]) as in [12], [4] and [11].
In our formulation, solving the dimensioning problem means *looking for the best number of sensors, with fixed communication range to be deployed in an area, to ensure a given property*, in the rest of this paper this property is *connectivity*. We solve the problem from this point of view and we also propose a novel formulation of the problem taking in consideration physical properties of the network. The physical model is similar to the one proposed in [1], [9] and [8] but with different metrics and results.
Another aspect of Homogeneous Topology Control is that it studies the relation between the transmitting range and the number of nodes in uniformly distributed sensor networks as in [7], [5], [4] and [14].
In sensor networks non-uniformity, in node distribution, is a new topic. We can find mention of it only in two recent works: [2] and [6].

## III. NETWORK MODEL

In our model, size of sensors is considered not influent (sensors are approximated by their coordinate point), the transmission area is circular and is centered on the sensor itself. We say that two sensors are neighbors if and only if they can communicate (their distance is less or equal than the transmission range).
In the paper sensors are identified with small cap 's' with an index (generic sensor is referred as $s_i$), transmission range is $r$, and the transmission area is $a = \pi r^2$. The expected neighborhood of sensor $s_i$ is $N(s_i)$ and denotes the *expected* set of sensors in the transmission range of $s_i$. $N(s_i)$ has a relative importance, the real important value is $|N(s_i)|$ that represents the *expected size* of such neighborhood. We denote with $A$ the total deployment area of the sensors and with $A_j$ a generic sub area of $A$ such that $A_j \subset A$. We also define $S$ as the set of all deployed sensors such that each $s_i \in S$ and with $S_j$ the *expected* sub set of $S$ of all the sensors $s_i$ such that $s_i$ is in the area $A_j$. The size of $S$ is denoted with $n$. As for $N(s_i)$ the real important value to consider is $|S_j|$ that denotes the *expected size* of $S_j$ sub set.
If we want to use more realistic communication range models (see [16]) we can consider directly the transmitting area $a$, as an approximation of transmitting area in the realistic radio model, without considering $r$.

## IV. BASIC MODEL

Our model is based on sensor density in the deployment area. If we consider a sensor $s_i$ and its *expected* neighborhood $N(s_i)$, we can define the *expected* density $d(a)$ of sensors in the area $a$, centered in $s_i$, as the size of $N(s_i)$ plus $s_i$ itself divided by the size of the communication area $a$ (defined as $\pi r^2$).

$$d(a) = \frac{1 + |N(s_i)|}{a} \tag{1}$$

In the case of uniform distribution of sensors $d(a) = d(A)$, where $d(A)$ represents the total density of the network, and we can state that:

$$\forall\ i,\quad d(A) = \frac{n}{A} = \frac{1 + |N(s_i)|}{a} = d(a) \tag{2}$$

From Eq. 2 we can obtain the total number of sensors to be deployed for a wished density, $\overline{K}$.

$$n = \frac{(1 + \overline{K})A}{a} \tag{3}$$

We can notice that Eq. 3 makes no more sense in areas where the distribution of sensors does not follow the uniform distribution, because for every distinct area $a$ the density $d(a)$ joins with the position of the sensor $s_i$ that defines that area.
In Fig. 1, we show the surface produced by Eq. 3. The surface is computed setting the value $A = 1$ and varying the value of $r$ in range $[0.1,\ 1.4]$ and the value $\overline{K}$ (in the Figure simply $K$) in range $[2,\ 9]$. The Figure shows that the surface has its maximum at the point with minimum range and maximum desired neighbors number and tends to grow very quickly for small values of $r$.

## V. GENERALIZATION OF THE MODEL

Eq. 3 gives information only about uniformly deployed sensors but not about other distributions because with not uniform distributions we cannot say that for every $A_i$ and $A_j$, with $A_i$ and $A_j$ of the same size but located in different places on the plane, the density of

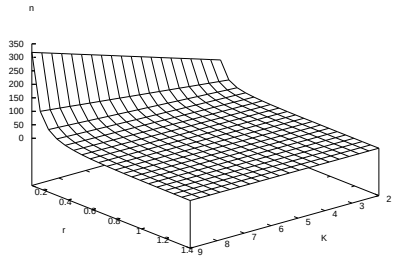



Fig. 1. Number of nodes for uniform distribution

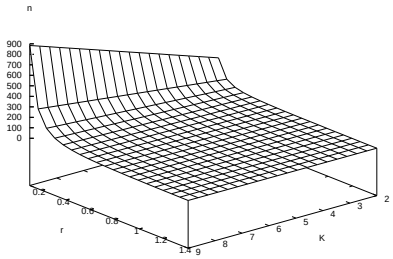



Fig. 2. Number of nodes for Gaussian with $\sigma_{xy} = 0.35$ in $A_{(0.9,0.9)}$

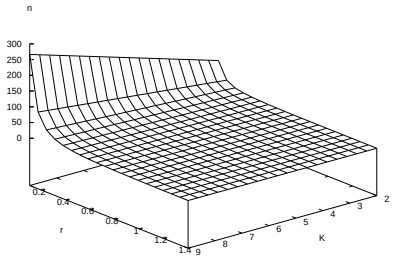



Fig. 3. Number of nodes for Gaussian with $\sigma_{xy} = 0.35$ in $A_{(0.5,0.5)}$

this sub areas are equal, thus $\frac{|S_i|}{A_i} \neq \frac{|S_j|}{A_j}$. For instance, Gaussian distributed sensors tend to be more dense near the mean point of the distribution and very sparse far from that point.

One way to generalize our model is to consider a reasonable small sub area of $A$, call it $A_j$, and approximate the distribution of the nodes in $A_j$ with an uniform distribution. Now the problem becomes to compare the distribution in the area $a$ with the distribution in sub area $A_j$. The use of $A_j$ instead of $A$ is fundamental also because $f(x, y)$ could be defined in all $\mathbb{R}^2$ and its area could be infinite.

Eq. 2 is changed to fit this idea. The total number of sensors $n$ is replaced by the *expected number* of nodes in sub area $A_j$ ($|S_j|$), that is $n \Pr[s_i \in S_j]$, that represents the total number of sensors times the probability of $s_i$ to be in sub set $S_j$ and so the probability to be deployed in sub area $A_j$, and the total area $A$ is replaced by the sub area $A_j$. This produces a new density Equation:

$$\frac{n \Pr[s_i \in S_j]}{A_j} \simeq \frac{1 + |N(s_i)|}{a} \tag{4}$$

The main difference from Eq. 2 is that the two terms of the equation are not strictly equal but approximately equal because we assumed that the distribution of the sensors in a sub area $A_j$ is uniform, that is good for $A_j$ small but becomes bad for large sub areas.

As for Eq. 2, we can derive from Eq. 4 an approximated estimation of the nodes to be deployed. However there is a difference in the use of the new Equation: the value $\overline{K}$ must be changed in $\overline{K_j}$ that represents the desired number of neighbors for the nodes that will fall in area $A_j$. For some $A_m$ different from $A_j$, but with the same size, the number $\overline{K_m}$ can be different from $\overline{K_j}$ because the distribution is not uniform.

As already done for Eq. 3 we replace the value $|N(s_i)|$ with the desired number of neighbors $\overline{K_j}$ and we move the terms to obtain $n$:

$$n \simeq \frac{(1 + \overline{K_j})A_j}{a \Pr[s_i \in S_j]} \tag{5}$$

Eq. 5 is fully compatible with Equation 3 and represents a special case of it: because in the uniform case if we take into account a sub area $A_j$ of $A$, $A_j$ must have the same density of $A$. In Fig. 3 and Fig. 2, we show a graphical representation of the Eq. 5 for the Gaussian distribution.

The Figures show the surfaces resulting from Eq. 5 in the case of an unitary square with the mean of the Gaussian in $(0.5,\ 0.5)$ and standard deviation on both axis equal to $\sigma_{xy} = 0.35$. The surface is computed varying the value of $r$ in range $[0.1,\ 1.4]$ and the value $\overline{K_j}$ (in the Figures simply $K$) in range $[2,\ 9]$. Sub areas $A_j$ used for the realization of the surfaces are two squares, with side of length $l = 0.2$. The square $A_{(0.5,0.5)}$ centered in the mean with sides parallel to the axis and the other square, $A_{(0.9,0.9)}$, is centered in point $(0.9\ , 0.9)$ with sides parallel to the axis, in order to be placed in the corner of the unitary square.

In Fig. 3, $A_j$ is centered in the mean of the distribution with $\Pr[s_i \in S_j] = 0.047$. In this case, the number of sensors to be dropped is small, also in the case of high values of $\overline{K_j}$ because the distribution has high values near the mean and a lot of sensors will be dropped there making easy to reach the value of $\overline{K_j}$ in that sub area even for low values of $n$. The shape of the Gaussian, high near the mean and low at the borders, is also the reason for the surface shown in Fig. 2 that represents the one produced by a sub area $A_j$ in the corner of the unitary square with $\Pr[s_i \in S_j] = 0.014$. In this case, with very high values of $\overline{K_j}$ the value of $n$ grows very

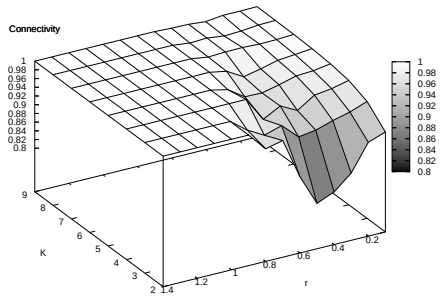



Fig. 4. Connectivity in uniform case

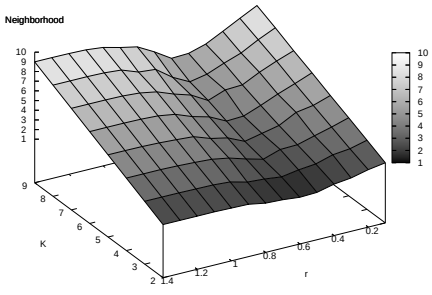



Fig. 5. Mean neighborhood in uniform case

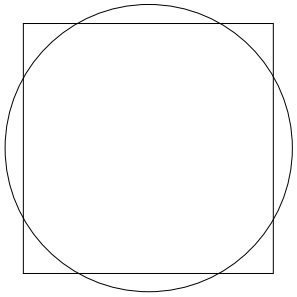


Fig. 6. Shape factor

quickly.

All the surfaces representing values distributed following a Gaussian distribution have a shape similar to the one presented in Fig. 1, because of the approximation of the distributions in sub areas $A_j$ with the uniform distribution.

We said before that the Eq. 5 is an approximation and now we study the error of this Equation at the variation of the parameter $A_j$ in relation with $a$.

We define the error $e$ as the absolute value of the difference between the two terms of Eq. 4:

$$
\begin{aligned}
e &= \left| \frac{n \Pr[s_i \in S_j]}{A_j} - \frac{1+\overline{K_j}}{a} \right| && (6)\\
&\leq \left| \frac{n \Pr[s_i \in S_j]}{A_j} \right| + \left| \frac{1+\overline{K_j}}{a} \right| && (7)\\
&= \frac{n \Pr[s_i \in S_j]}{A_j} + \frac{1+\overline{K_j}}{a} \\
&= \frac{n \int_{A_j} f(x,y) \mathrm{d}A_j}{A_j} + \frac{1+\overline{K_j}}{a} && (8)
\end{aligned}
$$

Starting from Equation 6 we can upper bound it with Equation 7 and remove from this equation the absolute values because the two terms are positive. Then in Equation 8 the probability term is substituted with its value that is the volume of the probability function $f(x,y)$ in sub area $A_j$. Now we can find another upper bound of Equation 8 using the following idea. The volume of $f(x,y)$ in sub area $A_j$ can be upper bounded by the volume of the solid that has as base the sub area $A_j$ and as height the maximum of the function $f(x,y)$ in sub area $A_j$ ($max_{A_j}\{f(x,y)\}$). At this point Equation 8 becomes:

$$
\begin{aligned}
e &\leq \frac{n \int_{A_j} f(x,y) \mathrm{d}A_j}{A_j} + \frac{1+\overline{K_j}}{a} \\
&\leq \frac{n \cdot A_j \cdot max_{A_j}\{f(x,y)\}}{A_j} + \frac{1+\overline{K_j}}{a} \\
&= n \cdot max_{A_j}\{f(x,y)\} + \frac{1+\overline{K_j}}{a} && (9)
\end{aligned}
$$

In Equation 8 after upper bounding the volume of the function $f(x,y)$ we can eliminate the term $A_j$ and finally obtain Equation 9. Equation 9 can be absolutely upper bounded using the maximum value of $f(x,y)$ in all $A$:

$$
\begin{aligned}
e &\leq n \cdot max_{A_j}\{f(x,y)\} + \frac{1+\overline{K_j}}{a} \\
&\leq n \cdot max\{f(x,y)\} + \frac{1+\overline{K_j}}{a} && (10)
\end{aligned}
$$

Eq. 10 gives an absolute upper bound to the error $e$ and gives a precious information about the error in relation with the shape of the distribution function. The error increases when the function is steep: for instance let us consider the Gaussian: if the standard deviation is low, the max of the distribution is high and the density of the function increases rapidly in a short space. If the standard deviation is high, the max is lower and the difference of density in the same span of space is lower. This implies that the approximation of that span with an uniform distribution introduces less error.

## VI. EXPERIMENTAL RESULTS

Experimental results are acquired using simulations that are organized as follows: we compute the number of sensors $n$ for the distributions and then we place the

$n$ sensors in the unitary square checking the expected $\overline{K}$ with the size neighborhood obtained by simulations. For the Gaussian distribution, we check the $\overline{K_j}$ with the size neighborhood in sub-area $A_j$ and in the whole area $A$.

### A. Uniform Distribution

Now we present the experimental results for the uniform distribution.

Fig. 4 shows the number of connected nodes over the total number of deployed nodes. As shown the value is in the order of one for the largest part of the tested combinations of deployed networks. We have, as we can expect, the worst results for low values of $\overline{K}$ because the desired structure of the network tend to be quite difficult to obtain in randomized network: for instance a $\overline{K} = 2$ means that the desired structure of the whole network is a ring, quite difficult structure to obtain randomly.

Fig. 5 shows the mean size of the neighborhood for each sensor deployed in $A$. The value, in the largest part of the experimental cases, is linear with the growth of $\overline{K}$ but around the value of $0.6$ we have a fall in the number of neighbors.

We call this effect the *shape factor*.

The shape factor is produced by the difference of shape between $A$, squared, and $a$, circular.

We explain the shape factor with an example: consider the sensor in the center of the unitary square with a communication range of $0.6$. The area reachable by this sensor has size larger than one but leaves the 4 regions at the corners uncovered (as shown in Fig. 6) . The shape effect is more evident with the growth of $\overline{K}$ because theoretically $a$ covers all $A$, but this is not true in practice because we have no topological coincidence between $A$ and $a$.

### B. Gaussian Distribution

For the non uniform distribution experiments we use a Gaussian distribution centered in $(0.5,\ 0.5)$, with both axis $\sigma_{xy} = 0.35$.

Experiments are performed varying $\overline{K_j}$ (in the Figures simply $K$) in range $[2,\ 9]$. The two sub-areas $A_j$ used for the experiments are both square, with side of length $l = 0.2$. In one case, that we called $A_{(0.5,\ 0.5)}$, $A_j$ is centered in the mean with sides parallel to the axis and the other, called $A_{(0.9,\ 0.9)}$, centered in point $(0.9,\ 0.9)$ with sides parallel to the axis, so to be placed in the corner of the unitary square.

According to considerations made at the end of the error analysis, we use $r = 0.1$ so that a sensor placed in the center of the sub-area $A_j$ has all its communication area inside $A_j$.

Fig. 7 shows the connectivity percentage of the whole network with our equation applied in $A_{(0.5,\ 0.5)}$. We can notice that the global connectivity grows with the growth of $\overline{K}$. We do not acquire the full connectivity of the network because near the max of the Gaussian we have a number of sensors that fits the requirements for $\overline{K}$ but far from the max the density of the sensors decrease and this bring to an unconnected network.

To understand this, we must see Fig. 8 that shows the connectivity percentage of the sub-area under the mean with the same Gaussian distribution and our equation applied to the square near the max of the distribution: in this Figure the connectivity percentage grows very quickly and goes to $1$ for values of $\overline{K} > 5$. If we discard from our experiment the value of $\overline{K} = 2$ the the connectivity is never below $0.94$.

Fig. 9, shows the connectivity percentage of the whole network with our equation applied to $A_{(0.9\ 0.9)}$, in the corner of the unitary square. We can notice that the global connectivity grows with a smoother curve if compared to the one in Fig. 7, and in a good interval that never falls under $0.91$, also for $\overline{K} = 2$. This happens because the number of neighbors fits the requirements given by a sub-area where the probability to be dropped is low: near the max of the Gaussian this large number guarantees high connectivity (higher than $\overline{K}$) and far from the max there is the right density of nodes to archive connectivity.

This is more evident if we look at Fig. 10 because the curve is similar to the one proposed in Figure 8: connectivity percentage keeps high values and grows to $1$ for values of $\overline{K} > 6$, with a percentage interval that starts at $0.93$.

Fig. 11 shows the mean neighborhood size of the whole network with our equation applied to the square under the mean of the distribution. We can notice that the mean size of neighborhoods grows with the growth of $\overline{K}$ but at a significantly lower rate. The mean neighborhoods size does not grow linearly because near the max of the Gaussian we have a number of sensors that fits the requirements for $\overline{K}$ but far from the max the density of the sensors decrease and this lowers the mean of neighborhood sizes. In fact, as shown in Fig. 12, we acquire that the mean neighborhood size in the sub-area near the max of the Gaussian the curve follows in a quasi-linear fashion the values of $\overline{K}$.

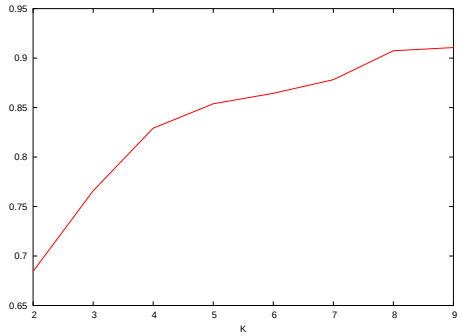



Fig. 7. Connectivity in Gaussian case with $\sigma_{xy} = 0.35$ with $A_j = A_{(0.5,0.5)}$ in whole network

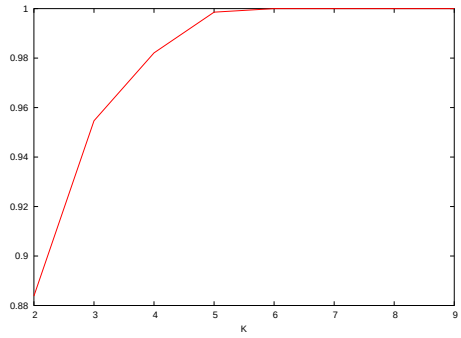



Fig. 8. Connectivity in Gaussian case with $\sigma_{xy} = 0.35$ with $A_j = A_{(0.5,0.5)}$ in $A_j$

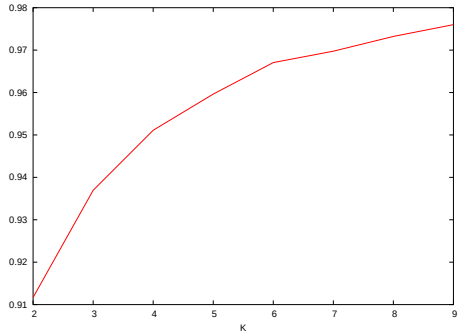



Fig. 9. Connectivity in Gaussian case with $\sigma_{xy} = 0.35$ with $A_j = A_{(0.9,0.9)}$ in whole network

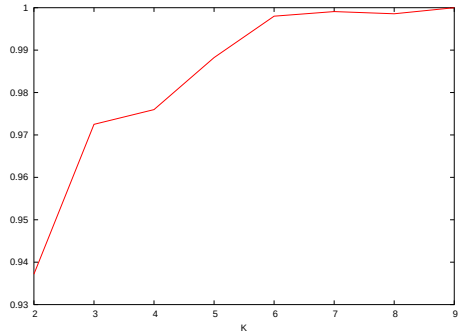



Fig. 10. Connectivity in Gaussian case with $\sigma_{xy} = 0.35$ with $A_j = A_{(0.9,0.9)}$ in $A_j$

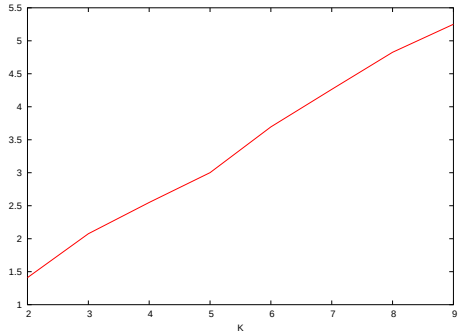



Fig. 11. Mean neighborhood in Gaussian case with $\sigma_{xy} = 0.35$ with $A_j = A_{(0.5,0.5)}$ in whole network

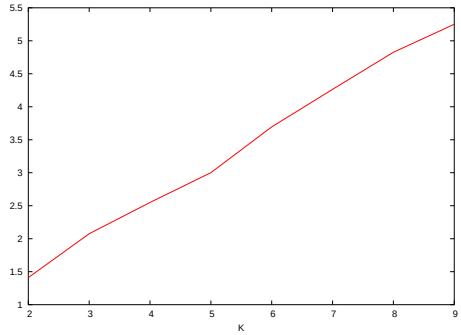



Fig. 12. Mean neighborhood in Gaussian case with $\sigma_{xy} = 0.35$ with $A_j = A_{(0.5,0.5)}$ in $A_j$

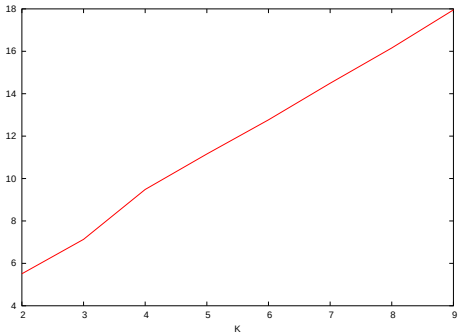



Fig. 13. Mean neighborhood in Gaussian case with $\sigma_{xy} = 0.35$ with $A_j = A_{(0.9,0.9)}$ in whole network

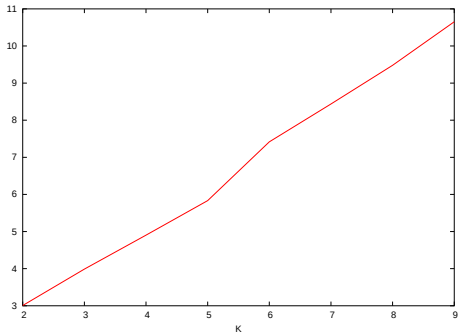
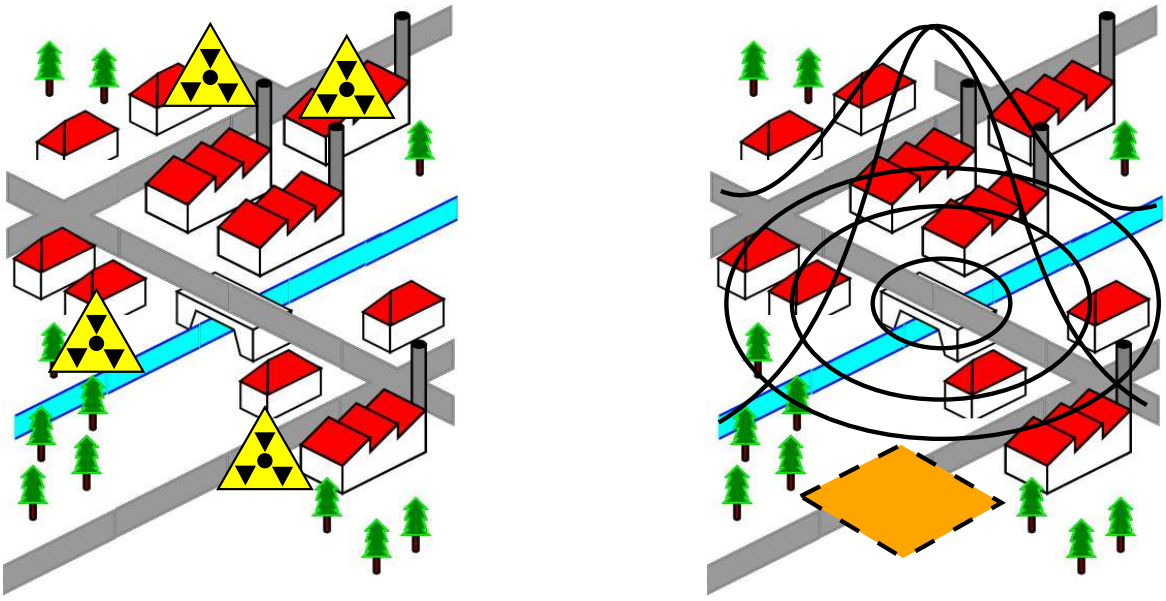



Fig. 14. Mean neighborhood in Gaussian case with $\sigma_{xy} = 0.35$ with $A_j = A_{(0.9,0.9)}$ in $A_j$



(a) Disaster Area

(b) Sensors Deployment

Fig. 15. Application scenario

Fig. 13 shows that when the number of sensors is computed respect to a sub-area not near the max, the mean size of the neighborhood of the whole network grows with $\overline{K}$ but is never greater than $\overline{K}$. This is because the total number of sensors in the network fits well the sub-area, but near the max we achieve a denser network.
Finally, Fig. 14 shows the mean size of neighborhoods in $A_{(0.9,\ 0.9)}$. This Figure shows results very similar to the ones in Fig. 12 with a quasi-linearity between the values of the curve and the values of $\overline{K}$.

## VII. APPLICATION SCENARIO

Suppose that in a urban area as the one shown in Fig. 15.a there is some kind of poisonous, or radioactive, substance released in the environment and the area is too large, or too dangerous, to be monitored with special squads of humans, even if well equipped with special clothes to operate safely. In cases like this, we are interested in dropping some sensors in the area to monitor the evolution of the situation. Such sensor drop can be performed, as instance, with airplanes.
As shown in Fig. 15.b sensors dropped from above, fall on the ground following a Gaussian distribution: the sensors density at the center of the deployment area is higher than on the borders of the same area.
If we are interested in monitoring all the area, we must drop enough sensors to ensure a connected network on all the region interested by the disaster. To perform such action we must know how many sensors must be dropped in the whole area.
To estimate such number, we establish that in one sub-area at the border of the deployment area, in Fig. 15.b the one in the square with dashed borders, we need a connectivity of $\overline{K}+1$ elements, or in other terms we need that each sensor $s_i$ in the sub area has $\overline{K}$ neighbors.
A simple numerical instance of the scenario could be the following.
We want to monitor the squared territory with side of $1Km$ with sensors that have a maximum transmitting range of $100m$. We will deploy the sensors using a Gaussian distribution with $\sigma_{xy} = 0.35$ and centered in the median point of the surface.
We also want to have a mean number of neighbors ($K_j$) at the borders of the area (the sub-area with dashed borders) at least $K_j = 6$ that implies, using our formula (or the chart in Fig. 2), to drop around $637$ sensors. We start from this value, performing simulations, to determine the right value of sensors to drop.
According to the chart in Fig. 9 the value $K_j = 6$ produce a connectivity of the whole network greater than $96\%$. On the other size the mean neighborhood size of the whole network in very high (around 12)

and we can expect higher values in the center of the distribution. A value so high will produce a network suffering of the channel contention problem.
To correct this problem we try with a lower value of $K_j$, and we try with $K_j = 3$. To obtain this mean neighborhood size according to our formula we need to drop around $364$ sensors.
Looking at the chart in Fig. 9 with this number of sensors we will guarantee a global connectivity greater than $93\%$ and a mean neighborhood size in the whole network around $6$.
The output values of our formula are indicative and represent starting points for the network deployment study: the network designer will decide the final number of sensors through many simulations and he will obtain the correct value for his application.

## VIII. CONCLUSIONS

In this paper, we analyzed the dimensioning of a sensor network as the problem of finding the minimum number of sensors to be dropped in an area to guarantee a desired mean connectivity.
We presented a novel formulation of the problem and discussed an approximate solution which uses a density equation. We also derived a theoretical bound on the error incurred and discussed simulation results.
We acquired this goal using a physical characteristic of the network, the density of the nodes.
The results that we archive are very good, and the evidence of that is given both analytically and experimentally. We achieve a network connectivity in the $97\%$ of the cases and we obtain with simulations a value of neighbors equal to $\overline{K}$ in the all the cases with few exceptions, due to some pathological problems of the networks as the impossibility of the creation of a connected network with $\overline{K} = 2$ and the shape factor in the approximation of sub-areas.
Also these pathological problems, and their consequences are studied and analyzed in the paper.

## IX. FUTURE WORKS

The study presented in this paper introduce a possible real solution for the network dimensioning.
One possible future development is to consider the case of networks deployed in a 3D environment, as undersea environments. This future development is interesting also for the evaluation of the physicals features, like slopes and the joined problem of sensor sliding. The problem concern the fact that some sensors can fall from the initial deployed position, so the final distribution could be different from the one estimated at the beginning.
Another future work is the study of our model and solution in the case of sensor networks that adopt sleeping patterns (see [15]).

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